Welcome

science foundation ireland
fondúireacht eolaíochta éireann

North-South Quantum Information Meeting 2008

23 talks
26 participants from UCC and QUB

→ pretty much doubled the number of participants!
(Vala, Slingerland, Pelucchi, McGettrick)


Belfast
Béal Feirste
벨파스트
Dublin
Baile Atha Cliath
더블린
Galway
Gaillimh
골웨이
Cork
Corcaigh
코르크

North South Supplement

- Supplement to Principle Investigator Grant currently held within Ultracold Quantum Gases Group in Cork
- runs until 31st May 2010
- exchange visits between QUB and UCC
- collaborative meetings
- establishing close connection and active collaboration
- very informal and flexible

Communication → QI^2

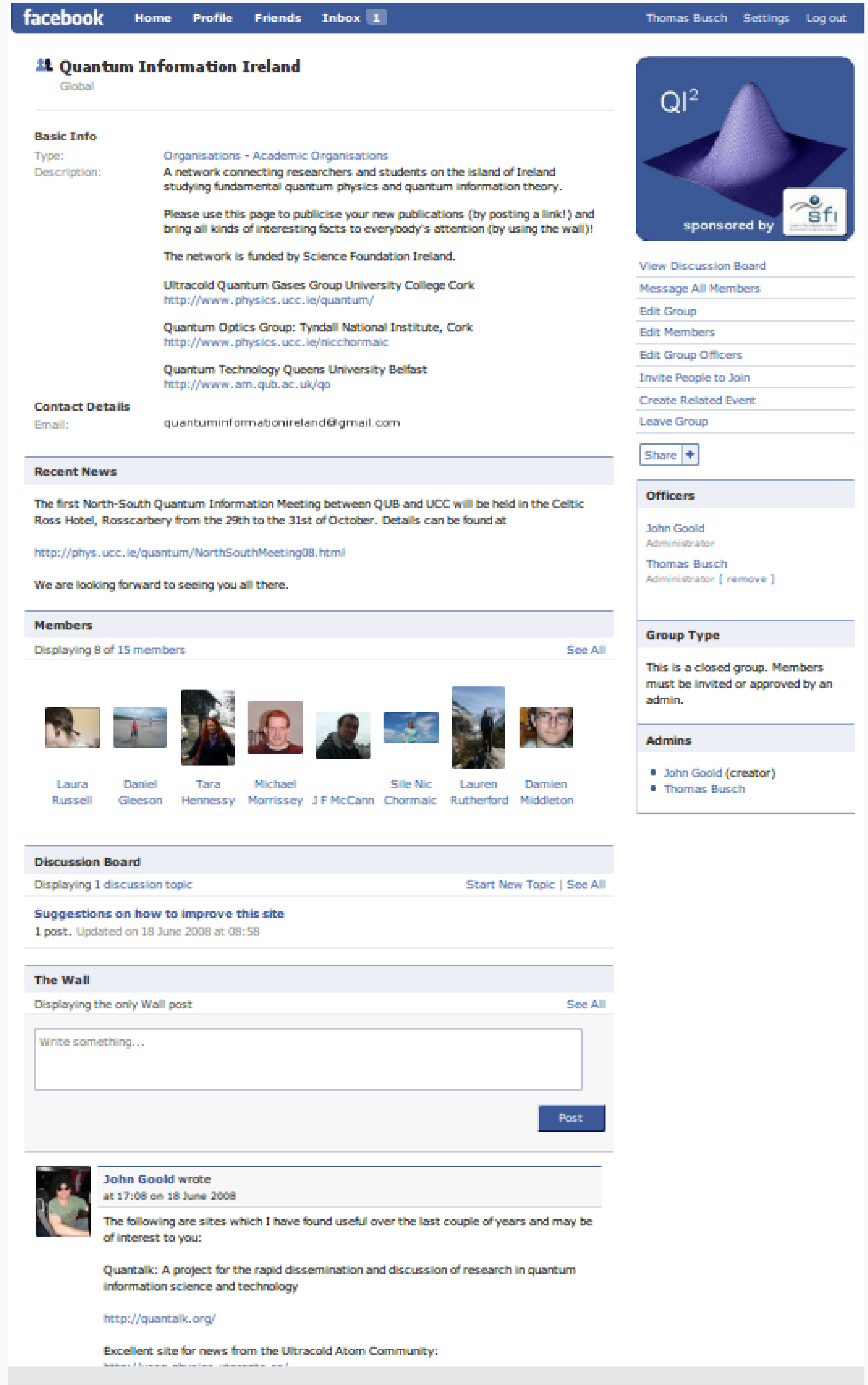
Idea: use facebook for networking

- free membership
- 36 members already
- (potential) worldwide visibility
- easy to maintain

Tools

- **News**
  - upcoming visits
  - conferences
- **Discussion board**
- **Wall**
  - job openings etc.
- **Posted Items**
  - links to papers, publications
- **Photos & Videos**
  - from visits, events

Any ideas welcome!



Programme

Monday

Lunch

Talks

After Dinner Discussions

Tuesday

Breakfast

Talks

Lunch

Talks

Soccer

Dinner

After Dinner Discussions

Wednesday

Breakfast

Talks

Implementing Ideas of Quantum Information in Ultracold Quantum Gases

Thomas Busch
Ultracold Quantum Gases Group
University College Cork

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NDP
NATIONAL DEVELOPMENT PLAN

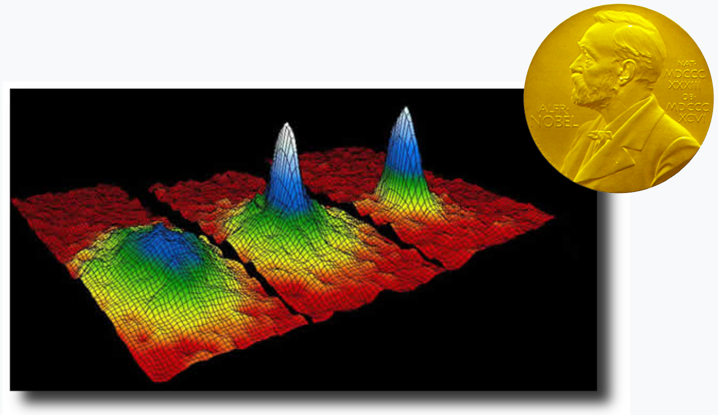
embarkinitiative
Investing in People and Ideas

Centre for QuantumIT
University College Cork

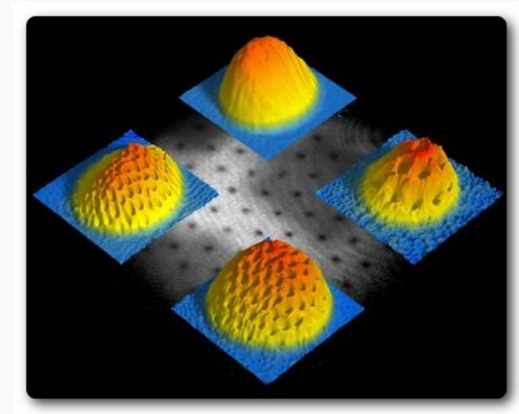
UCC
University College Cork

Cold Atoms

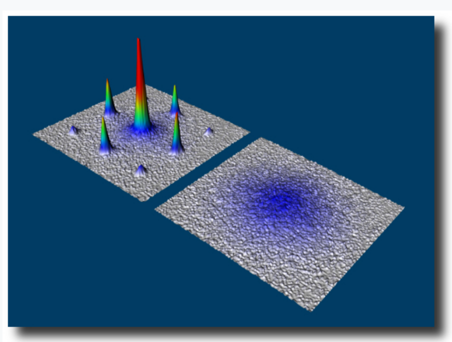
New states of matter:



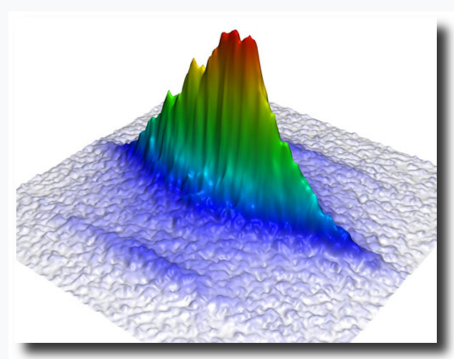
1995 - Bose-Einstein Condensation



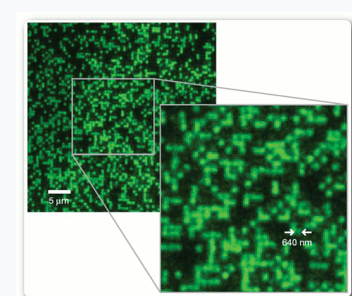
2001 – Quantised Vortices



2002 - Mott Transitions



2004 – Tonks Gas



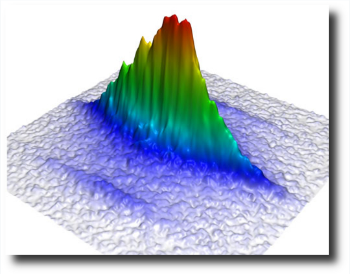
2009 – Single Atom Addressibility

Overview

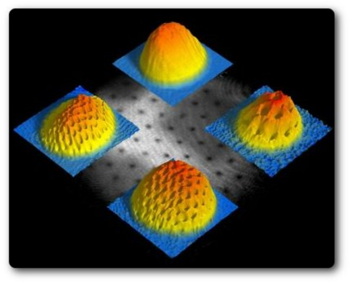
Usually: entanglement of internal states of atoms (P Zoller Industries Ltd)

Here: other degrees of freedom (position, momentum, phases, modes, etc...)

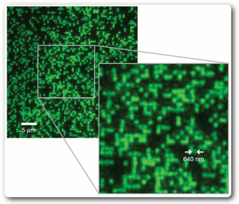
1. One-Dimensional Systems



2. Angular Momentum States



3. Centre of Mass States



One Dimensional Gases

Waveguides are different to **Free Space**

(a) fewer modes available for the particles

→ free space estimates for collisional effects no longer valid

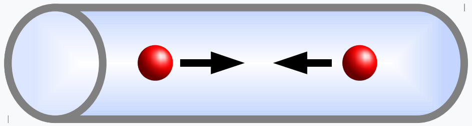
(b) assume atoms are trapped by an axially symmetric 2D harmonic potential of frequency $\omega_\perp$

→ atomic motion is cooled down below the transverse vibrational energy $\hbar\omega_\perp$ →

$$\Psi_I = e^{ik_z z}\phi_0(x,y)$$

$$\Psi_F = f(z)\phi_0(x,y)$$

(c) atomic motion along the z-axis is free



interaction can be modelled by pseudo-potential

→ $$U(r) = g\delta(\vec{r})\frac{\partial}{\partial r}(r\cdot)$$

(regularisation operator removes 1/r divergences from scattered wave)

