(1) A force $F_1 = 8 \text{ N}$ is exerted horizontally on block A, which has a mass of 4.5 kg. The coefficient of static friction between A and the table is $\mu_s = 0.20$, and the corresponding coefficient of kinetic friction is $\mu_k = 0.15$.

(a) Does the block move? What is the friction force felt by the block? Now a force $F_2 = 25 \text{ N}$ is exerted horizontally on block A. (b) What is the minimum mass of block C to keep A from sliding? (c) Block C is suddenly lifted off. What is the acceleration of block A?

(a) The maximum possible force of static friction between A and the table is $F_{s,\text{max}} = \mu_s N = \mu_s M_A g = (0.20)(4.5 \text{ kg})(9.8 \text{ m/s}^2) = 8.82 \text{ N}$. This means that if a force is applied horizontally on block A, the block will move if this force is at least 8.82 N, and will not move if this force is less than 8.82 N. Therefore, the block does not move due to the action of $F_1$. Since the block is not moving, the sum of forces in the horizontal direction must be zero, and so the force of static friction is exactly equal and opposite to $F_1$: 8.0 N to the left.

(b) The maximum possible force of static friction between A and the table when block C is sitting on block A is $F_{s,\text{max}} = \mu_s N = \mu_s (M_A + M_C) g$. In order for the two blocks not to move under the action of $F_2$, the mass of C must be large enough that $F_2$ is smaller than the maximum possible force of static friction. The corresponding minimum mass for C is given by

$$
F_2 - \mu_s (M_A + M_C) g = 0
$$

$$
M_C = \frac{F_2}{\mu_s g} - M_A
$$

$$
= \frac{25 \text{ N}}{0.20(9.8 \text{ m/s}^2)} - 4.5 \text{ kg}
$$

$$
M_C = 8.25 \text{ kg}
$$

(c) The acceleration of A after C is lifted off will be given by

$$
F_2 - \mu_k M_A g = M_A a
$$

$$
a = \frac{F_2}{M_A} - \mu_k g
$$

$$
= \frac{25 \text{ N}}{4.5 \text{ kg}} - 0.15(9.8 \text{ m/s}^2)
$$

$$
a = 4.08 \text{ m/s}^2$$
A 110-gram hockey puck sent sliding over ice with an initial speed of 6.0 m/s is stopped in 15 m by the frictional force on it from the ice. (a) What is the magnitude of the frictional force on the ice? (b) What is the coefficient of kinetic friction between the puck and the ice?

(a) Since the frictional force is constant, the acceleration will be constant, and we can use our equations for constant acceleration. We know the initial velocity (6.0 m/s), the final velocity (0 m/s) and the distance travelled (15 m). Therefore,

\[ v^2 = v_0^2 + 2a(x - x_0) \]

\[ a = \frac{v^2 - v_0^2}{2(x - x_0)} \]

\[ a = \frac{0^2 - 6.0 \text{ m/s}^2}{2(15 \text{ m})} = -1.2 \text{ m/s}^2 \]

The magnitude of the frictional force will be

\[ F_k = m|a| = (0.110 \text{ kg})(1.2 \text{ m/s}^2) = 0.132 \text{ N} \]

(b) The frictional force is given by

\[ F = \mu_k N \]

In the vertical direction, there are two forces, the puck’s weight and the normal force, which must add to zero:

\[ N - mg = 0 \rightarrow N = mg \]

Therefore,

\[ F = \mu_k N = \mu_k mg \]

\[ \mu_k = \frac{F}{mg} = \frac{ma}{mg} = \frac{a}{g} \]

\[ \mu_k = \frac{1.2 \text{ m/s}^2}{9.8 \text{ m/s}^2} = 0.12 \]

(3) A pig slides down a 35° incline in twice the time that it would take him to slide down a frictionless 35° incline. What is the coefficient of kinetic friction between the pig and the incline?

