

3 Study of the Radioactive Decay of a Short-lived Radioactive Source

(Reference: Mansfield and O'Sullivan pp. 666–672; Krane pp. 384–387)

The decrease in the activity of a radioactive isotope is characterized by its half-life, which is the time required for one-half of the nuclei of a sample to decay. The number of nuclei in a sample cannot be counted directly, but when one-half of the sample has decayed, the activity or the rate of emission of nuclear radiation will also have decreased by one-half. Thus, by monitoring the sample with a Geiger counter, when the count rate has decreased by one-half, one half-life has elapsed.

In this experiment, the half-life of the radioactive isotope $^{137}_{56}\text{Ba}^*$ will be determined.

Theory

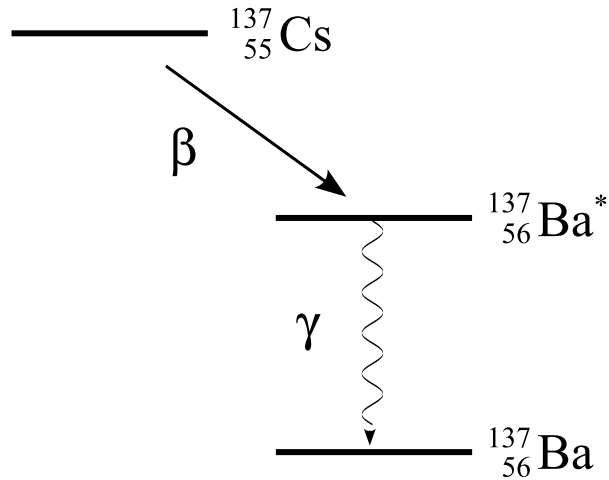
To measure radioactive decay in a reasonable amount of time, it is necessary to have a short-lived radioactive source. One way to produce such a source is by collecting a short-lived radioactive product (*daughter*) of some radioactive source with a longer decay time, and hence longer shelf life.

The mini-generator used in this experiment contains a minute quantity of $^{137}_{55}\text{Cs}$ which has a half-life of about 30 years and decays by β -emission into $^{137}_{56}\text{Ba}^*$ (an excited state of the $^{137}_{56}\text{Ba}$ nucleus). $^{137}_{56}\text{Ba}^*$, in turn, decays with a short half-life into stable $^{137}_{56}\text{Ba}$ by emission of a 0.661 MeV γ -particle (see illustration below).

The mini-generator normally consists of $^{137}_{55}\text{Cs}$ and $^{137}_{56}\text{Ba}^*$ in equilibrium (i.e. the rate at which $^{137}_{56}\text{Ba}^*$ is produced by decay of $^{137}_{55}\text{Cs}$ equals the rate at which $^{137}_{56}\text{Ba}^*$ decays to $^{137}_{56}\text{Ba}$). The Ba can be washed out of the mini-generator by elution with 0.04 NHCl and collected in a beaker, leaving the generator as almost pure $^{137}_{55}\text{Cs}$. The eluate in the beaker consists of $^{137}_{56}\text{Ba}^*$ and no Cs and hence is a γ -ray source of short half-life.

The number of $^{137}_{56}\text{Ba}^*$ nuclei, N , at any time t is given by the radioactive decay equation (see for example Mansfield and O'Sullivan pp. 666)

$$N = N_0 e^{-\lambda t} , \tag{3.1}$$



where N_0 is the number of $^{137}_{56}\text{Ba}^*$ nuclei present when the timing commenced ($t = 0$) and λ is the disintegration constant of $^{137}_{56}\text{Ba}^*$, which is directly related to its half-life time $T_{1/2}$

$$T_{1/2} = \frac{0.693}{\lambda}. \quad (3.2)$$

Since every decaying atoms creates exactly one γ , the recorded count rate C is directly proportional to N so that we can write

$$C = C_0 e^{-\lambda t}, \quad (3.3)$$

where C_0 is the count rate when $t = 0$. Thus

$$\ln C = \ln C_0 - \lambda t. \quad (3.4)$$

Error analysis

It can be shown that the standard deviation of a count rate C is given by \sqrt{C} . Thus the error on a count rate of 10.000 is 100 (1%) whereas the error in a count rate of 100 is 10 (10%).