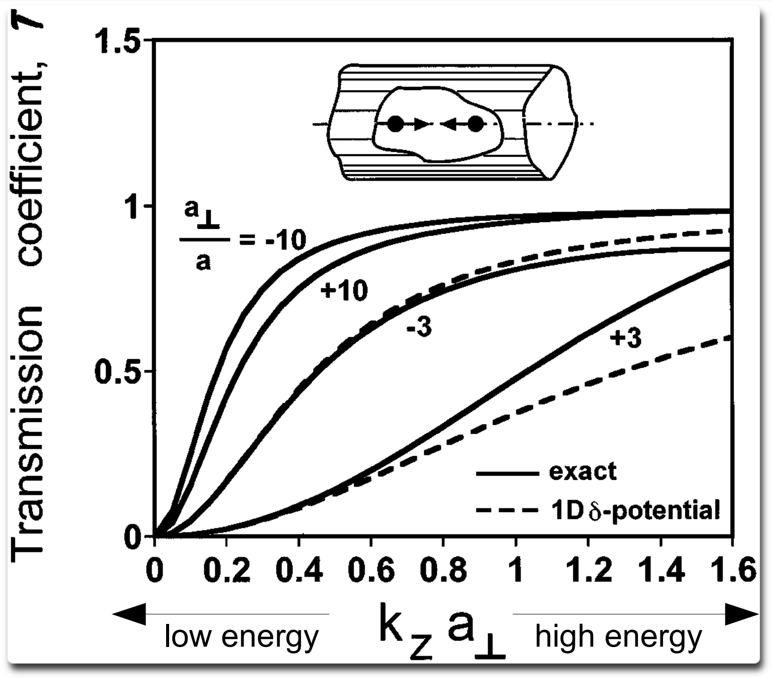
$$g = \frac{2\pi\hbar^2 a}{\mu}$$

s-wave scattering length

reduced mass

Binary Collisions between Cold Atoms

But: during the scattering event virtual excitations to higher states have to be taken into account!



atoms cannot pass each other

atoms pass each other easily

Strongly Interacting Limit
(Girardeau Gas)
coherence low

Weakly Interacting Limit
(GPE Limit)
coherence high

M. Olshanii, Phys. Rev. Lett. ***81****, 938 (1998)*

Strongly Interacting Limit

1. N neutral, bosonic atoms with point-like interactions

$$H = \sum_{j=1}^{N} -\frac{\hbar^2}{2m}\frac{d^2}{dx_j} + V(x_1, \ldots, x_N, t) + a\sum_{i<j}^{N} \delta(|x_i - x_j|)$$

2. assume a $\rightarrow \infty$ and replace the interaction term by a constraint

$$\Psi = 0 \qquad \text{if} \qquad |x_i - x_j| = 0 \qquad i \neq j$$

3. equivalent to the Pauli exclusion principle!

➡ *Solve fermionic problem and symmetrise!*

Bose-Fermi Mapping Theorem

An exactly solvable, strongly interacting, experimentally realistic, many-particle problem!

Bose-Fermi Mapping

So, we need:

1. a system where the single particle eigenfunctions are known (and where they are *nice!*)

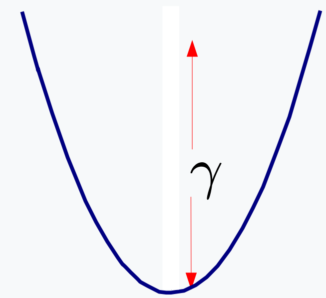
   → free space, box, harmonic oscillator,...

2. a system where the Slater determinant can be calculated (analytically)

   → probably best if eigenfunctions were polynomials

The δ-split Harmonic Oscillator

$$H_0 = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{1}{2}m\omega^2 x^2 + \gamma\delta(x)$$



- the *odd* eigenfunctions of the HO are still good eigenfunctions!
- the *even* ones have to be found

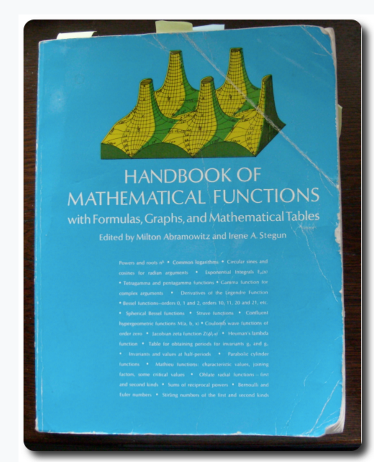
Scaling: length in ground state sizes $a_0 = \sqrt{\frac{\hbar}{2m\omega}}$

energies in ground state energies $\epsilon_0 = \hbar\omega$

for $\kappa = 0$

$$\left(-\frac{d^2}{dx^2} + \frac{1}{4}x^2 + \tilde{\gamma}\delta(x) + \epsilon_n\right)\phi_n(x) = 0$$

For $x > 0$ this is Whittakers equation!



The δ-split Harmonic Oscillator

x > 0

$$U(\epsilon_n, x) = \cos\left(\frac{\pi}{4} + \frac{\pi\epsilon_n}{2}\right) Y_1 - \sin\left(\frac{\pi}{4} + \frac{\pi\epsilon_n}{2}\right) Y_2$$

$$Y_1 = \frac{\Gamma\left(\frac{1}{4} - \frac{1}{2}\epsilon_n\right)}{\sqrt{\pi}2^{\frac{1}{4}+\frac{1}{2}\epsilon_n}} e^{\frac{1}{4}x^2} M\left(\frac{1}{4} + \frac{1}{2}\epsilon_n, \frac{1}{2}, \frac{1}{2}x^2\right)$$

$$Y_2 = \frac{\Gamma\left(\frac{3}{4} - \frac{1}{2}\epsilon_n\right)}{\sqrt{\pi}2^{-\frac{1}{4}-\frac{1}{2}\epsilon_n}} e^{-\frac{1}{4}x^2} xM\left(\frac{3}{4} + \frac{1}{2}\epsilon_n, \frac{3}{2}, \frac{1}{2}x^2\right)$$

for any value of γ!

x < 0 since we are looking for the even eigenfunctions

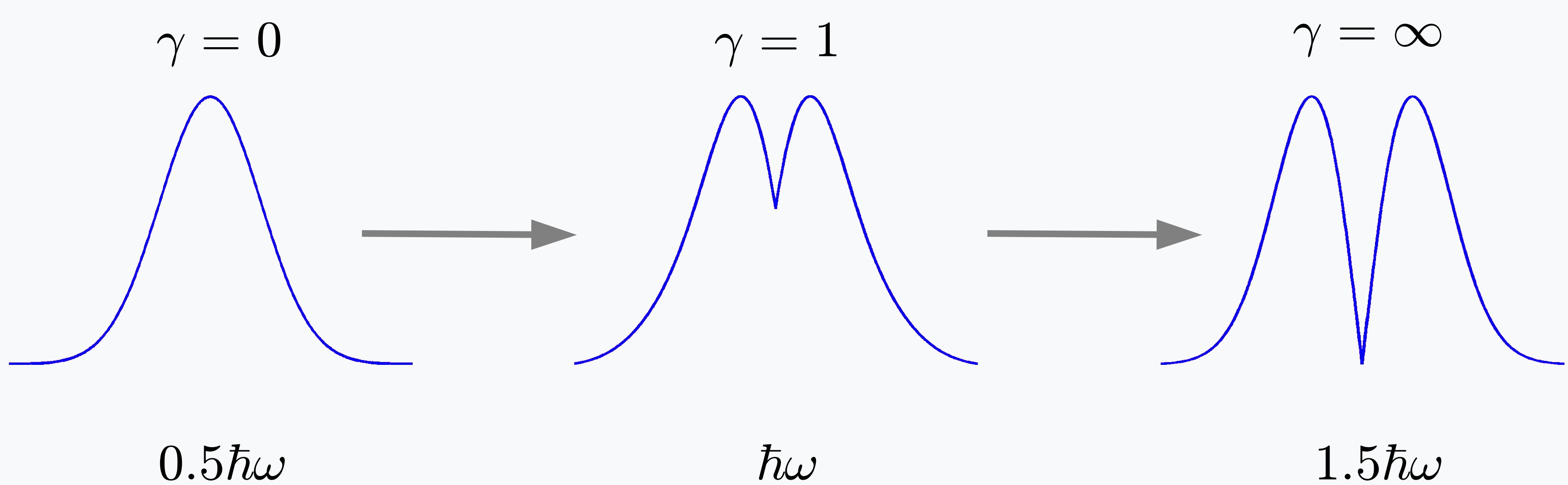
$$\phi_n(x) = CU(\epsilon_n, |x|)$$

x = 0 evaluate the continuity condition:

$$\frac{d}{dx}\phi_n(0^+) - \frac{d}{dx}\phi_n(0^-) = \tilde{\gamma}\phi_n(0)$$

Ground State Eigenfunction

With increasing central potential height the magnitude at the centre of the even eigenfunctions decreases:

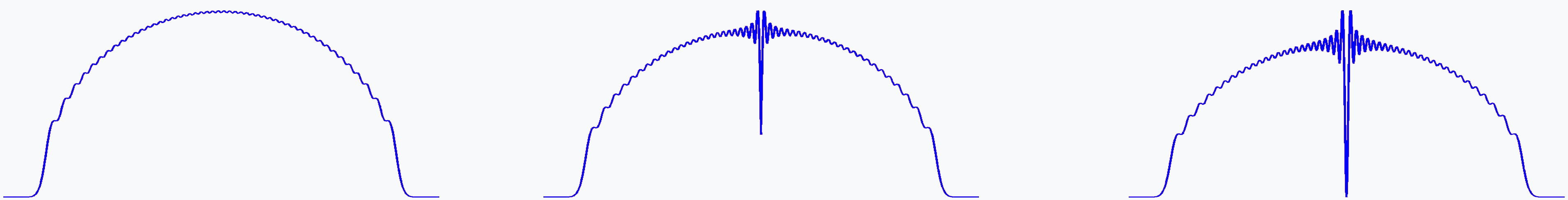


- same functional behaviour for all other even states
- for $\gamma = \infty$ even and odd states become degenerate

Many Particles in a δ-split trap

Next: calculate the Slater determinant...

$$\psi_F(x_1,\ldots,x_N) = \frac{1}{\sqrt{N!}} \det_{(n,j)=(0,1)}^{N-1,N} \phi_n(x_j) \qquad \text{\& symmetrise:}$$



$\gamma = 0$ $\qquad$ $\gamma = 3$ $\qquad$ $\gamma = \infty$

Because we know the ground state is real:

$$\Rightarrow \quad \psi_B(x_1,\ldots,x_N) = |\psi_F(x_1,\ldots,x_n)|$$

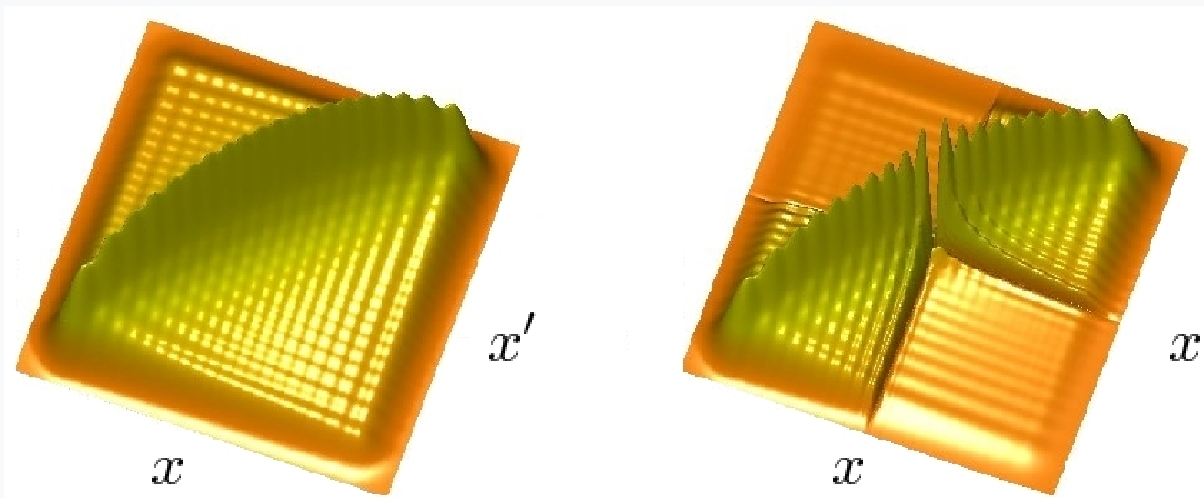
$$\Rightarrow \quad \rho_B(x_1,\ldots,x_n) = \rho_F(x_1,\ldots,x_n)$$

Bosons and fermions become (kind of) indistiguishable!