There are three forces acting on the pig: the normal force, its weight and the friction force. The normal force \( N \) is purely perpendicular to the ramp, the frictional force \( F_k \) is purely up the ramp and the weight has a component \( mg \sin(35) \) down the ramp and \( mg \cos(35) \) perpendicular to the ramp, opposite the normal force. There is no acceleration perpendicular to the ramp, and we’ll call the acceleration down the ramp \( a_f \) (for acceleration with friction):

\[ N - mg \cos(35) = 0 \quad \text{perpendicular to the ramp} \]

\[ mg \sin(35) - F_k = ma_f \quad \text{along the ramp} \]

\[ mg \sin(35) - \mu_k N = ma_f \]

Solving for \( N \) in the first equation, plugging into the third equation and solving for \( a_f \) yields

\[ a_f = g(\sin(35) - \mu_k \cos(35)) \]

If there is no friction, this is equivalent to \( \mu_k = 0 \), and the corresponding acceleration \( a_{nf} \) will be

\[ a_{nf} = g \sin(35) \]
Now use these two accelerations to find the time for the pig to slide down the ramp of length \( x \) with and without friction, \( t_f \) and \( t_{nf} \):

\[
x = v_0 t + \frac{1}{2} a f t^2 = \frac{1}{2} a f t^2 \quad v_0 = 0
\]
\[
x = \frac{1}{2} a_{nf} t_{nf}^2 = \frac{1}{2} a_{nf} t_{nf}^2
\]

But we are told that \( t_f = 2 t_{nf} \):

\[
\frac{1}{2} a_f [2 t_{nf}]^2 = \frac{1}{2} a_{nf} t_{nf}^2
\]
\[
4 a_f = a_{nf}
\]
\[
4 g (\sin(35) - \mu_k \cos(35)) = g \sin(35)
\]
\[
4 \mu_k g \cos(35) = 3 g \sin(35)
\]
\[
\mu_k = \frac{3}{4} \tan(35) = 0.52
\]

(4) A dart is thrown horizontally with an initial speed of 10 m/s toward the bull’s eye on a dart board. It hits at a point directly below the bull’s eye 0.19 s later. (a) How far below the bull’s eye does the dart hit? (b) How far away from the dart board is the dart released? (c) What are the horizontal and vertical components of the dart’s velocity when it hits the board?

We have the following two sets of equations for the dart’s motion in the \( x \) and \( y \) directions (we take the initial position of the dark to be \( x = 0, y = 0 \), and will take \(+y\) to be upward and \(+x\) to be in the direction of motion of the dart):

\[
x = x_0 + v_{0x} t \rightarrow x = 10 \text{ m/st}
\]
\[
v_x = v_{0x} \rightarrow v_x = 10 \text{ m/s}
\]
\[
y = y_0 + v_{0y} t - \frac{1}{2} g t^2 \rightarrow y = -\frac{1}{2} g t^2
\]
\[
v_y = v_{0y} - g t \rightarrow v_y = -g t
\]

(a) Because we know the time the dart is in flight, we can find the \( y \) value corresponding to this time:

\[
y = -\frac{1}{2} g t^2 = -\frac{1}{2} (9.8 \text{ m/s}^2)(0.19 \text{ s})^2
\]
\[
y = -0.177 \text{ m} = -17.7 \text{ cm}
\]

The dart hits 17.7 cm below the bull’s eye.

(b) Knowing the time of flight, we can also find the total distance to the board:

\[
x = 10 \text{ m/st} = 10 \text{ m/s}(0.19 \text{ s}) = 1.9 \text{ m}
\]

(c) We can also find the horizontal and vertical components of the dart’s velocity when it hits the board:

\[
v_x = 10 \text{ m/s}
\]
\[
v_y = -g t = -(9.8 \text{ m/s}^2)(0.19 \text{ s}) = -1.86 \text{ m/s}
\]
(5) A supply airplane diving at an angle of $35.0^\circ$ with the horizontal releases a package of supplies at an altitude of 130 m. The package hits the ground 3.5 s after being released. (a) What is the speed of the plane? (b) How far did the package travel horizontally during its flight?

