PY3101 Optics

Huygens-Fresnel Principle

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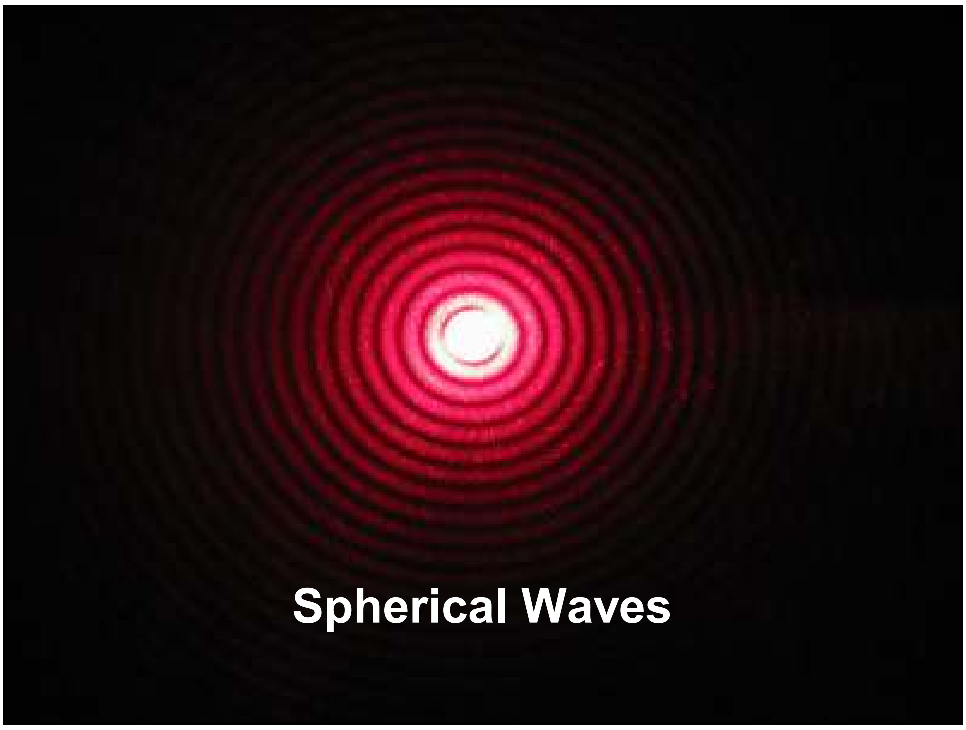
Learning objectives

- The formulation of spherical waves
- Huygens' Principle
- Interference and coherence
- The Huygens-Fresnel Principle
- Application of these principles to find:
  - The Law of Rectilinear Propagation
  - The Law of Reflection
  - The Law of Refraction (Snell's Law)



Spherical waves

In this case, it is convenient to use spherical polar coordinates. The wave vector **k** has constant magnitude and always points away from the centre of radiation. Thus we put