Reduced Single Particle Density Matrix

The self correlations are given by:

$$\rho(x, x') = \int \psi_B(x, x_2, \ldots, x_N) \times \psi_B(x', x_2, \ldots, x_n)\, dx_2 \ldots dx_N$$



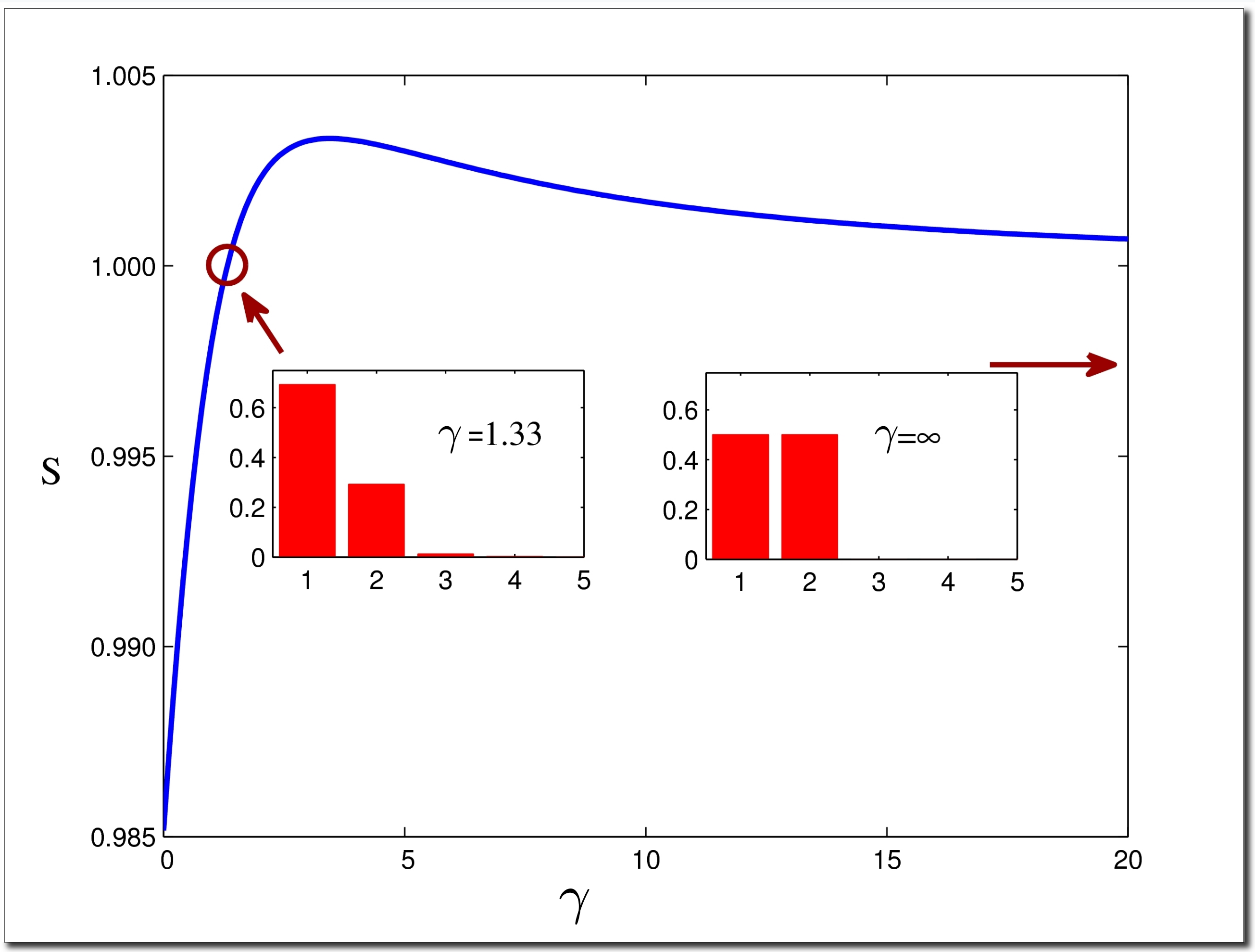
no barrier

high barrier

classical result: $\rho(x, x') = \delta(x - x')$

Two Particle Entanglement

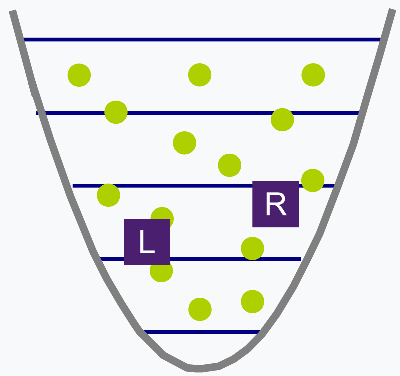
$S(\rho) = -\mathrm{Tr}(\rho \ln \rho)$ von Neumann entropy (only for a two particle system though...)



Indistinguishability?

How about many particle entanglement?

Idea:



- let two particles interact with the gas in two different regions of the trap
- in second quantisation the regions can be described as modes

$$|\phi_G\rangle \sim |L\rangle + |R\rangle \longrightarrow |\phi_{LR}\rangle \sim |10\rangle + |01\rangle$$

- calculate the entanglement of the state of the two sensors

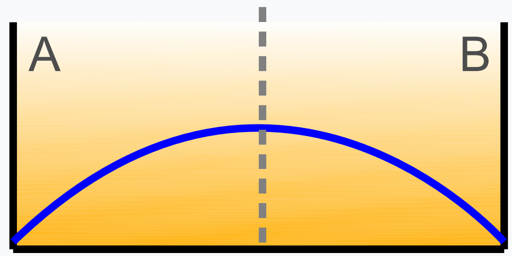
Why is that interesting?

For ideal Bose gas:

increase in entanglement $\longleftrightarrow$ BEC transition temperature

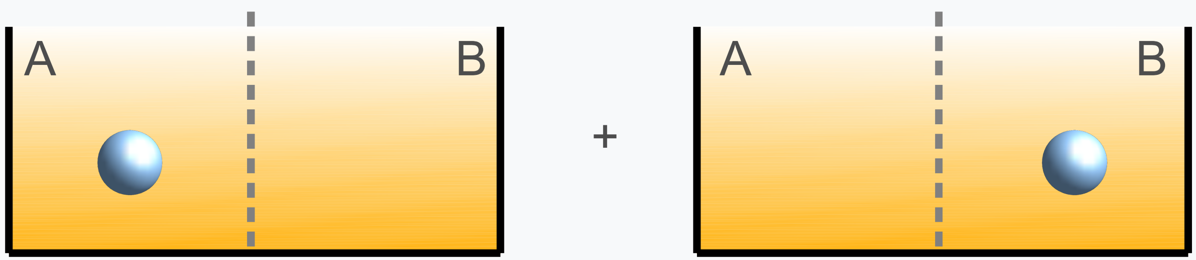
Spatial Mode Entanglement

1st Quantisation



Single particle is in a superposition between left and right

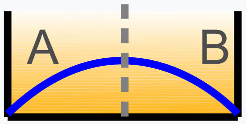
2nd Quantisation



non-local particle number entanglement between modes A and B

$$|\psi\rangle_{AB} = \frac{1}{\sqrt{2}}(|1\rangle_A|0\rangle_B + |0\rangle_A|1\rangle_B)$$

Spatial Mode Entanglement



Language: non-relativistic quantum field theory

→ construct mode operators

$$\hat{\psi}^{\dagger}_{A,B} = \int_{A,B} dx\; g(x)\hat{\psi}^{\dagger}(x)$$

bosonic quantum field operator

mode function

$$\int |g(x)|^2 = 1 \qquad \left[\hat{\psi}_i, \hat{\psi}^{\dagger}_j\right] = \delta_{ij}$$

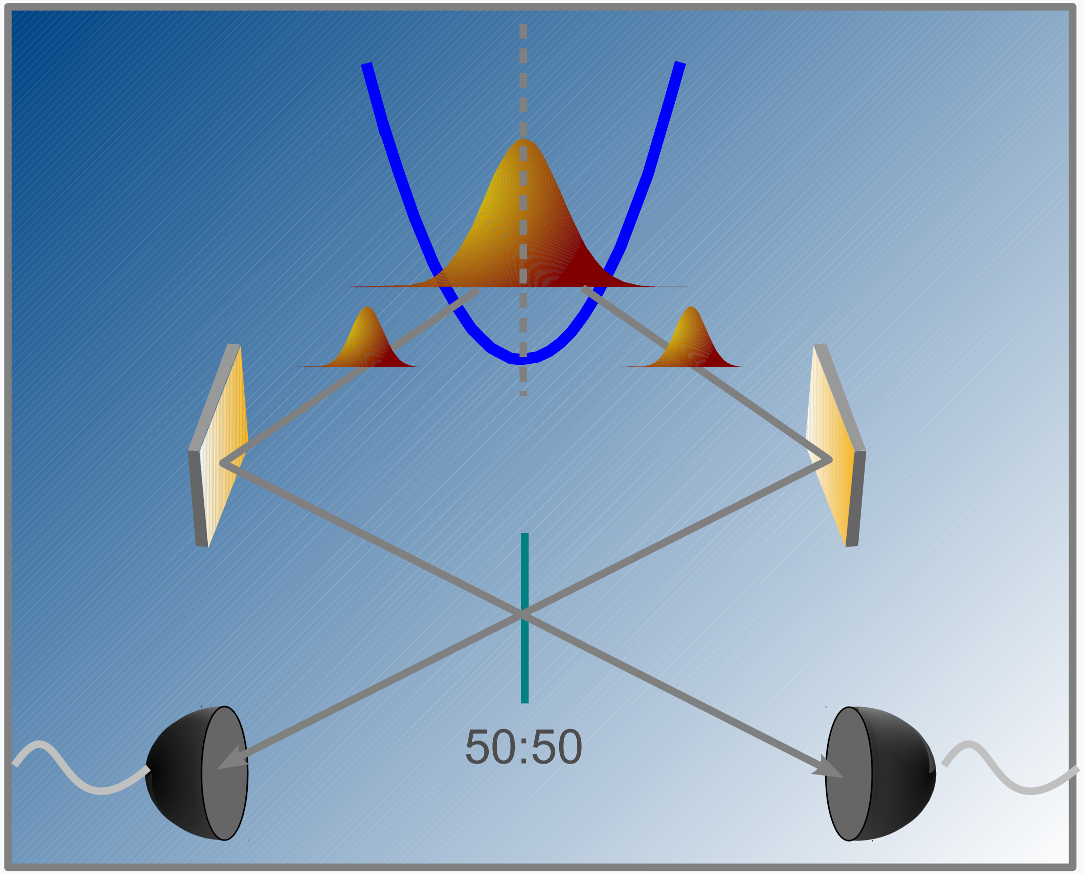
→ number of particles in the gas $N = \mathrm{tr}\left[\hat{\psi}^{\dagger}_A \hat{\psi}_A\ \rho\right] + \mathrm{tr}\left[\hat{\psi}^{\dagger}_B \hat{\psi}_B\ \rho\right]$