This means that the percentage error attached to count rates towards the end of the ten minutes period over which readings are recorded is substantially greater than the percentage error attached to count rates at the beginning of the period. Readings at the beginning should therefore be given more weight (i.e. smaller error bars) when determining the slope of the graph.

The length of an error bar $\Delta(\ln C)$ for a particular value of $\ln C$ may be determined in terms of the error bar in C , $\Delta C = \sqrt{C}$, through differentiation. Using the derivative

rule

$$d(\ln x) = \frac{dx}{x}, \quad (3.5)$$

we obtain

$$\Delta(\ln C) = \frac{\Delta C}{C} = \frac{1}{\sqrt{C}}. \quad (3.6)$$

Use *Origin 7.0*, as described below, to calculate an error bar for each value of C in the $\ln C$ against t plot, and use this, again as described below, to determine the error in the slope which is used to determine $T_{1/2}$.

Procedure

Ask the instructor to show you how to elute the mini-generator (since the elute is a liquid radioactive source in an open beaker **great care must be taken to avoid spilling!**). Wear latex gloves handling the mini-generator. After the experiment pour the liquid down the drain and flush with plenty of water.

Experiment (A): Measure the dark count. Use the Geiger counter and AWARE software package, as described below, and record the count rate C at ten seconds intervals for 10 minutes. Determine the average and standard deviation (using Origin 7.0). Discuss the results in your report.

Experiment (B): Generate a Ba^* solution in a beaker using the mini-generator and place the beaker on the Geiger counter. Measure the count rate C at ten seconds intervals for 15 minutes. Subtract the dark count rate from the data and determine the half-life of the Ba^* decay. Plot the decay curve semi-logarithmically ($\ln C$ against t , using Origin 7.0) with error bars and show the fitted curve (see eq. (3.4)). Discuss your results in your report.

Experiment (C): Place an aluminium plate provided on the Geiger counter. Generate a Ba^* solution in a beaker using the mini-generator and place the mini-generator on the aluminium plate on top of the Geiger counter. Measure the count rate C at ten seconds intervals for 15 minutes. Subtract the count rate at the start of the curve from your data. Fit the function,

$$C(t) = C_0(1 - e^{-\lambda t}) \quad (3.7)$$

to the measured data. Determine the half-life from λ and compare with experiment (B). Plot your data $C(t)$ with error bars and show the fitted curve. Discuss your results in your report.

Safety

The minigenerator is a sealed radioactive source that contains radioactive cesium with a half-life of about 30 years. Because this source is sealed, you will not come into contact with this cesium unless the minigenerator is broken. Therefore, please be careful handling this piece of equipment. The radioactive barium that you will be working with has a very short half-life. By the time the lab is over, more than 10 half-lives will have passed, at which point the radioactivity levels are equal to the background radioactivity normally present in the air. Therefore, this is very safe to work with as long as the following safety precautions are followed:

- No food or drinks are allowed in the lab area, please leave all food outside the lab room.
- Safety goggles and gloves must be worn at all times when working with the radioisotope generators.
- The eluting solution contains a weak concentration of HCl, this can cause burns on your skin and clothing, please be careful to not spill it and wear gloves and goggles at all times.
- Please notify your lab instructor of any spills or accidents immediately.
- Please dispose of your used gloves in the trash and wash your hands thoroughly before leaving the lab.

Using the AWARE software package

To open the AWARE software package double click the 'Shortcut to AW-RADW' icon on the desktop.