$$\mathbf{k} = k\mathbf{e}_r$$

and

$$\mathbf{r} = r\mathbf{e}_r$$

so that the equation for surfaces of constant phase becomes

$$kr - \omega t = \phi_0 .$$

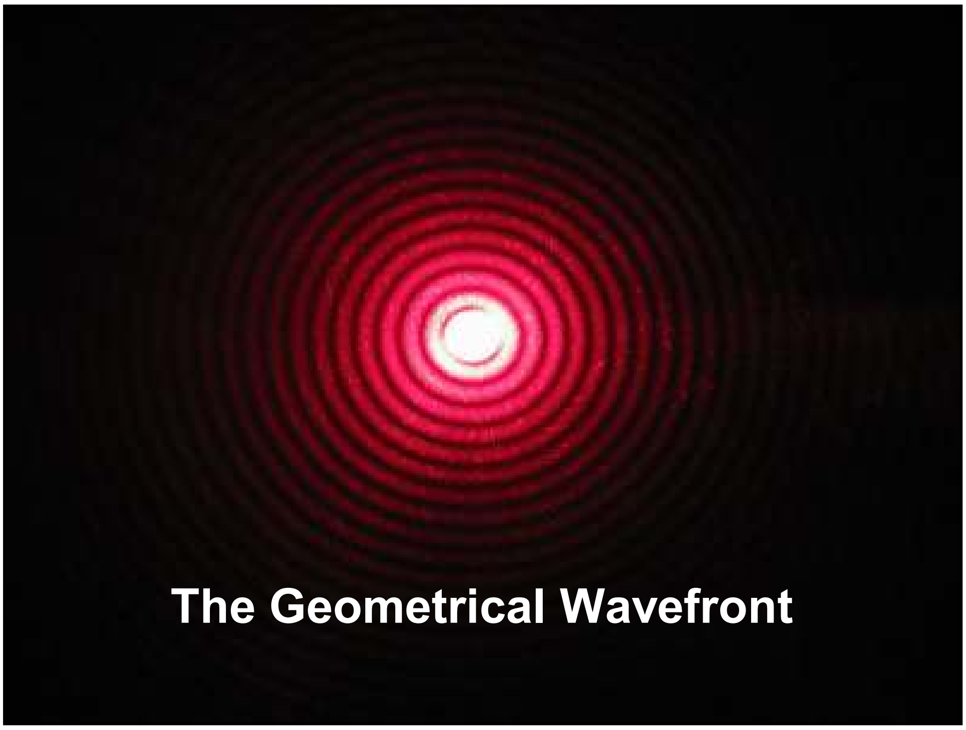
Spherical waves

Since the intensity of an EM wave is proportional to the squared modulus of the amplitude, by the **conservation of energy**, the amplitude must vary as $1/r$.

Moreover, the requirement that the amplitude be finite at $r = 0$ means that the spherical wave must be of the form

$$E(r,t) = \frac{E_r}{r} e^{-i\omega t} \sin kr.$$

Spherical waves

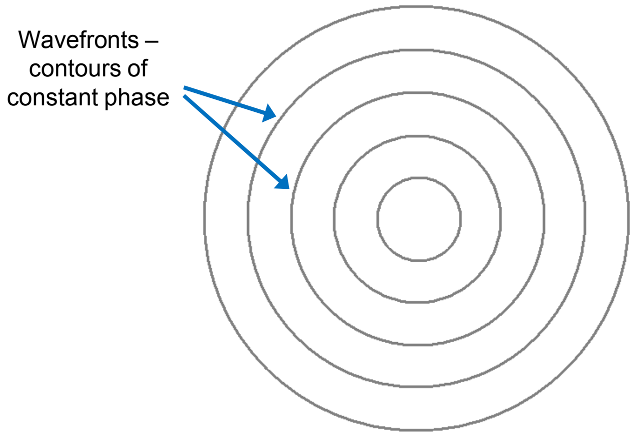


Geometric wavefront

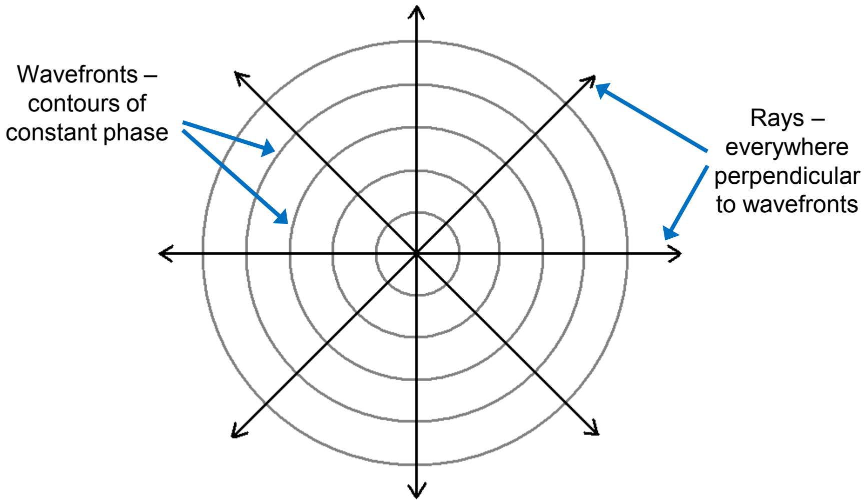
> *A* ***geometric wavefront*** *is the surface in space containing all points in an optical field that have the same* ***phase****.*