→ N particle BEC split in the middle is described therefore as

$$|\Psi\rangle = \frac{1}{\sqrt{N!}}\left(\frac{\hat{\psi}^{\dagger}_A}{\sqrt{2}} + \frac{\hat{\psi}^{\dagger}_B}{\sqrt{2}}\right)^N |0\rangle = \frac{1}{\sqrt{2^N}}\sum_{n=0}^{N}\frac{\sqrt{N!}}{\sqrt{n!(N-n)!}}|n, N-n\rangle$$

Interference Detection Scheme (Global Measurement)

joint measurement
of the two modes



$$\hat{\psi}_C^\dagger = \frac{1}{\sqrt{2}}\left(\hat{\psi}_A^\dagger + \hat{\psi}_B^\dagger\right)$$

$$\hat{\psi}_D^\dagger = \frac{1}{\sqrt{2}}\left(\hat{\psi}_A^\dagger - \hat{\psi}_B^\dagger\right)$$

$$N_C = \mathrm{tr}\left[\hat{\psi}_C^\dagger \hat{\psi}_C \ \rho\right]$$

$$N_D = \mathrm{tr}\left[\hat{\psi}_D^\dagger \hat{\psi}_D \ \rho\right]$$

assume a fixed total particle number

⇒ pure, separable state cannot show total destructive interference

Ch. Simon, Phys. Rev. A **66**, 052323 (2002)

Interference Detection Scheme

Calculate detector outcomes:

$$N_C = \mathrm{tr}\left[\hat{\psi}_C^\dagger \hat{\psi}_C \ \rho\right] = \frac{1}{2}\left(\mathrm{tr}[\hat{\psi}_A^\dagger \hat{\psi}_A \ \rho] + \mathrm{tr}[\hat{\psi}_B^\dagger \hat{\psi}_B \ \rho] + 2\mathrm{tr}[\hat{\psi}_A^\dagger \hat{\psi}_B \ \rho]\right) = \frac{N}{2} + \epsilon_{AB}$$

$$N_D = \mathrm{tr}\left[\hat{\psi}_D^\dagger \hat{\psi}_D \ \rho\right] = \frac{1}{2}\left(\mathrm{tr}[\hat{\psi}_A^\dagger \hat{\psi}_A \ \rho] + \mathrm{tr}[\hat{\psi}_B^\dagger \hat{\psi}_B \ \rho] - 2\mathrm{tr}[\hat{\psi}_A^\dagger \hat{\psi}_B \ \rho]\right) = \frac{N}{2} - \epsilon_{AB}$$

$$\epsilon_{AB} = \int_A dx \int_B dx' \ g(x)g(x') \ \rho^{(1)}(x, x')$$

reduced single particle density matrix

fully separable state: $\rho_{\mathrm{sep}} = \sum_i p_i |n_i\rangle\langle_i|_A \otimes |N - n_i\rangle\langle N - n_i|_B$

$$\epsilon_{AB} = 0$$

general state (of **fixed** N): $\epsilon_{AB} \neq 0$

→ measure of spatial coherence → good measure for entanglement for N=2

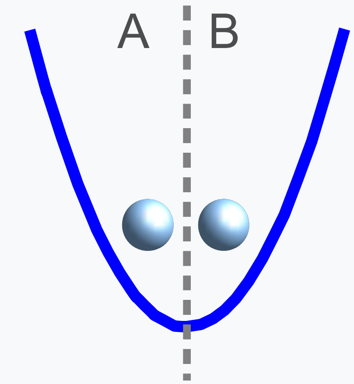
I. Bloch, T.W. Hänsch and T. Esslinger, Nature **403**, 166 (2000)

L. Heany, PhD Thesis, University of Leeds (2008)

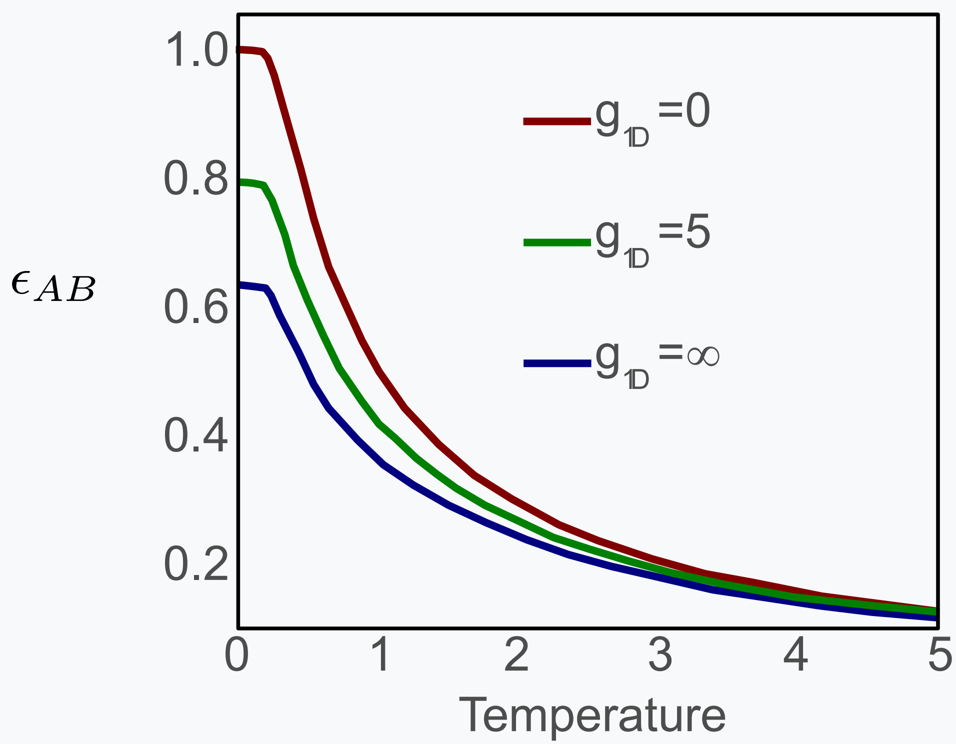
Cold Boson Pair

Boson pair Hamiltonian (1D)

$$H = \sum_{i=1}^{2} \left(-\frac{\hbar^2}{2m}\frac{d^2}{dx_i^2} + \frac{1}{2}m\omega^2 x_i^2 \right) + g_{1D}\delta(|x_i - x_j|)$$



same Hamiltonian, different interpretation!

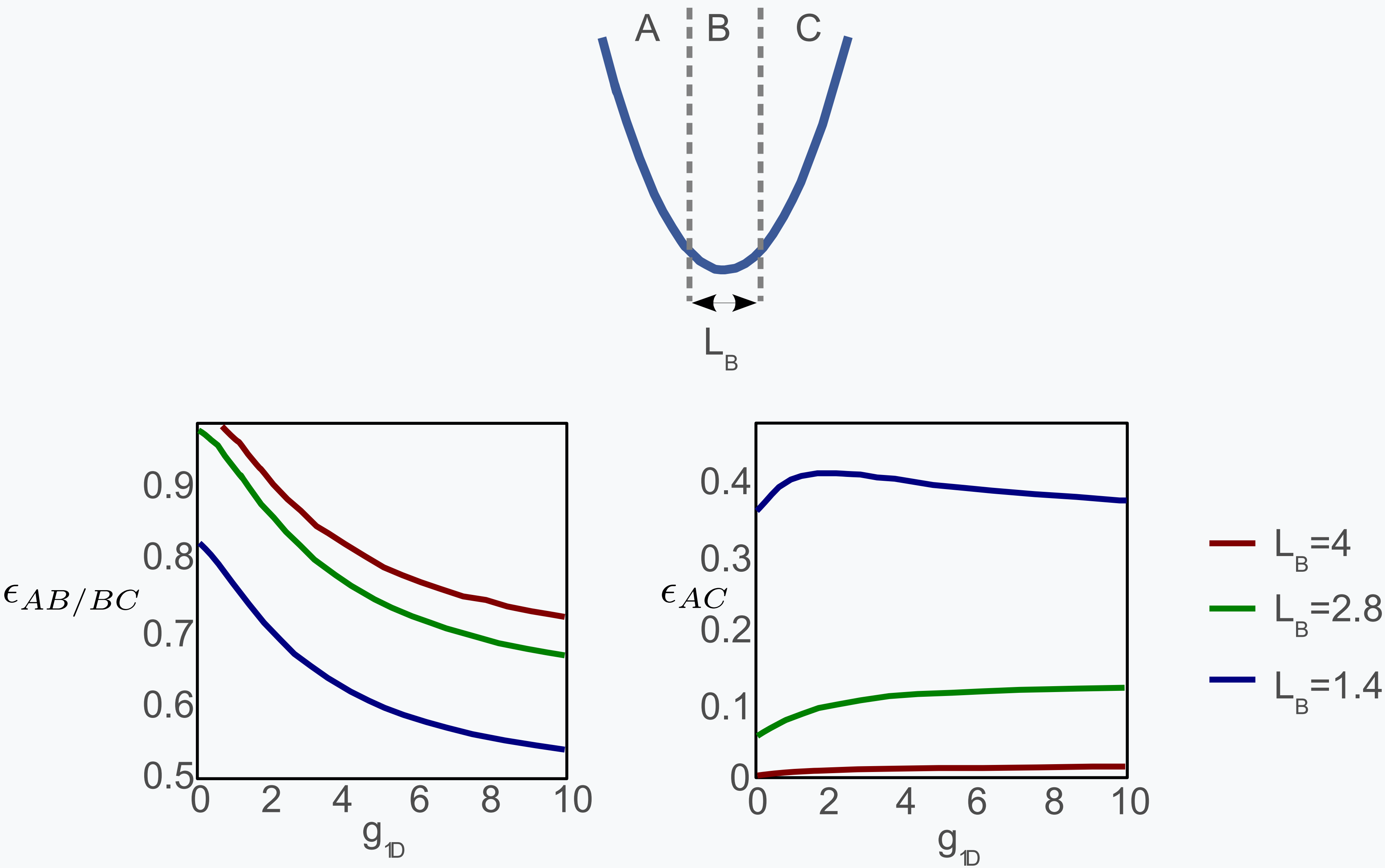


entanglement finite even at strong interactions

D.S. Murphy, J.F. McCann, J. Goold and TB, Phys. Rev. A **76**, 053616 (2007)