Before starting to count the following menu points should be set:

- *Output Options – Screen Options – Draw ASCII File Format to Screen*
- *Output Options – ASCII Text File Options – ASCII Delimiter Character – Tab*
- *Output Options – ASCII Text File Options – ASCII File time code – s-Seconds since 1/1/1970*

In order to start collecting data select

- *Rad Collection – Express start collection of Rad data*

The computer will now prompt you for a *Select Aware Binary Rad Data File*. Click on *Cancel*. Next the computer will prompt you to enter a file name for your data in the form *Aware ASCII Output File*. Create your own *file name.dat*. When you click *OK* in this window the program will start collecting data.

To stop the count select

- *Rad Collection – Stop Collection Of Rad Data.*

Optionally: If during collection of data you want to extract some of the data already collected by the counter, select

- *screen – copy – copy screen to clipboard most recent at bottom*

This will copy all the information displayed on the entire screen, including comments and status lines. The data can then be pasted into Notepad or an Origin7.0 worksheet for further analysis.

Using Origin7.0

To open Origin7.0, double click on the 'shortcut to Origin70.exe' icon on the desktop.

Import the data into Origin7.0 by selecting

- *File – Import – Single ASCII*

The computer will now prompt you for a file name. Select your *file name.dat*.

To calculate the background average:

Import the background radiation data as described above. Highlight column $B(Y)$ by clicking on the $B(Y)$ column title. Then select

- *statistics – descriptive statistics – statistics on column*

Origin now will create a window displaying all the statistics of the column including mean and standard deviation.

To fit curve to log plot:

To open a new worksheet, select

- *File – New*

Origin will now prompt you to select a type of window. Select *Worksheet* and then click *OK*.

Import the data as described above. Add three new columns by selecting

- *Column – Add New Columns*

Origin7.0 will now prompt you for the number of new columns you need. Enter *3* and click *OK*.

Highlight column 'C(Y)' and then select

- *Column – Set column values*

Origin will now prompt you to set a value for column C(Y). In the box provided type *col(B) – background value* and click *OK*. Highlight column C(Y) and select

- *Edit – Copy*

Next highlight column B(Y) and select

- *Edit – Paste*

The time in column A(X) is the number of seconds since 1/1/1970. To make these times more relevant, highlight the box A(X)1 and select

- *Edit – Copy*

Then highlight column A(X) and select

- *Column – Set Column Values*

In the prompt box type *col(A) - 'Ctrl v'*, where holding down *Ctrl* and pressing *v* pastes the time from A(x)1, and then click *OK*. Next highlight column A(x) and select

- *Column – Set Column Values*

In the prompt box type *col(C) – 'the first value in column C(Y)'* and click *OK*. Column A(X) should now contain data starting at a time of zero and increase in ten second intervals.

Highlight column D(Y) and select

- *Column – Set Column Values*

and click *OK*. In the prompt box type *ln(col(B))*.

Highlight column E(Y) and select

- *Column – Set Column Values*

In the prompt box type $1/\text{sqrt}(\text{col}(B))$ and click *OK*. While E(Y) is still highlighted select

- *Column – Set As Y Error*

Highlight column C(Y) and select

- *Column – Set Column Values*

In the prompt box type $\text{col}(A)$ and click *OK*. While Column C(Y) is still highlighted select

- *Column – Set As X.*

Curve fitting:

To fit curve to data highlight columns C(X), D(Y) and E(Y) by placing the mouse on the C(X) column title, holding the mouse button down and moving the mouse across as far as the E(Y) column title.

Select

- *Analysis – Non-linear Curve fit – Advanced Fitting Tool*

A new window will now open. On this window select

- *Function – New*

In the new window change *Number Of Parameters* to 2 in the drop down menu option. In the definition text box type $P1+P2*x$.

Select the

- *Action - Dataset*

menu option. Click *Your file name_D* in the available datasets list box. Click on *Assign*. Select

- *Option – Control*

from the menu options. Choose *Arbitrary dataset* from the *Weighting Method* drop down menu. Click *Your file name_E* in the available Datasets list box.

Select the

- *Action – Fit*

menu option. Enter an initial estimate for P1, P2, P1.2 and in the respective *Value* boxes. Click on *100 iter.* repeatedly until the message *chi-sqr is not reduced* is displayed in the large text box. Click *Done*.