> *A* ***ray*** *is a path through space that is everywhere perpendicular to the wavefront.*

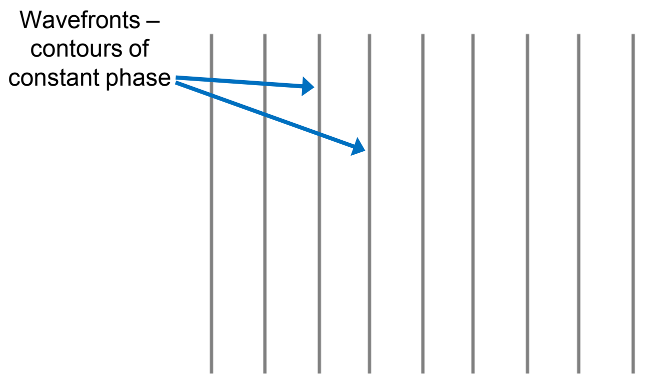
Geometric wavefront - spherical



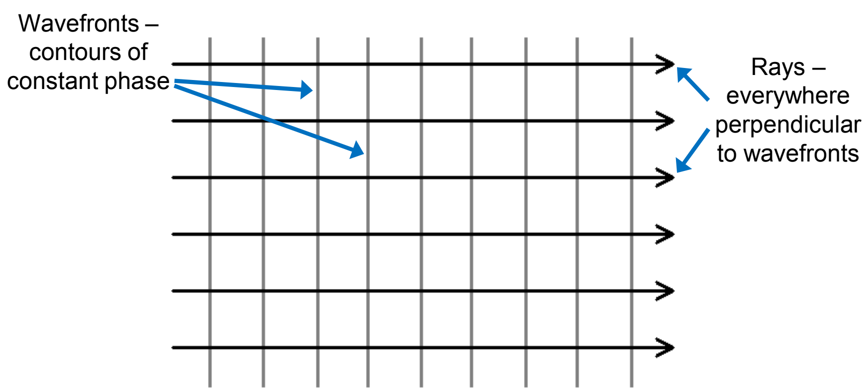
Geometric wavefront - spherical

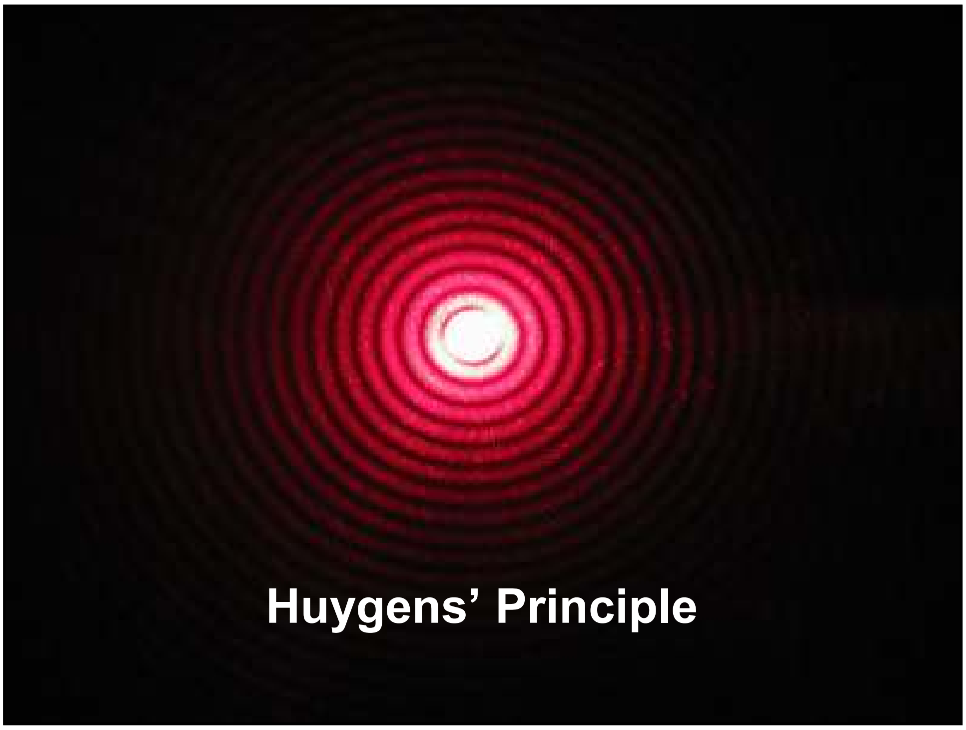


Geometric wavefront - plane