J. Goold, L. Heany, TB and V. Vedral, Phys. Rev. A **80**, 022338 (2009).

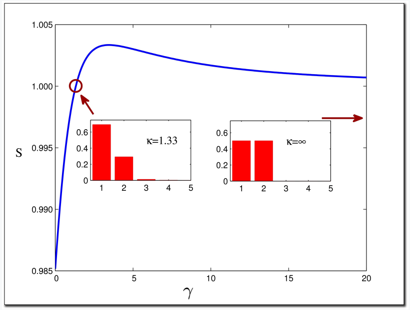
Cold Boson Pair



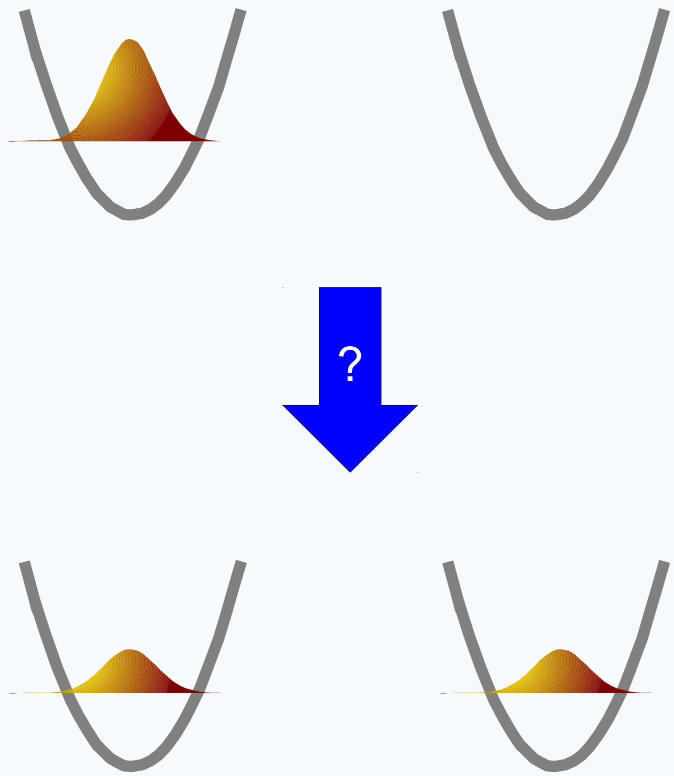
tuning the interaction parameter modifies the distribution of entanglement

Summary (1)

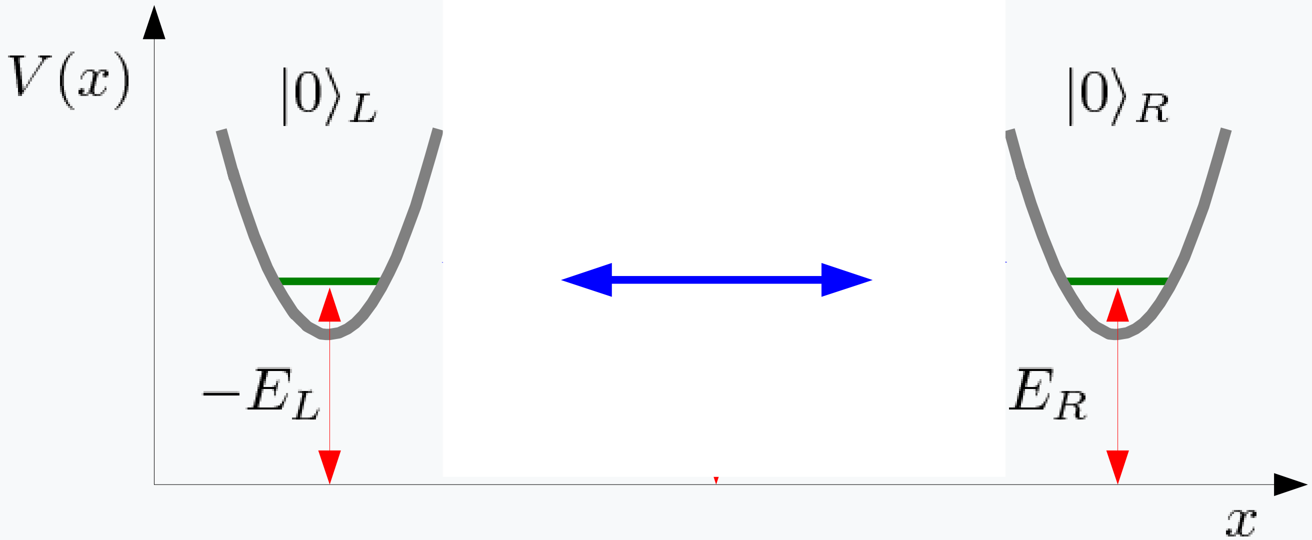
- ***One dimensional quantum gases***
  allow to realise and test concepts
  of quantum information and calculate
  exact numbers



Centre of Mass States



Spatial STIRAP



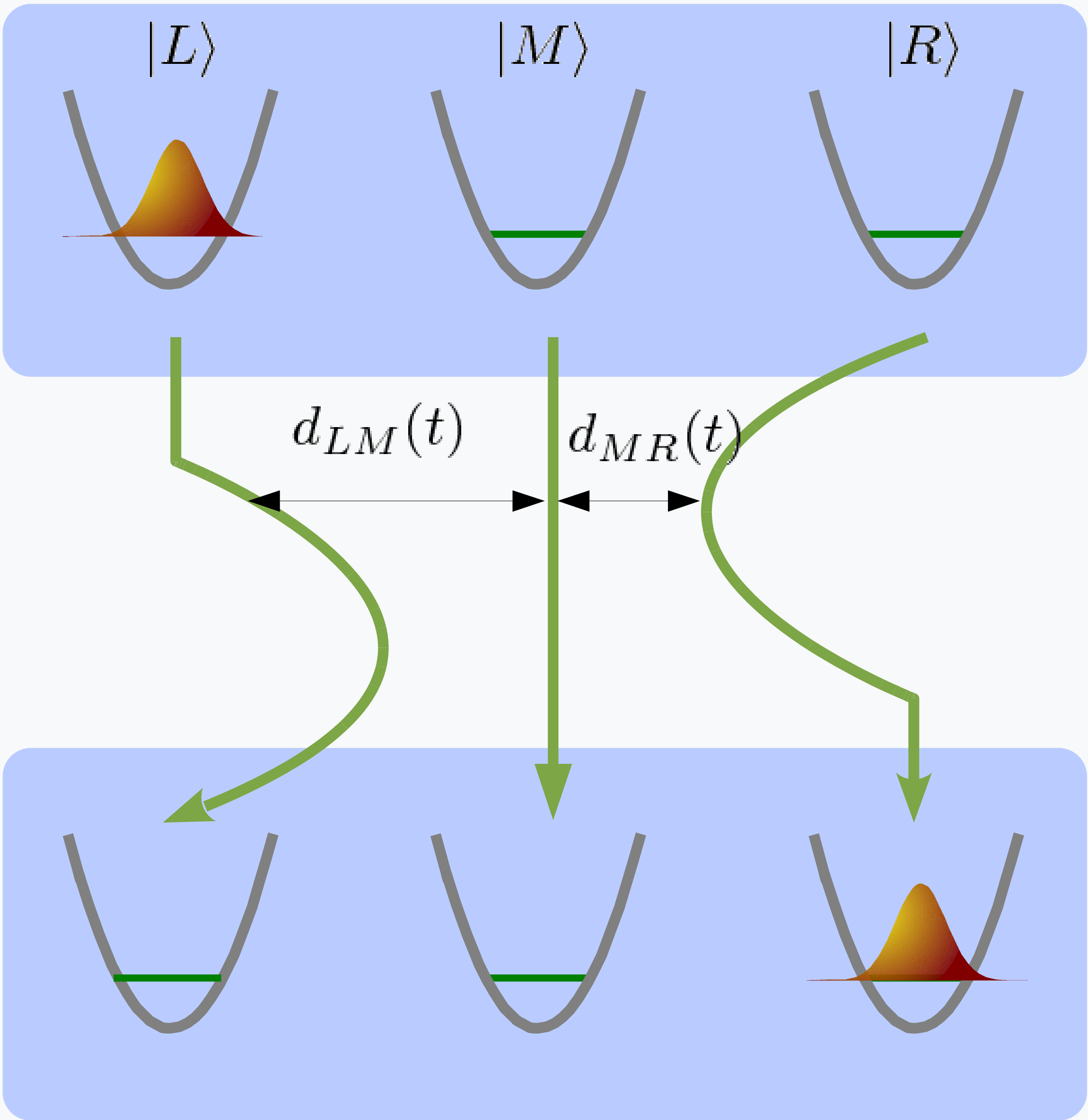
Direct tunneling leads to Rabi-type oscillations!

Atom Optical STIRAP

Eckert *et al.* PRA **70**, 023606 (2004)

Counterintuitive Tunneling

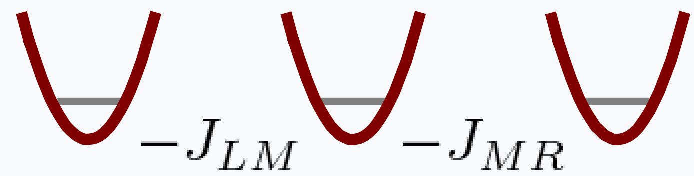
- 100% tansfer
- never any population in $|M\rangle$



see also:
Greentree *et al.*,
PRB **70**, 235317 (2004)

Three-Level Atom Optics

$$H = \begin{pmatrix} 0 & -J_{ML} & 0 \\ -J_{LM} & 0 & -J_{MR} \\ 0 & -J_{MR} & 0 \end{pmatrix}$$



- three level system possesses *dark state*

$$|\psi(\theta)\rangle = \cos\theta|L\rangle - \sin\theta|R\rangle \qquad \tan\theta = \frac{J_{LM}}{J_{MR}}$$

- STIRAP-like techniques can be developed

Counterintuitive scheme:

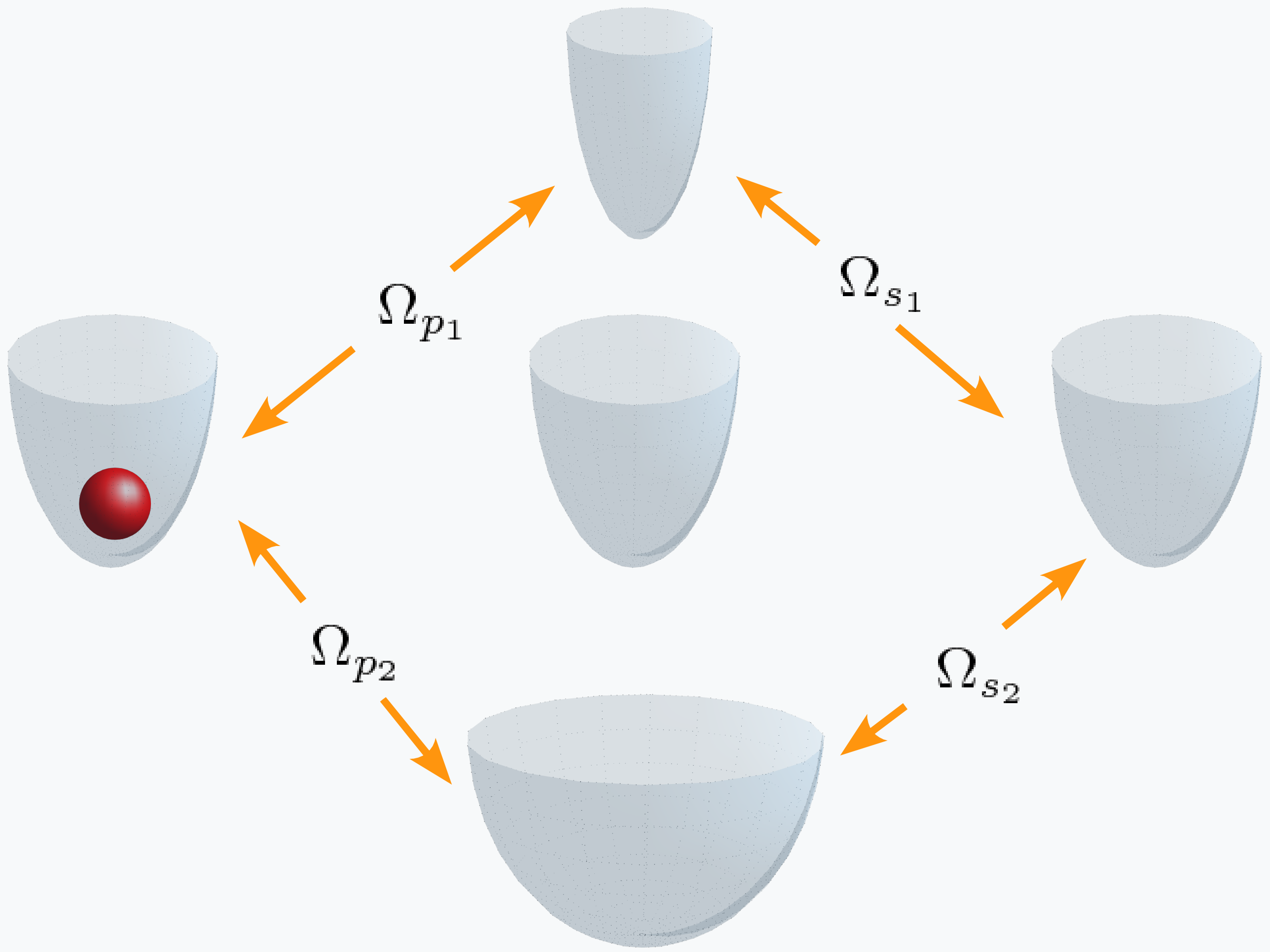
$$\tan\theta : 0 \longrightarrow \infty$$

