

# 2 The Balmer Series of Hydrogen – Determination of the Rydberg Constant

(Reference: Mansfield and O’Sullivan pp. 572–579)

Observing that atoms only emit light at definite spectral frequencies demonstrates electrons in atoms only populate distinctive (quantized) energy levels. In the Bohr model of the atom radiation is emitted anytime an electron makes a transition from a higher lying energy level into a lower lying one. For the hydrogen atom it turns out that for some transitions to the  $n = 2$  level from higher lying levels the wavelength of the radiation emitted is in the visible range. The series is called the Balmer series.

## Theory

Bohr’s third postulate states that electromagnetic radiation is produced whenever an electron makes a transition from an energetically higher state into a lower state. The frequency of the radiation produced in the transition is proportional to the energy difference between these two states

$$\nu = \frac{E_i - E_f}{h} = \frac{E_0}{h} \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right), \quad (2.1)$$

where  $n_i$  and  $n_f$  are the quantum number associated with the initial and the final state, respectively. Rewriting this relation in terms of the wavelength, gives the definition of the Rydberg constant

$$\frac{1}{\lambda} = R_\infty \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right), \quad (2.2)$$

with

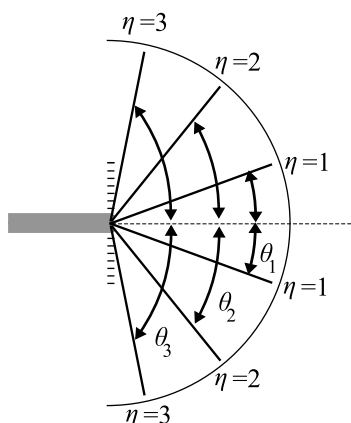
$$R_\infty = \frac{m_e e^4}{64\pi^3 \hbar^3 \epsilon_0^2 c}. \quad (2.3)$$

The derivation of eq. (2.3) however assumes that the mass of the nucleus is infinitely large, which is of course not the case. Therefore the experimental value of the Rydberg constant has to be adjusted to take into account the finite mass of the nucleus. This is done by replacing the electron mass by an reduced mass via

$$m_r = \frac{m_e m_n}{m_e + m_n} = \frac{1836}{1837} m_e , \quad (2.4)$$

so that the experimentally measured Rydberg constant for hydrogen is given by

$$R_H = \frac{1836}{1837} R_\infty . \quad (2.5)$$



To measure the wavelength of the emitted light we use diffraction of light at a transmission grating. If parallel light of wavelength  $\lambda$  is incident along the normal of a diffraction grating, then the transmitted light exhibits a diffraction pattern which consists of a number of spatially separated intensity maxima at angles  $\theta$  given by

$$d \sin \theta = \eta \lambda , \quad (2.6)$$

where  $\eta$  is defined as the order of the maximum and  $d$  is the spacing between the grating rulings. Therefore knowing  $d$  and measuring  $\theta$  for a each  $\eta$  allows the determination of the wavelengths in the spectrum.

## Procedure

Measure the wavelength of the lines of the H Balmer series as accurately as possible, using measurements of images on both sides of the straight-through position of the spectrometer and in as many orders  $\eta$  as are observable.

## The spectrometer

The set up and adjustment of the spectrometer are described below. There is no need to measure  $d$ , the spacing between successive ruling of the diffraction grating, as this can be calculated from the ruling density of the grating (which is 600 lines per mm).

**Please take care not to touch the ruled area of the grating!**

## Adjustment of the spectrometer

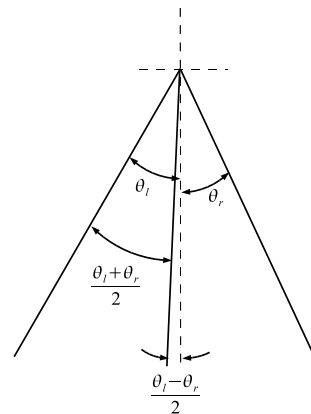
**Eye-piece:** Place a piece of well-illuminated white paper in front of the telescope and adjust the eye piece until the cross-hairs are clearly in sharp focus. Rotate the telescope barrel until one of the cross-hairs is vertical. The eye-piece should not be re-adjusted during the experiment.