Geometric wavefront - plane





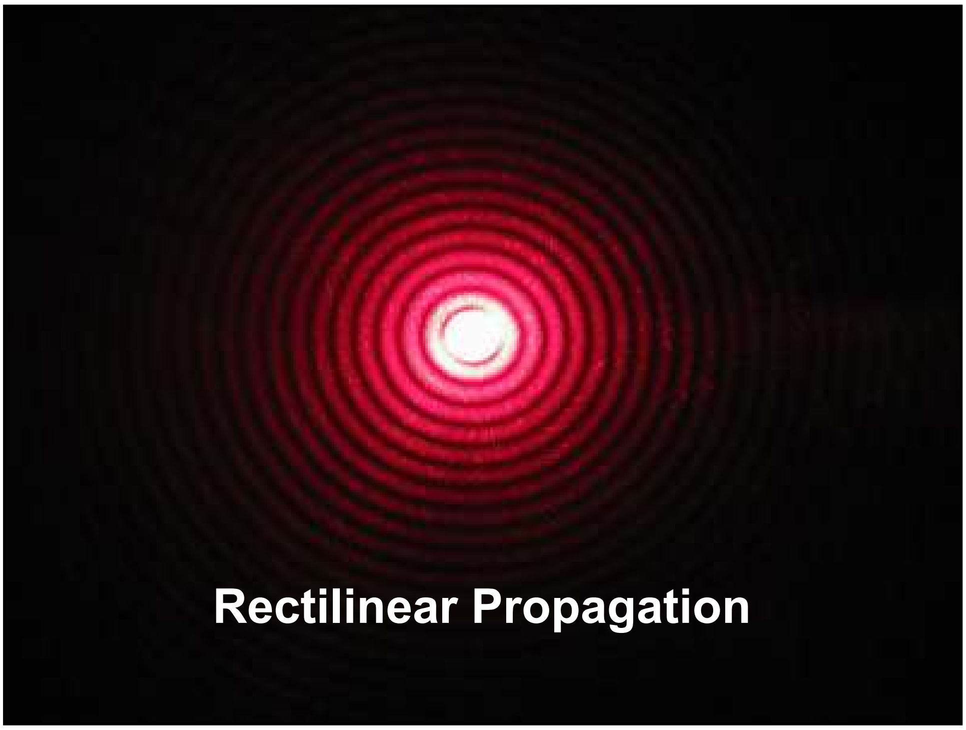
Huygens' Principle

Each point on a wavefront acts as a source of secondary, spherical wavelets.

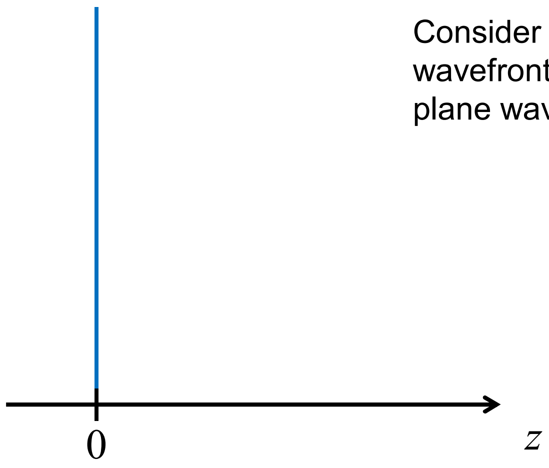
At a later time, t, a new wavefront is constructed from the sum of these wavelets.