$$\theta : 0 \longrightarrow \frac{\pi}{2}$$

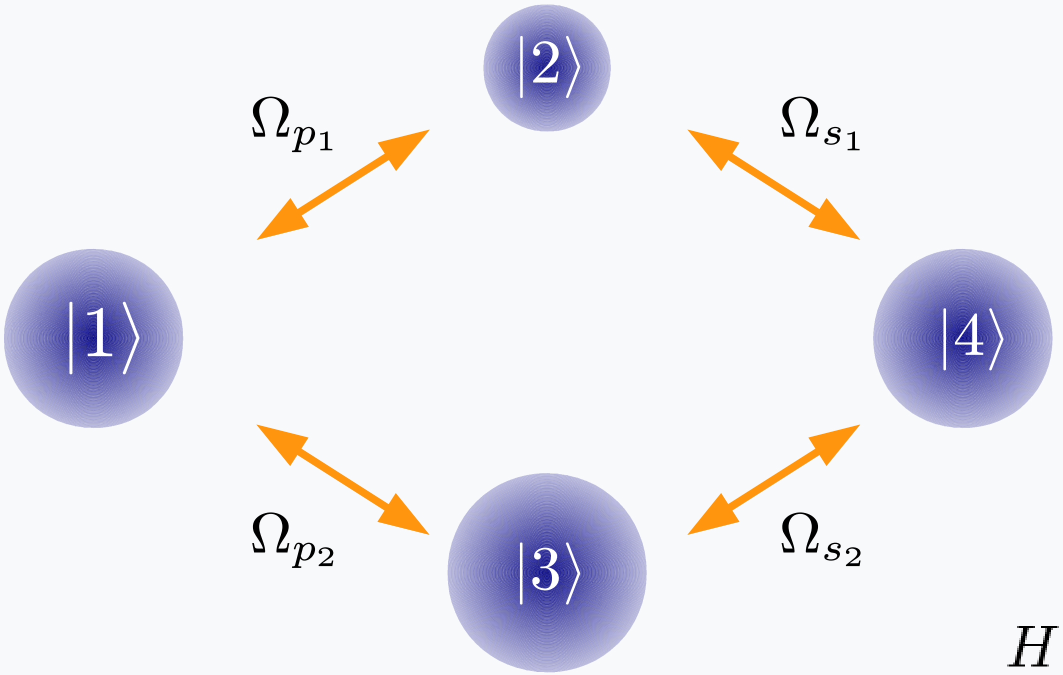
$$\cos\theta : 1 \longrightarrow 0$$

$$\sin\theta : 0 \longrightarrow 1$$

Phase Gate



Phase Gate



$$H = \begin{pmatrix} 0 & \Omega_{p_1} & \Omega_{p_2} & 0 \\ \Omega_{p_1} & \Delta & 0 & \Omega_{s_1} \\ \Omega_{p_2} & 0 & -\Delta & \Omega_{s_2} \\ 0 & \Omega_{s_1} & \Omega_{s_2} & 0 \end{pmatrix}$$

For $\Omega_{p_1} = \Omega_{p_2}$, $\Omega_{s_1} = \Omega_{s_2}$ two states with eigenvalue zero

$$|\phi_1\rangle = \cos\theta|1\rangle - \sin\theta|4\rangle$$

$$|\phi_2\rangle = \sin\theta\sin\varphi|1\rangle + \cos\theta\sin\varphi|4\rangle + \frac{1}{\sqrt{2}}\cos\varphi(|2\rangle - |3\rangle)$$

Phase Gate

If initial state co-incides with one of these

→ adiabatic mixing depending on the geometric phase (Berry phase)

$$\Psi_a(t) = \sum_b^2 B_{ab}(t)\phi_b(t)$$

non-Abelian phase matrix

→ no dynamical phase picked up due to zero eigenvalue

→ phase can be mapped into population distributions

→ easy to measure

Population Distributions

$$\Psi_a(t) = \sum_b^2 B_{ab}(t)\phi_b(t)$$

$$\rightarrow \quad \frac{d}{dt}B_{ab}(t) = -\sum_c A_{bc}(t)B_{ca}(t)$$ with $$A_{bc}(t) = \left\langle \phi_b(t) \middle| \frac{d}{dt} \middle| \phi_c(t) \right\rangle$$

$$\rightarrow \quad B(t) = T\ \exp\left[-\int_{-\infty}^{t} A(t')\, dt'\right]$$

depends on choice of basis
(Hilbert-space structure)

$$B(\infty) = \begin{pmatrix} \cos\ \gamma_f & \sin\ \gamma_f \\ -\sin\ \gamma_f & \cos\ \gamma_f \end{pmatrix}$$

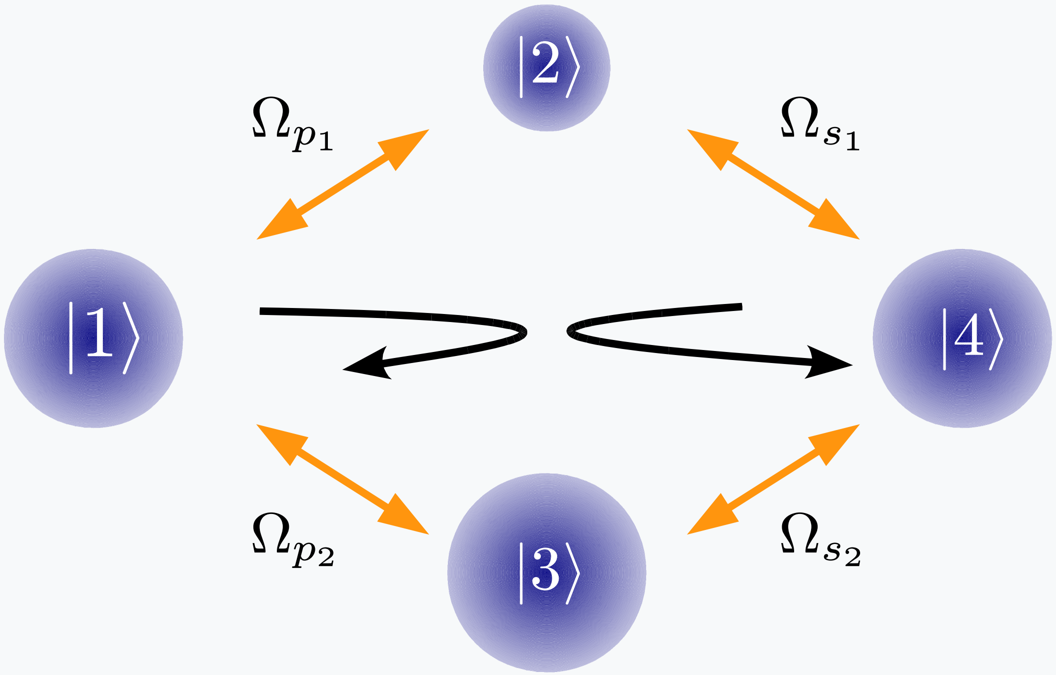
$$\gamma_f = \oint_{-\infty}^{t} \frac{d\theta}{dt'} \sin\varphi\ dt'$$

$$\phi'_a(t) = \sum_b U_{ab}(t)\phi_b(t)$$

$$A'(t) = U(t)A(t)U(t)^{-1} + \dot{U}(t)U(t)^{-1}$$

non-Abelian gauge potential

Phase Gate



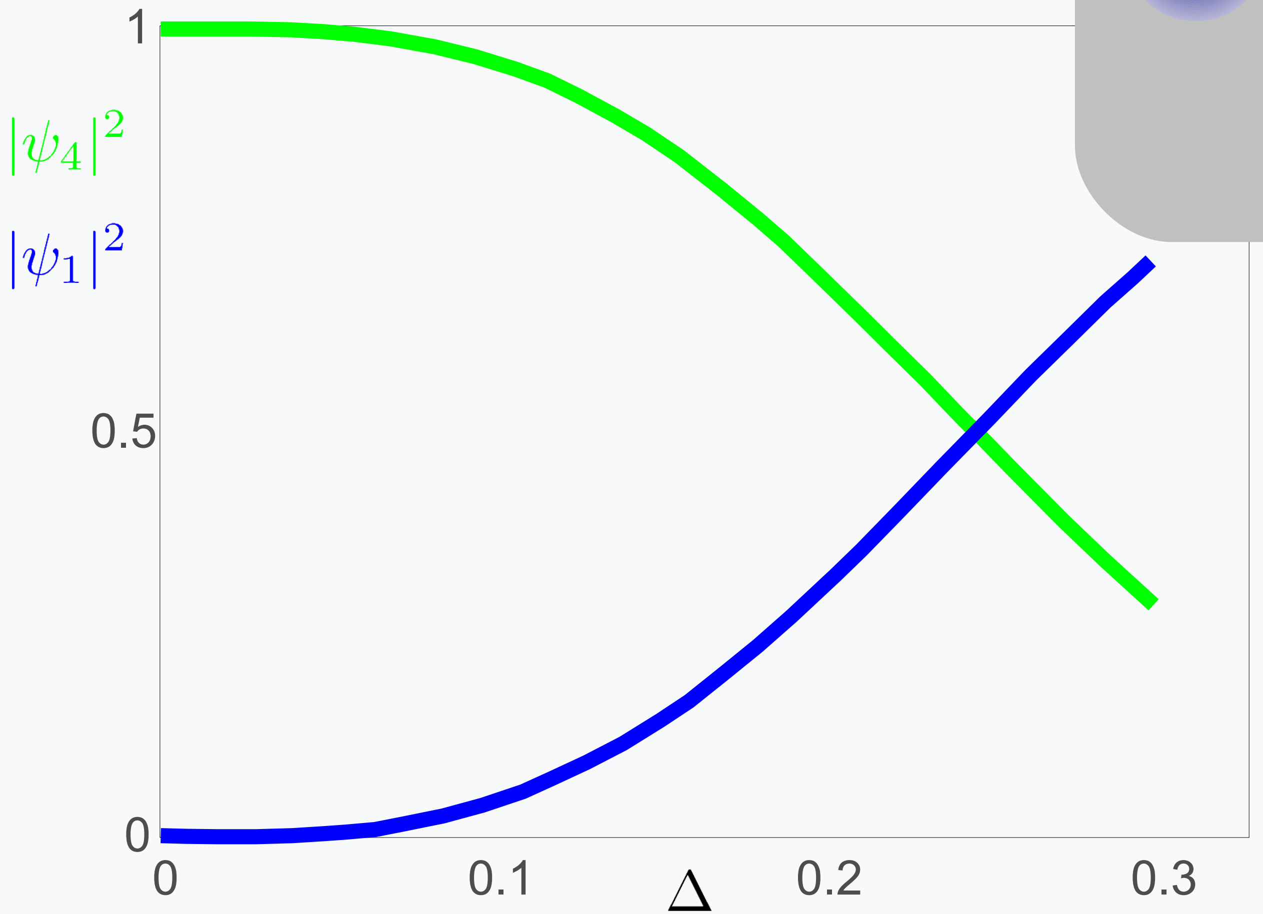
$$|\psi(\infty)\rangle_{\mathrm{CI}} = \sin\gamma|1\rangle - \cos\gamma|4\rangle$$

