OLLSCOIL NA hEIREANN, CORCAIGH
THE NATIONAL UNIVERSITY OF IRELAND, CORK

COLAISTE NA hOLLSCOILE, CORCAIGH
UNIVERSITY COLLEGE, CORK

SUMMER EXAMINATION 2006

Process Engineering I
Civil and Environmental Engineering I
Microelectronic Engineering I

PY1007: Physics for Engineers I

Prof. I. McGovern
Prof. J. McInerney
Dr. Th. Busch

Solutions

Question 1

(a) Kinetic energy, potential energy and heat energy

$$E_{kin} = \frac{1}{2}mv^2 \qquad , \qquad E_{grav} = mgh \tag{1}$$

[3 marks]

(b) The initial velocity is given by $v_0 = 6.7\ \text{ms}^{-1}$ and the angle is given by $\alpha = 45°$.

$$x = v_0 \cos(\alpha)t \tag{2}$$

$$y = v_0 \sin(\alpha)t - \frac{1}{2}gt^2 \tag{3}$$

The vertical height distance is 1 m, therefore we get

$$1\ \text{m} = v_0 \sin(45)t - \frac{1}{2}gt^2 \tag{4}$$

$$\Rightarrow \qquad t = \frac{\frac{2}{g}v_0 \sin(\alpha) \pm \sqrt{\frac{4}{g^2}v_0^2 \sin^2(\alpha) - 4\frac{2}{g}}}{2} = 0.483\ \text{s} \pm 0.171\ \text{s} \tag{5}$$

This describes a parabolic curve and since we are looking for the situation when the ball moves downward, the larger solution is the one of interest, $t = 0.654$ s. During that time the ball has travelled the following x-distance

$$x = v_0 \cos(\alpha)t = 3.1\ \text{m} \tag{6}$$

[4 marks]

(c) Since energy has to be conserved we have

$$\frac{1}{2}mv_i^2 = \frac{1}{2}mv_f^2 + mgh \tag{7}$$

$$\Rightarrow \qquad v_f = \sqrt{v_i^2 - 2gh} = 5.03 \text{ ms}^{-1} \tag{8}$$

[3 marks]

OR

(a) The equation of conservation of momentum in both spatial directions are given by

$$mv_0 + 0 = mv_1 \cos\theta_1 + 2mv_2 \cos\theta_2 \qquad \text{for } p_x \tag{9}$$

$$0 + 0 = mv_1 \sin\theta_1 - 2mv_2 \sin\theta_2 \qquad \text{for } p_y \tag{10}$$

and the equation for the energy is given by

$$\frac{1}{2}mv_1^2 + \frac{1}{2}2mv_2^2 = \frac{1}{2}\left[\frac{1}{2}mv_0^2\right] \tag{11}$$

[3 marks]

(b) From the equation for the energy we know

$$4v_2^2 = v_0^2 - 2v_1^2 \qquad \text{and from the question} \tag{12}$$

$$4v_2^2 = v_0^2 - 2v_0 v_1 \cos\theta_1 + v_1^2 \tag{13}$$

Equating the right hand sides gives therefore

$$3v_1^2 = 2v_0 v_1 \cos\theta_1 \tag{14}$$

$$\Rightarrow \qquad v_1 = \frac{2}{3}v_0 \cos\theta_1 = \frac{2}{3}\frac{3\sqrt{2}}{5}v_0 = \frac{2\sqrt{2}}{5}v_0 \tag{15}$$

[4 marks]

(c) From the equation given in part (b) we get

$$4v_2^2 = v_0^2 - 2v_0\frac{2\sqrt{2}}{5}v_0\frac{3\sqrt{2}}{5} + \frac{8}{25}v_0^2 \tag{16}$$

$$= v_0^2\left(1 - \frac{24}{25} + \frac{8}{25}\right) \tag{17}$$

$$= \frac{9}{25}v_0^2 \tag{18}$$

$$\Rightarrow \qquad v_2 = \frac{3}{10}v_0 \tag{19}$$

[3 marks]

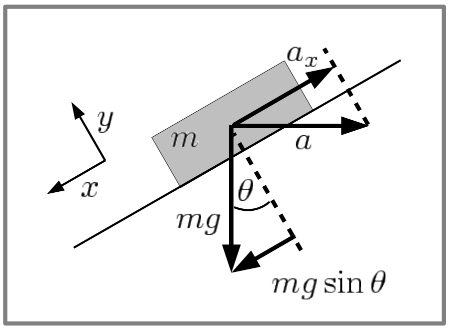
Question 2

(a) The *inward* force given by gravity has to be compensated be the *outward* force due to radial acceleration

$$mg\sin\theta = ma_x = m\frac{v^2}{r}\cos\theta \tag{20}$$

The angle is therefore given by

$$\theta = \arctan\left(\frac{v^2}{gr}\right) = 77^\circ \tag{21}$$



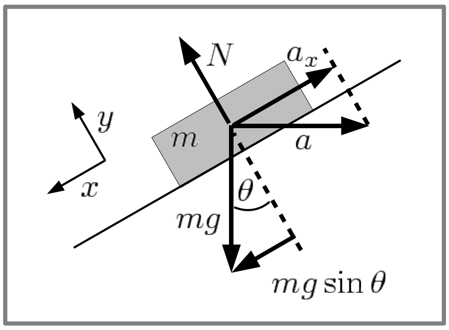
[3 marks]