**Telescope:** Turn the telescope towards some distant object. Adjust the length of the telescope barrel until the distant object is in sharp focus with no parallax between the image of the cross-hairs and the image of the distant object. The telescope is now focused (i.e. a beam of parallel light will come to focus on the cross-hairs) and should not be re-adjusted

**Collimator:** Illuminate the slit of the collimator with light from the sodium lamp. It is helpful if the slit is opened fairly wide initially. Turn the telescope into line with the collimator and view the image of the slit. Vary the distance between the slit and the lens of the collimator until the image of the slit is in sharp focus with no parallax between it and the image of the cross-hairs. Adjust the width of the slit to get a narrow image which should be parallel to the vertical crosshair. These adjustments ensure that the light emerges from the collimator as a parallel beam.

When the slit is observed directly with the telescope, its image should lie fairly centrally in the field of view. Place the grating on the spectrometer table and observe the diffracted images on both sides of the straight-through position. If necessary, adjust the leveling screws on the spectrometer table so that the tops and bottoms of the images are horizontal. This ensures that the lines of the grating are now parallel to the slit.

It is now necessary to set the grating at right angles to the ray from the collimator. Note the straight-through position and then turn the telescope to observe a suitable order image on the right-hand side and find the angle  $\theta_r$  between its position and the straight-through position. Doing the same for the same order image on the left-hand side will give an angle  $\theta_l$ . If  $\theta_r$  is equal to  $\theta_l$  then the grating is at right angle to the rays. If not, and  $\theta_l$  is larger than  $\theta_r$ , then rotate the turntable through an angle  $\frac{\theta_l - \theta_r}{2}$  (see diagram). Then repeat the procedure and the new values of the angles should be much



more equal. Repeat the procedure until the different order images are symmetrically situated relative to the straight-through position.

## Analysis

Calculate the wavelength for each order of the spectrum you have observed and determine the average value and the error bar (assume errors in  $d$  and  $\theta$ ). Plot a graph of  $\frac{1}{\lambda}$  against  $\frac{1}{n_i^2}$ , where  $n_i = 3$  for the red line,  $n_i = 4$  for the blue-green line,  $n_i = 5$  for the violet line and  $n_i = 6$  for the deep violet line (which may just be visible). The gradient of this graph gives you the value for the Rydberg constant (see eq. (2.2)). The integer  $n_f$  in eq. (2.2) can be determined from the intercept in the  $\frac{1}{n_i^2}$  axis, i.e. when  $\frac{1}{\lambda} = 0$  and  $\frac{1}{n_i^2} = \frac{1}{n_f^2}$ .

Compare your result for  $R_H$  to the theoretical value from eq. (2.5) using  $m_e = 9.109 \times 10^{-31}\text{kg}$ ,  $e = 1.602 \times 10^{-19}\text{C}$ ,  $\epsilon_0 = 8.854 \times 10^{-12}\text{F/m}$ ,  $\hbar = 1.055 \times 10^{-34}\text{Js}$  and  $c = 2.998 \times 10^8\text{m/s}$ .

Use your measured value of  $R_H$  to calculate the wavelength of the series limit of the H Balmer series (i.e. corresponding to  $n_i = \infty$ ). Then use the equation

$$E = hf = \frac{hc}{\lambda}, \quad (2.7)$$

to determine the corresponding transition energy.

**Note:** Your measurements have been made in air. To achieve a better agreement between the experimental and theoretical value, you should divide your result of  $R_H$  by the refractive index of air before comparing with theory (which applies to vacuum).

$$\frac{c_{\text{vac}}}{c_{\text{air}}} = n = 1.000292 \quad (\text{refractive index of air}) \quad (2.8)$$

$$\Rightarrow \frac{c_{\text{vac}}}{f} = n \frac{c_{\text{air}}}{f} \quad (2.9)$$

$$\Rightarrow \lambda_{\text{vac}} = n \lambda_{\text{air}} \quad (2.10)$$