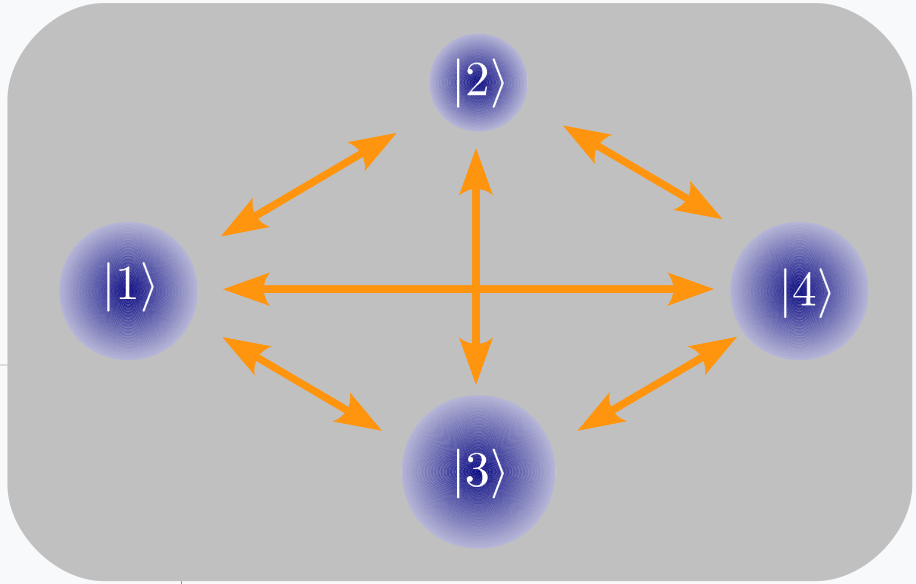
$$|\psi(\infty)\rangle_{\mathrm{I}} = -\sin\gamma|1\rangle + \cos\gamma|4\rangle$$

$\longrightarrow$ any arbitrary superpostion state can be prepared

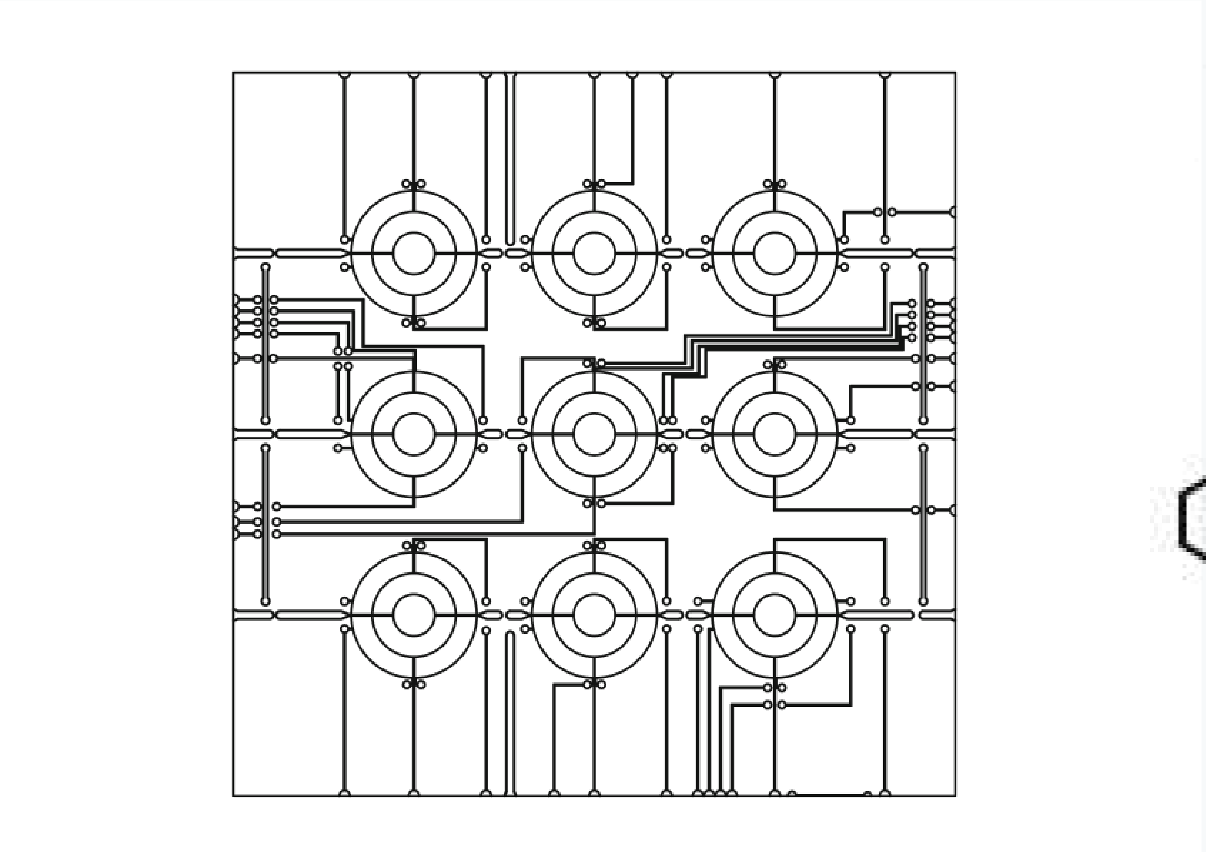
Counter-Intuitive STIRAP

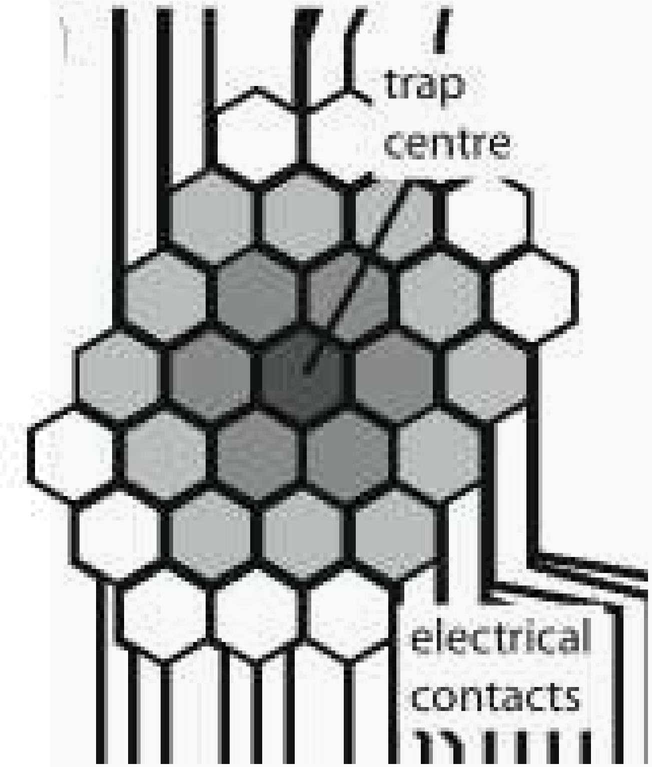
Full numerical integration of the
two-dimensional Schrödinger equation:





Experimental Realisation

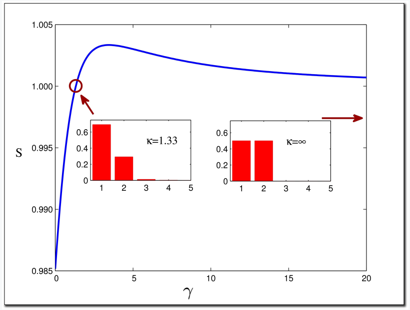




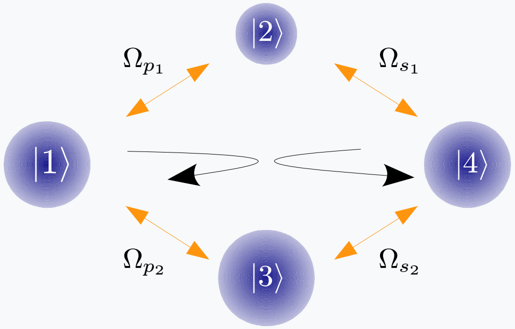
Ferdinand Schmidt-Kaler, University of Ulm

Summary (2)

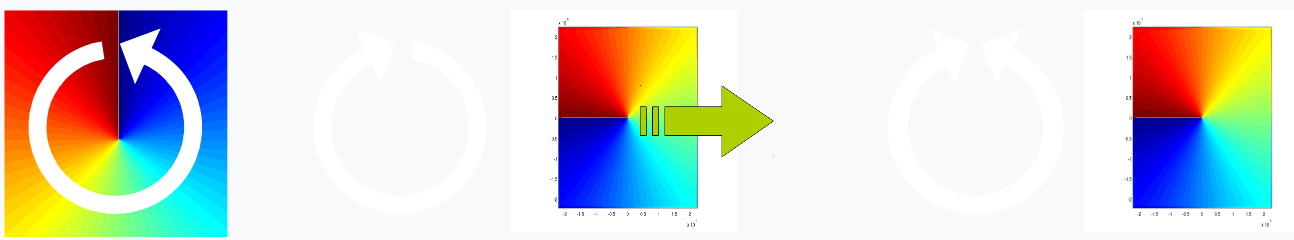
- ***One dimensional quantum gases*** *allow to realise and test concepts of quantum information and calculate* *exact numbers*



- ***STIRAP techniques*** allows for *high fidelity* manipulations of single atoms



Geometric Qubits in Ultracold Atoms



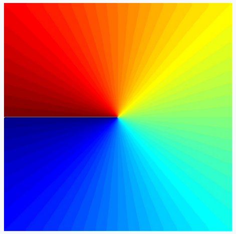
two level systems based on atomic phases

Vortices in BEC

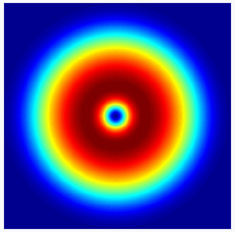
vortices in superfluid Bose-Einstein condensates have a long lifetime!

$$\psi = \sqrt{\rho} e^{-iS(\theta)} = \sqrt{\rho} e^{-in\theta} \longrightarrow \quad \text{phase:} \quad 0 \to 2\pi$$

$\longrightarrow$ angular momentum is quantised!



