3 A4: Study of the Radioactive Decay of a Short-lived Radioactive Source

(Reference: Mansfield and O'Sullivan pp. 666–672; Krane pp. 384–387)

The decrease in the activity of a radioactive isotope is characterized by its half-life, which is the time required for one-half of the nuclei of a sample to decay. The number of nuclei in a sample cannot be counted directly, but when one-half of the sample has decayed, the activity or the rate of emission of nuclear radiation will also have decreased by one-half. Thus, by monitoring the sample with a Geiger counter, when the count rate has decreased by one-half, one half-life has elapsed.

In this experiment, the half-life of the radioactive isotope ${}^{137}_{56}\mathrm{Ba}^*$ will be determined.

Theory

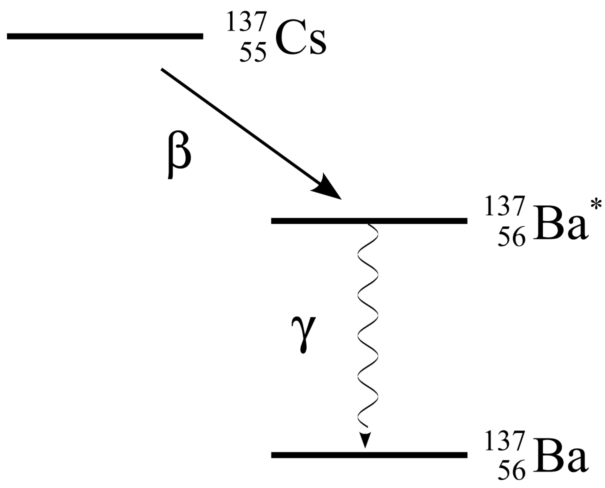
To measure radioactive decay in a reasonable amount of time, it is necessary to have a short-lived radioactive source. One way to produce such a source is by collecting a short-lived radioactive product (*daughter*) of some radioactive source with a longer decay time, and hence longer shelf life.

The mini-generator used in this experiment contains a minute quantity of ${}^{137}_{55}\mathrm{Cs}$ which has a half-life of about 30 years and decays by β-emission into ${}^{137}_{56}\mathrm{Ba}^*$ (an excited state of the ${}^{137}_{56}\mathrm{Ba}$ nucleus). ${}^{137}_{56}\mathrm{Ba}^*$, in turn, decays with a short half-life into stable ${}^{137}_{56}\mathrm{Ba}$ by emission of a 0.661 MeV γ-particle (see illustration below).

The mini-generator normally consists of ${}^{137}_{55}\mathrm{Cs}$ and ${}^{137}_{56}\mathrm{Ba}^*$ in equilibrium (i.e. the rate at which ${}^{137}_{56}\mathrm{Ba}^*$ is produced by decay of ${}^{137}_{55}\mathrm{Cs}$ equals the rate at which ${}^{137}_{56}\mathrm{Ba}^*$ decays to ${}^{137}_{56}\mathrm{Ba}$). The Ba can be washed out of the mini-generator by elution with 0.04 NHCl and collected in a beaker, leaving the generator as almost pure ${}^{137}_{55}\mathrm{Cs}$. The eluate in the beaker consists of ${}^{137}_{56}\mathrm{Ba}^*$ and no Cs and hence is a γ-ray source of short half-life.

The number of ${}^{137}_{56}\mathrm{Ba}^*$ nuclei, N, at any time t is given by the radioactive decay equation (see for example Mansfield and O'Sullivan pp. 666)

$$N = N_0 e^{-\lambda t} \,, \tag{3.1}$$



where N_0 is the number of $^{137}_{56}Ba^*$ nuclei present when the timing commenced ($t = 0$) and λ is the disintegration constant of $^{137}_{56}Ba^*$, which is directly related to its half-life time $T_{1/2}$

$$T_{1/2} = \frac{0.693}{\lambda}. \qquad (3.2)$$

Since every decaying atoms creates exactly one γ, the recorded count rate C is directly proportional to N so that we can write

$$C = C_0 e^{-\lambda t}, \qquad (3.3)$$

where C_0 is the count rate when $t = 0$. Thus

$$\ln C = \ln C_0 - \lambda t. \qquad (3.4)$$

Error analysis

It can be shown that the standard deviation of a count rate C is given by $\sqrt{C}$. Thus the error on a count rate of 10.000 is 100 (1%) whereas the error in a count rate of 100 is 10 (10%).

This means that the percentage error attached to count rates towards the end of the ten minutes period over which readings are recorded is substantially greater than the percentage error attached to count rates at the beginning of the period. Readings at the beginning should therefore be given more weight (i.e. smaller error bars) when determining the slope of the graph.

The length of an error bar $\Delta(\ln C)$ for a particular value of $\ln C$ may be determined in terms of the error bar in C, $\Delta C = \sqrt{C}$, through differentiation. Using the derivative rule

$$d(\ln x) = \frac{dx}{x} \; , \tag{3.5}$$

we obtain

$$\Delta(\ln C) = \frac{\Delta C}{C} = \frac{1}{\sqrt{C}} \; . \tag{3.6}$$

Use *Origin 7.0*, as described below, to calculate an error bar for each value of C in the $\ln C$ against t plot, and use this, again as described below, to determine the error in the slope which is used to determine $T_{1/2}$.

Procedure

Ask the instructor to show you how to elute the mini-generator (since the elute is a liquid radioactive source in an open beaker **great care must be taken to avoid spilling!**). Wear latex gloves handling the mini-generator. After the experiment pour the liquid down the drain and flush with plenty of water.

Experiment (A): Measure the dark count. User the Geiger counter and AWARE software package, as described below, and record the count rate C at ten seconds intervals for 10 minutes. Determine the average and standard deviation (using Origin 7.0). Discuss the results in your report.

Experiment (B): Generate a Ba* solution in a beaker using the mini-generator and place the beaker on the Geiger counter. Measure the count rate C at ten seconds intervals for 15 minutes. Subtract the dark count rate from the data and determine the half-life of the Ba* decay. Plot the decay curve semi-logarithmically (ln C against t, using Origin 7.0) with error bars and show the fitted curve (see eq. (0.19)). Discuss your results in your report.

Experiment (C): Place an aluminium plate provided on the Geiger counter. Generate a Ba* solution in a beaker using the mini-generator and place the mini-generator on the aluminium plate on top of the Geiger counter. Measure the count rate C at ten seconds intervals for 15 minutes. Subtract the count rate at the start of the curve from your data. Fit the function,

$$C(t) = C_0(1 - e^{-\lambda t}) \tag{3.7}$$

to the measured data. Determine the half-life from λ and compare with experiment (B). Plot your data $C(t)$ with error bars and show the fitted curve. Discuss your results in your report.