(b) The friction force, F_R, compensates partly for the gravitational force

$$mg\sin\theta - F_R = ma_x \tag{22}$$

and is given by

$$F_R = \mu_R N \tag{23}$$



Therefore

$$mg\sin\theta - \mu_R(mg\cos\theta) = m\frac{v_{\max}^2}{r}\cos\theta \tag{24}$$

$$\Rightarrow \qquad v_{\max} = \sqrt{\frac{rg(\sin\theta - \mu_R\cos\theta)}{\cos\theta}} = 75\ \text{ms}^{-1} = 270\ \text{km h}^{-1} \tag{25}$$

[4 marks]

(c) Conservation of energy means that energy can neither created or destroyed. In the case of a car this means that its kinetic energy is transformed into heat energy via the friction of the tires on the ground. **[3 marks]**

OR

(a) (i) The stone makes 0.4 revolutions per second, which corresponds to 2.51 radians per second

$$v_{tan} = r\omega = (0.75\ \text{m})(2.51\ \text{rad s}^{-1}) = 1.9\ \text{ms}^{-1} \tag{26}$$

(ii)

$$\omega = \omega_0 + \alpha t \tag{27}$$

$$2.51\ \text{rad s}^{-1} = 0 + \alpha(2\ \text{s}) \qquad \Rightarrow \qquad \alpha = 1.26\ \text{rad s}^{-2} \tag{28}$$

(iii)

$$a_c = r\omega^2 = (0.75\ \text{m})(2.51\ \text{rad s}^{-1})^2 = 4.73\ \text{ms}^{-2} \tag{29}$$

[4 marks]

(b) The string breaks when

$$mr\omega^2 = 11\ \text{N} \tag{30}$$

$$(1.3\ \text{kg})(0.75\ \text{m})\omega^2 = 11\ \text{N} \tag{31}$$

$$\omega = 3.36\ \text{rad s}^{-1} \tag{32}$$

Therefore the time is given by

$$\omega = \omega_0 + \alpha t \tag{33}$$

$$3.36\ \text{rad s}^{-1} = 0 + 1.26\ \text{rad s}^{-2} t \qquad \Rightarrow \qquad t = 2.67\ \text{s} \tag{34}$$

[3 marks]

(c) The angular momentum is given by

$$|L| = |r \times p| = |r \times (mv)| = m|rv|\sin\theta = 1.85\ \text{kg m}^2\ \text{s}^{-1} \tag{35}$$

where $\theta = 90°$ is the angle between the radius vector and the tangential velocity vector. **[3 marks]**

Question 3

(a) The expression for a simple harmonic, traveling wave in one direction is given by

$$y(x,t) = A\sin(\omega t - kx + \phi) \tag{36}$$

where A is the amplitude, ϕ is the phase, ω is the frequency and k is the wavenumber. **[4 marks]**

(b) Dispersion means that the speed of a travelling wave depends on its frequency

$$v = f\lambda = \left(\frac{\omega}{2\pi}\right)\left(\frac{2\pi}{k}\right) = \frac{\omega}{k} \tag{37}$$

[3 marks]

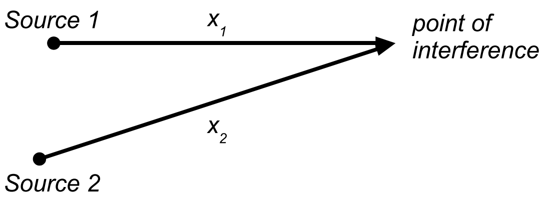
(c) The superposition principle says that two waves impinging on one point have to be added.

$$y = y_1 + y_2 = A\sin(\omega t - kx_1 + \phi) + A\sin(\omega t - kx_2 + \phi) \tag{38}$$

The conditions for constructive and destructive interference are

$$(x_1 - x_2) = \left(n + \frac{1}{2}\right)\lambda \qquad \text{destructive} \tag{39}$$

$$(x_1 - x_2) = n\lambda \qquad \text{constructive} \tag{40}$$



[3 marks]

OR

(a) The First Law of Thermodynamics is given by

$$\Delta Q = \Delta U - W = \Delta U - p(\Delta V) \tag{41}$$

and it describes the fact that the change in internal energy is given by the heat added *to* the system minus the work done *by* the system. **[3 marks]**

(b) The average kinetic energy and the temperature are related by

$$\langle E_{\mathrm{kin}} \rangle = \frac{3}{2} N k_B T = 6.21 \times 10^{-21} J \tag{42}$$

and the average velocity

$$v = \sqrt{\frac{2E_{\mathrm{kin}}}{m_{He}}} = 1367 \ \mathrm{ms}^{-1} \tag{43}$$

[4 marks]

(c) The equipartition theorem says that the internal energy of a particle is divided equally between all degrees of freedom. A diatomic molecule has 7 degrees of freedom: three spatial, two rotational and two vibrational. **[3 marks]**