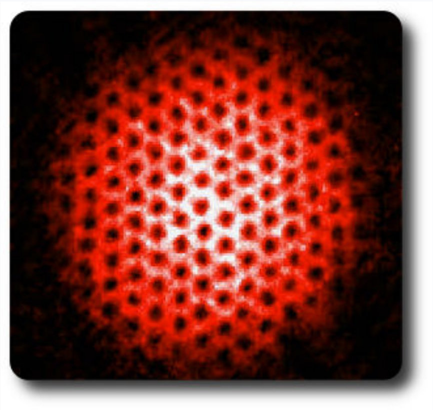
phase

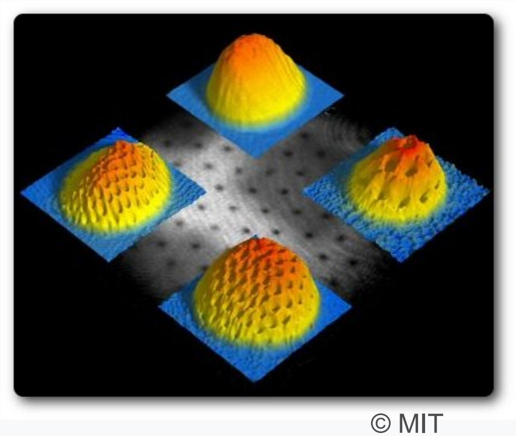


density

GPE: $$H = -\frac{\hbar^2}{2m}\nabla^2 + V(r) - \Omega L + U|\psi|^2$$

$\longrightarrow$ for $\Omega > \Omega_c$ ground state of the condensate carries vortices

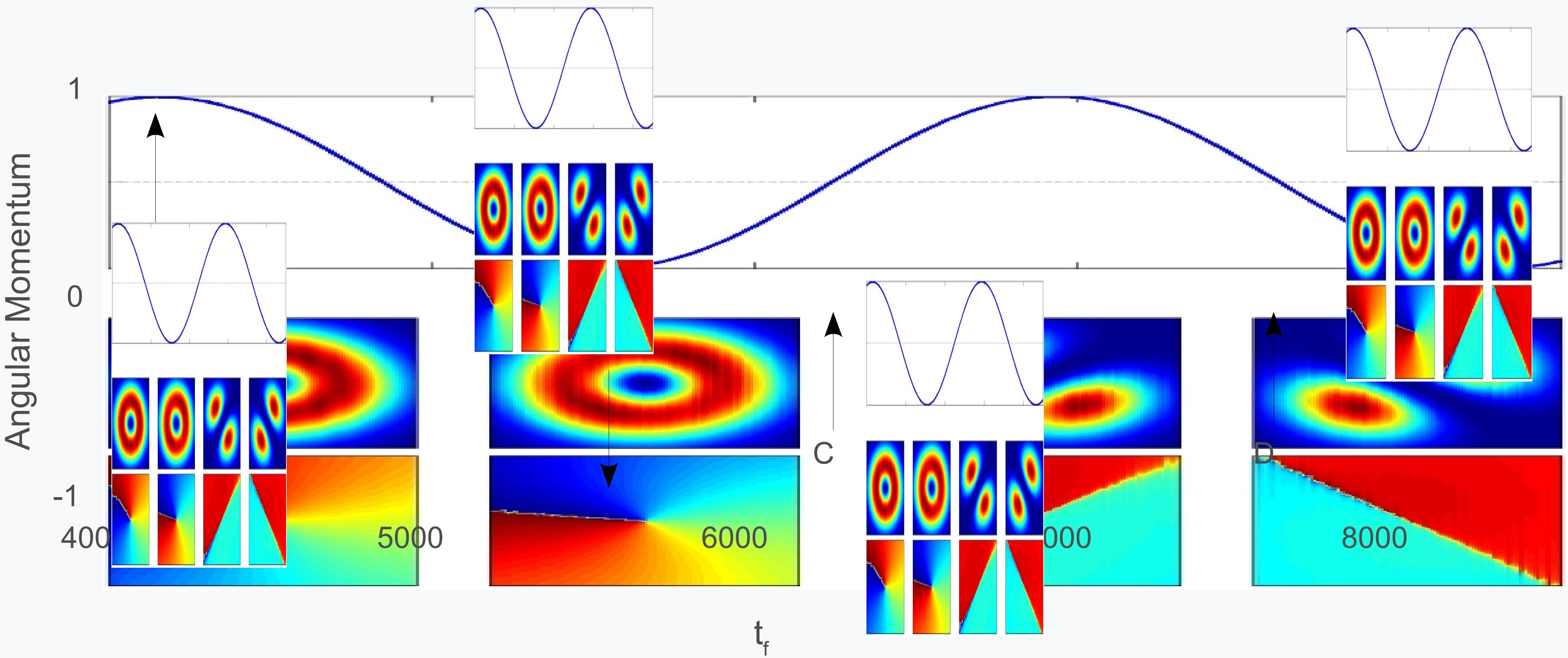




Vortex Superposition States

Try STIRAP for atom with *one unit of positive* angular momentum:

$$H = \begin{pmatrix} 0 & J_{LM} & 0 \\ J_{LM} & 0 & J_{MR} \\ 0 & J_{MR} & 0 \end{pmatrix}$$



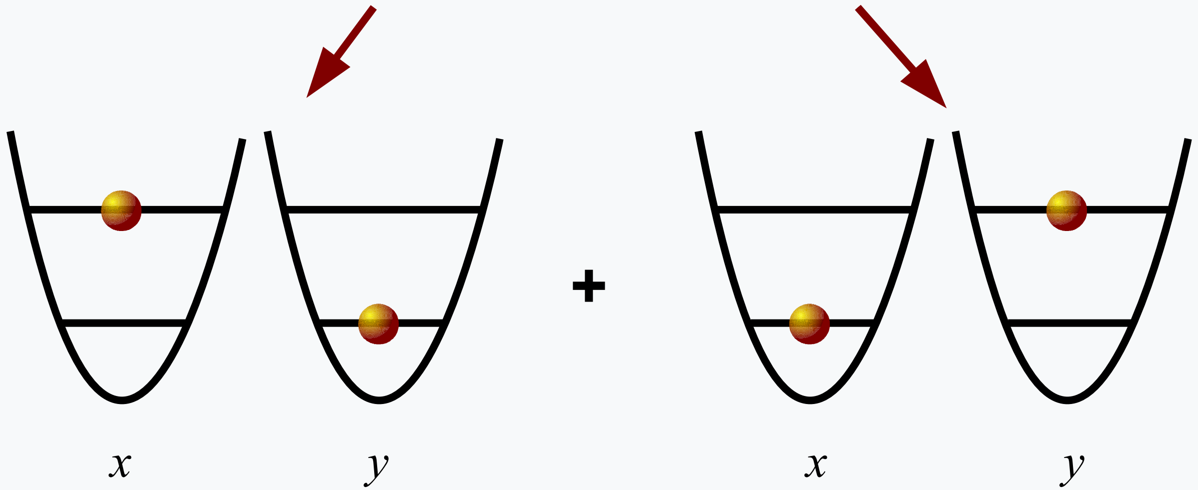
Angular momentum not conserved!

Collaboration with Sarah Croke, Perimeter Institute

Vortex Superposition States

Consider initial state with one unit of angular momentum: $e^{i\theta} = x + iy$

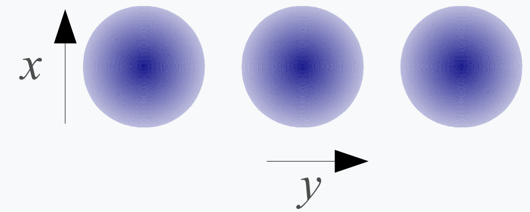
$$\psi = \psi_1(x)\psi_0(y) + i\psi_0(x)\psi_1(x)e^{-i\theta_0}$$



Adiabaticity:

$$H = \begin{pmatrix} \epsilon_0 & J_{LM} & 0 \\ J_{LM} & \epsilon_0 & J_{MR} \\ 0 & J_{MR} & \epsilon_0 \end{pmatrix} \qquad H = \begin{pmatrix} \epsilon_1 & J_{LM} & 0 \\ J_{LM} & \epsilon_1 & J_{MR} \\ 0 & J_{MR} & \epsilon_1 \end{pmatrix}$$

Vortex Superposition States



Carry out adiabatic STIRAP process:

$$\psi(x,y,t_f) = e^{i\gamma}\left[\psi_1(x)\psi_0(y) + i\psi_0(x)\psi_1(y)e^{i\theta}\right]$$

with $\gamma = \frac{3}{2}\omega t_f + \int_0^{t_f} \epsilon_0(t')dt'$

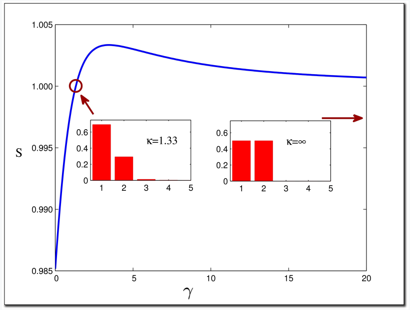
$$\theta = \omega t_f + \int_0^{t_f} [\epsilon_0(t') - \epsilon_1(t')]dt'$$

→ energy eigenvalues in the y-direction slightly change during the STIRAP process due to overlapping trapping potentials

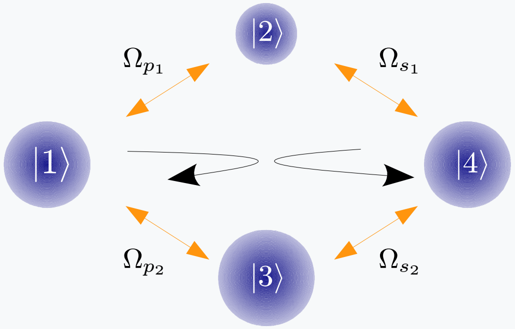
→ deterministic phase engineering possible!

Summary

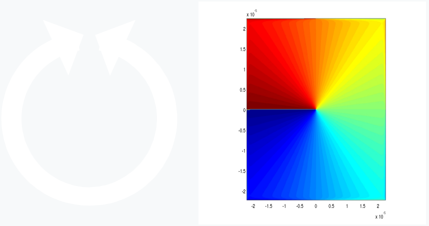
- ***One dimensional quantum gases*** *allow to realise and test concepts of quantum information and calculate exact numbers*



- ***STIRAP techniques*** allows for *high fidelity* manipulations of single atoms



- ***Orbital angular momentum*** *of atoms can be used to create long-living geometrical quantum bits*



Das Team

Quantum Technologies Group

Prof. Myungshik Kim
Dr. Jim McCann
Dr. Mauro Paternostro

Quantum Information Group (Leeds) &
Centre for Quantum Technologoes (Singapore)

Dr. Libby Heany
Prof. Vlatko Vedral

Perimeter Institute

Dr. Sarah Croke