Huygens' Principle

- **May be used to derive:**
  - **The Law of Rectilinear Propagation**
  - **The Law of Reflection**
  - **The Law of Refraction**

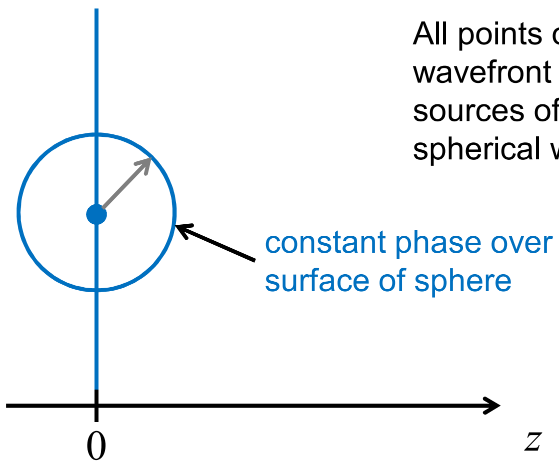


Huygens' Principle – rectilinear propagation



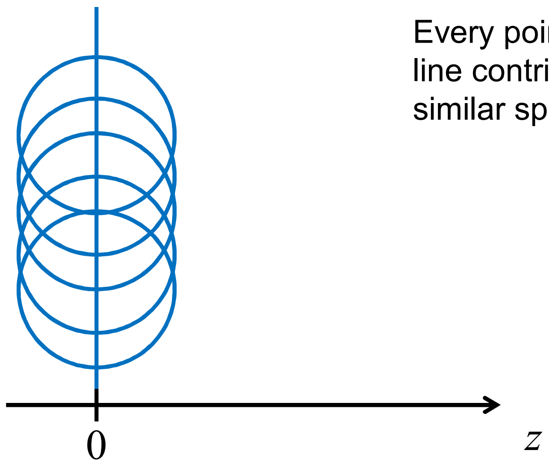
Consider the wavefront of a plane wave at $z = 0$

Huygens' Principle – rectilinear propagation



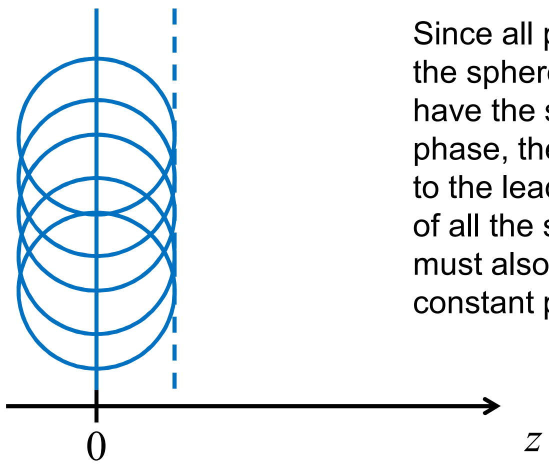
All points on the wavefront act as sources of spherical wavelets

Huygens' Principle – rectilinear propagation



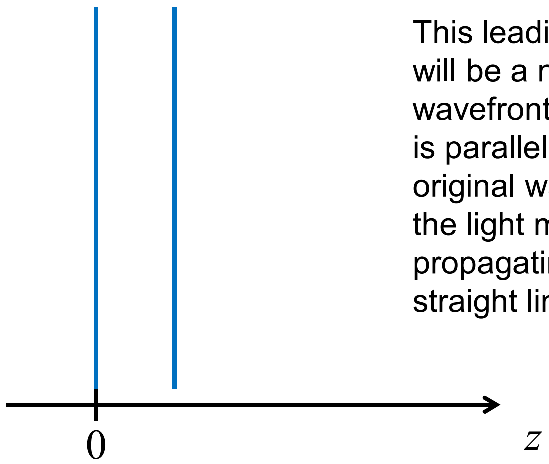
Every point on the line contributes a similar sphere

Huygens' Principle – rectilinear propagation



Since all points on the spheres must have the same phase, the tangent to the leading edge of all the spheres must also be at a constant phase.