We have the following two sets of equations for the package’s motion in the $x$ and $y$ directions (we take the initial position of the package when it is released to be $x = 0, y = 0$, and will take $+y$ to be upward and $+x$ to be in the horizontal direction of motion of the plane):

\[
\begin{align*}
  x &= x_0 + v_{0x} t = x = v \cos(35) \text{ m/s} \\
  v_x &= v_{0x} = v \cos(35) \text{ m/s} \\
  y &= y_0 + v_{0y} t - \frac{1}{2} g t^2 = y = -v \sin(35) t - \frac{1}{2} g t^2 \\
  v_y &= v_{0y} - gt = v_y = -v \sin(35) - gt
\end{align*}
\]

(a) We know that $y = -130$ m when $t = 3.5$ s. We wish to find $v$ – we can do this using the equation for $y$:

\[
\begin{align*}
  y &= -v \sin(35) t - \frac{1}{2} g t^2 \\
  y_f &= -v \sin(35) t_f - \frac{1}{2} g t_f^2 \\
  v \sin(35) t_f &= -y_f - \frac{1}{2} g t_f^2 \\
  v &= \frac{-y_f - \frac{1}{2} g t_f^2}{\sin(35) t_f} \\
  &= \frac{-(-130 \text{ m}) - \frac{1}{2} (9.8 \text{ m/s}^2)(3.5 \text{ s})^2}{\sin(35)(3.5 \text{ s})} \\
  v &= 34.8 \text{ m/s}
\end{align*}
\]

(b) Knowing $v$ and $t_f$, we can also find the horizontal distance travelled before the package hit the ground:

\[
\begin{align*}
  x_f &= v \cos(35) \text{ m/st} = (34.8 \text{ m/s}) \cos(35)(3.5 \text{ s}) \\
  x_f &= 99.8 \text{ m}
\end{align*}
\]

(6) The Sun has a radius of $6.96 \times 10^8$ m and the material at its equator rotates about its axis once every 26 days. What is the linear velocity of material at the Sun’s equator due to its rotation?

The linear velocity will be the equal to

\[
v = \frac{2\pi R}{T} = \frac{2\pi (6.96 \times 10^8 \text{ m})}{26 \text{ d}}
\]

We must simply express the period $T$ in seconds to get the speed in m/s:

\[
\begin{align*}
  T &= 26 \text{ d} \times \frac{24 \text{ hrs}}{\text{d}} \times \frac{3600 \text{ s}}{\text{hr}} = 2.25 \times 10^6 \text{ s} \\
  v &= \frac{2\pi (6.96 \times 10^8 \text{ m})}{2.25 \times 10^6 \text{ s}} = 1.94 \times 10^3 \text{ m/s} = 1.94 \text{ km/s}
\end{align*}
\]
(7) On a French TGV train, the magnitude of the acceleration experienced by the passengers is to be limited to 0.050g. (a) If such a train is going around a curve at a speed of 216 km/hr what is the smallest radius of curvature that the curve can have without exceeding the maximum allowed acceleration on the passengers? (b) With what speed does the train go around a curve with a 1.00 km radius if the acceleration exerted on the passengers is at its maximum allowed value?

(a) Since the train is moving in a circular arc, we have

\[ a_c = \frac{v^2}{r} \rightarrow r = \frac{v^2}{a_c} \]

We can see that the maximum centripetal acceleration, 0.050g, corresponds to the minimum radius. The train’s speed and acceleration are

\[ v = 216 \text{ km/hr} \times \frac{1000 \text{ m}}{1 \text{ km}} \times \frac{1 \text{ hr}}{3600 \text{ s}} = 60.0 \text{ m/s} \]
\[ a_c = 0.050g = 0.050(9.8 \text{ m/s}^2) = 0.490 \text{ m/s}^2 \]

Therefore, the minimum radius is

\[ r = \frac{v^2}{a_c} = \frac{(60.0 \text{ m/s})^2}{0.490 \text{ m/s}^2} \]
\[ r = 7.35 \times 10^3 \text{ m} = 7.35 \text{ km} \]

(b) Again we have

\[ a_c = \frac{v^2}{r} \rightarrow v = \sqrt{ra_c} \]
\[ v = \sqrt{(1.00 \times 10^3 \text{ m})(0.490 \text{ m/s}^2)} = 22.1 \text{ m/s} \]

(8) A puck of mass \( m \) slides in a circle on a frictionless table while attached to a hanging mass \( M \) by a cord through a hole in the table. With what speed must the puck move to keep the hanging mass stationary?

