Controlled Creation of Spatial Superposition States for Single Atoms.

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Controlled Creation of Spatial Superposition States for Single Atoms.

- Basic concepts of optical Stirap and the atom optical counterpart.

- Creation of symmetrically selective spatial superpositions.

- Expansion to a two-dimensional configuration allowing preparation of any arbitrary superposition state.
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Introduction

The physics of three level optical systems has a 1:1 analogue in atom optical systems.

\[ |1\rangle \overset{\Omega_p(t)}{\longrightarrow} |2\rangle \overset{\Omega_s(t)}{\longrightarrow} |3\rangle \]

\[ \Delta_1 \quad \Delta_2 \]

\[ |0\rangle_L \quad |0\rangle_M \quad |0\rangle_R \]

\[ d_L(t) \quad d_R(t) \quad x \]

Electronic states \( |1\rangle \equiv |0\rangle_L \) \( |2\rangle \equiv |0\rangle_M \) \( |3\rangle \equiv |0\rangle_R \)

Center of mass states

Optical transitions \( \Omega_p \equiv - J_{LM} \) \( \Omega_s \equiv - J_{MR} \)

Tunneling transitions

Freq. detunings \( \Delta_1 \equiv E_M - E_L \) \( \Delta_2 \equiv E_M - E_R \)

Eigenvalue diff.s

Introduction

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**Electronic states**

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**Center of mass states**

**Optical transitions**

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**Tunneling transitions**

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**Introduction**

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- **Electronic states**: $|1\rangle \equiv |0\rangle_L$, $|2\rangle \equiv |0\rangle_M$, $|3\rangle \equiv |0\rangle_R$
- **Center of mass states**
- **Optical transitions**: $\Omega_p \equiv -J_{LM}$, $\Omega_s \equiv -J_{MR}$
- **Tunneling transitions**
- **Freq. detunings**: $\Delta_1 \equiv E_M - E_L$, $\Delta_2 \equiv E_M - E_R$
Introduction

The physics of three level optical systems has a 1:1 analogue in atom optical systems.

\[|1\rangle, |2\rangle, |3\rangle\]

\[\Omega_p(t), \Omega_s(t)\]

\[\Delta_1, \Delta_2\]

\[|0\rangle_L, |0\rangle_M, |0\rangle_R\]

\[d_L(t), d_R(t)\]

\[\Delta_1 \equiv E_M - E_L, \quad \Delta_2 \equiv E_M - E_R\]

\[\Omega_p \equiv -J_{LM}, \quad \Omega_s \equiv -J_{MR}\]

electronic states

optical transitions
tunneling transitions
center of mass states
eigenvalue diff.s
In resonance the system possesses a dark state:

\[ H = \begin{pmatrix} 0 & -J_{ML} & 0 \\ -J_{LM} & 0 & -J_{MR} \\ 0 & -J_{RM} & 0 \end{pmatrix} \rightarrow |\Psi(\theta)\rangle = \cos(\theta)|0\rangle_L - \sin(\theta)|0\rangle_R \]

\[ \tan(\theta) = \frac{J_{LM}}{J_{MR}} \]

Adiabatic Tuning of the mixing angle, \( \theta \), from 0 to \( \pi \), allows transport of the atom from \( |0\rangle_L \rightarrow |0\rangle_R \).
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Equal Amplitude Superposition states

**Aim:** create a spatial superposition with full control over the symmetry.

\[ |0\rangle_L \quad |0\rangle_M \quad |0\rangle_{RL} \quad |0\rangle_{RR} \]

Allowing for trapping frequency \( \omega_R \)

\[ H = \begin{pmatrix}
0 & -J_{LM}(t) & 0 & 0 \\
-J_{LM}(t) & 0 & -J_{MR}(t) & 0 \\
0 & -J_{MR}(t) & \omega - \omega_R & -J_R \\
0 & 0 & -J_R & \omega - \omega_R 
\end{pmatrix} \]

Aim: create a spatial superposition with full control over the symmetry.

condition for dark state: \[ \omega - \omega_R = \pm J_R \]

Mixing Angle: \[ \tan(\theta) = \sqrt{2} \frac{J_{LM}}{J_{MR}} \]

**Equal Amplitude Superposition states**

**Aim:** create a spatial superposition with full control over the symmetry.

One can find the eigenstates:

\[
|\phi^{\pm}\rangle = \cos(\theta)|0\rangle_L - \sin(\theta)\left[\left(|0\rangle_{RL} \pm |0\rangle_{RR}\right)/\sqrt{2}\right]
\]

\[
= \cos(\theta)|0\rangle_L - \sin(\theta)|0\rangle_{RL}^{\pm}
\]

Aim: create a spatial superposition with full control over the symmetry.

One can find the eigenstates:

\[ |\phi^{\pm}\rangle = \cos(\theta) |0\rangle_L - \sin(\theta) \left( (|0\rangle_{RL} \Box |0\rangle_{RR}) / \sqrt{2} \right) \]

\[ = \cos(\theta) |0\rangle_L - \sin(\theta) |0\rangle_R \]

**Equal Amplitude Superposition states**

**Aim:** create a spatial superposition with full control over the symmetry.

\[ |\phi^{\pm}\rangle = \cos(\theta) |0\rangle_L - \sin(\theta) \left( |0\rangle_{R_L} \pm |0\rangle_{R_R} \right) / \sqrt{2} \]

\[ = \cos(\theta) |0\rangle_L - \sin(\theta) |0\rangle_R \]

How can this be used ??
Single atom interferometer

Separate the wells of the double trap, apply a phase, and repeat stirap process in reverse.

Phase imprint changes symmetry of state.

Time of phase imprint
Single atom interferometer

\[
\phi = 0 \quad \phi = \pi
\]

Time of phase imprint

Only part with correct initial symmetry is in resonance and will be transferred back to the lhs.
Coherent tunneling

- Ion doped optical waveguides
- Numerical simulation
- Fluorescence pattern recorded on a CCD camera
- High Fidelities

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 Arbitrary Superposition States

Choose a 2-D system:

\[
H = \begin{pmatrix}
0 & \Omega_{p1} & \Omega_{p2} & 0 \\
\Omega_{p1} & -\Delta_1 & 0 & \Omega_{s1} \\
\Omega_{p2} & 0 & -\Delta_2 & \Omega_{s2} \\
0 & \Omega_{s1} & \Omega_{s2} & 0
\end{pmatrix}
\]

Dark State solutions exist for

\[
|\omega_{1,4} - \omega_2| = |\omega_{1,4} - \omega_3| = \omega_0
\]

\[
|\phi_1\rangle = \cos(\theta)|1\rangle - \sin(\theta)|4\rangle
\]

\[
|\phi_2\rangle = \sin(\theta)\sin(\phi)|1\rangle + \frac{1}{\sqrt{2}}\cos(\phi)(|2\rangle - |3\rangle)
\]

\[
+ \cos(\theta)\sin(\phi)|4\rangle
\]

with,

\[
\tan(\phi) = \frac{\omega_0}{\sqrt{2(\Omega_{p(t)}^2 + \Omega_{s(t)}^2)}}
\]

\[
\tan(\theta) = \frac{\Omega_{p(t)}}{\Omega_{s(t)}}
\]

Arbitrary Superposition States

During the positioning sequence the two dark states will couple non-adiabatically:

\[
\langle \phi_2 | \dot{\phi}_1 \rangle = -\frac{d\theta}{dt} \sin \phi
\]

\[
|\Phi(t)\rangle = \sum_{i,j=1,2} B_{i,j} |\phi(t)\rangle
\]

\[
B(\infty) = \begin{pmatrix}
\cos(\gamma_f) & \sin(\gamma_f) \\
-sin(\gamma_f) & \cos(\gamma_f)
\end{pmatrix}
\]

\[
\gamma_f = \int_{-\infty}^{t} \frac{d\theta}{dt} \sin \phi dt
\]

Arbitrary Superposition States

The final state of the system is dependent on the detunings of the middle traps:

\[ |\psi(\infty)\rangle = \sin(\gamma_f) |1\rangle - \cos(\gamma_f) |4\rangle \]

Any arbitrary superposition can be created:

![Graph showing the relationship between \(|\psi^2|\) and \(\Delta\omega_0\) for states |1\rangle and |4\rangle.](image)
Arbitrary Superposition States
**Experimental Systems**

G. Raymond - Lab. Charles Fabry de l'Institut d'Optique

Strongly confining optical dipole can store and manipulate single atoms.

**Microscopic dipole traps**

Active Optical Element, with a programmable phase grating.

The diffraction pattern of the Spatial Light Modulator optically traps the BEC.
Outlook

Single-qubit gate \rightarrow two-qubit gate.

Multi-particle systems, interacting and non-interacting particles, \rightarrow non-linear systems.

Extension to waveguides, using photonic and atomic systems.