The upward tension in the cord must balance the downward force of gravity acting on \( M \). This tension will also be equal to the centripetal force acting on \( m \):

\[ T - Mg = 0 \quad \text{balance of forces for } M \]
\[ T = \frac{mv^2}{r} \quad \text{centripetal force on } m \]
\[ \frac{mv^2}{r} - Mg = 0 \]
\[ v = \sqrt{\frac{M}{m}rg} \]
(1b) For women’s volleyball, the top of the net is 2.24 m above the floor and the court measures 9.0 m by 9.0 m on each side of the net. Using a jump serve, a player strikes the ball at a point that is 3.0 m above the floor and a horizontal distance of 8.0 m from the net. If the initial velocity of the ball is horizontal, what minimum magnitude must it have if the ball is to clear the net, and what magnitude can it have if the ball is to strike the floor inside the back line on the other side of the net?

Take the point where the volleyball is hit to be \((x = 0, y = 3.0 \text{ m})\). We also know that \(v_{0,x} = v_0\) and \(v_{0,y} = 0\). Our equations for \(x\) and \(y\) as a function of time are then

\[
\begin{align*}
\text{constant velocity} & \\
\text{constant acceleration}
\end{align*}
\]

\[
\begin{align*}
x &= v_0 t \\
y &= 3.0 \text{ m} - \frac{1}{2}gt^2
\end{align*}
\]

In order to clear the net, we must have \(y = 2.24 \text{ m}\) when \(x = 8.0 \text{ m}\). Use the relation for \(y\) to find the time, then plug this into the relation for \(x\) to find \(v_0\).

\[
t = \sqrt{\frac{2.24 \text{ m} - 3.0 \text{ m}}{-g/2}} = 0.394 \text{ s}
\]

\[
v_0 = \frac{x}{t} = \frac{8.0 \text{ m}}{0.394 \text{ s}} = 20.3 \text{ m/s}
\]

Thus, the minimum speed the volleyball must have is 20.3 m/s. For the ball to land on the line, we must have \(x = 17.0 \text{ m}\) when \(y = 0\). Using the same procedure, we find

\[
t = \sqrt{\frac{0 \text{ m} - 3.0 \text{ m}}{-g/2}} = 0.782 \text{ s}
\]

\[
v_0 = \frac{x}{t} = \frac{17.0 \text{ m}}{0.782 \text{ s}} = 21.7 \text{ m/s}
\]

Thus, the server must hit the volleyball so that it has a speed no less than 20.3 m/s and no more than 21.7 m/s.

(2b) A gardener wishes to pile a cone of sand onto a circular area in his yard. The radius of the circle is \(R\), and no sand is to spill onto the surrounding area. If \(\mu_s\) is the coefficient of static friction between each layer of sand along the slope and the sand beneath it (along which it must slip), show that the greatest volume of sand that can be stored in this manner is \(\pi\mu_s R^3/3\). The volume of a cone is \(Ah/3\), where \(A\) is the base area and \(h\) is the cone’s height.
A 1000-kg boat is traveling at 90 km/hr when its engine is shut off. The magnitude of the frictional force between the boat and water is proportional to the speed of the boat: \( f_k = 70v \), with \( v \) in m/s and the force in Newtons. How long does it take for the boat to slow to 45 km/hr?

We can find a relation between the time and the change in the velocity as follows:

\[
F = -kv = ma = m \frac{dv}{dt}
\]

\[
dt = -\frac{m}{k} \frac{dv}{v}
\]

Integrating both sides of this equation, we obtain

\[
\int_{t_i}^{t_f} dt = -\frac{m}{k} \int_{v_i}^{v_f} \frac{dv}{v}
\]

\[
t_f - t_i = -\frac{m}{k} \ln \frac{v_f}{v_i} = \frac{m}{k} \ln \frac{v_i}{v_f}
\]

Inserting \( m = 1000 \text{ kg} \), \( k = 70 \text{ Ns/m} \), \( v_i = 90 \text{ km/hr} \) and \( v_f = 45 \text{ km/hr} \), we find

\[
\Delta t = \frac{1000}{70} \ln \frac{90}{45} = \frac{1000}{70} \ln 2 = 9.90 \text{ s}
\]