Huygens' Principle – rectilinear propagation



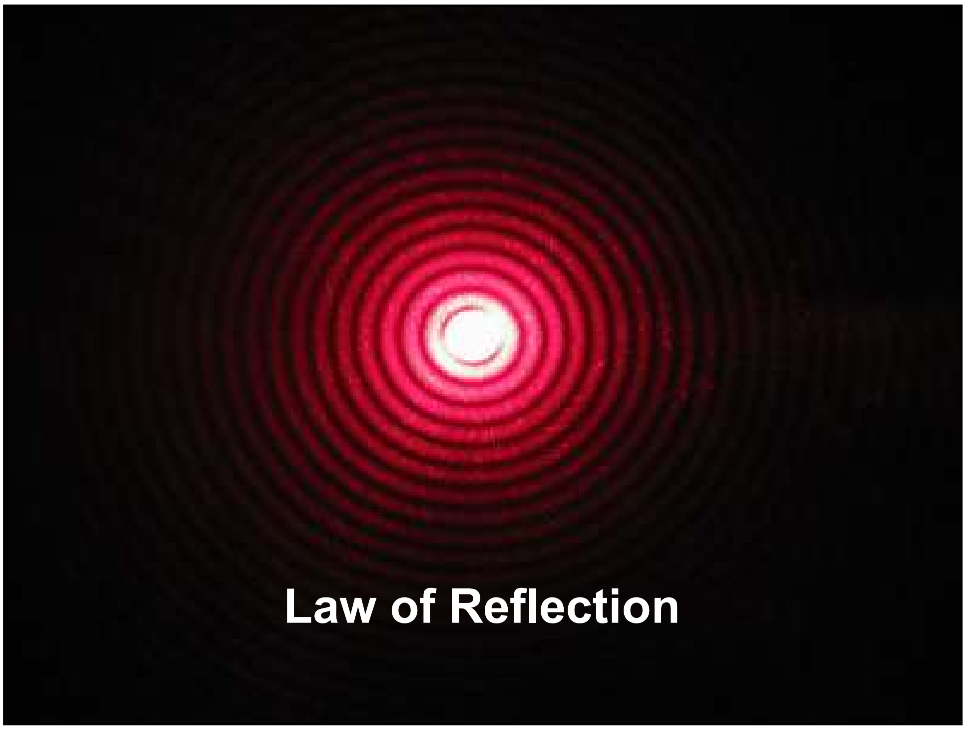
This leading edge will be a new wavefront. Since it is parallel to the original wavefront, the light must be propagating in a straight line.

Huygens' Principle – rectilinear propagation

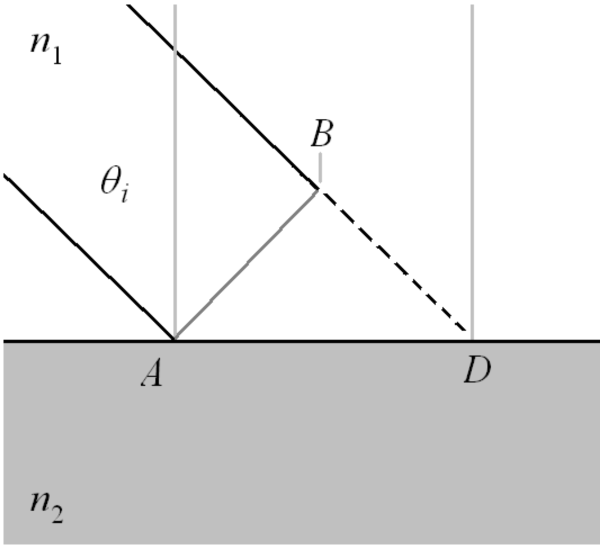
We have used Huygens' Principle to prove the Law of Rectilinear Propagation for a plane wave.

Note: **we did not invoke the Principle of Superposition to prove this result.**

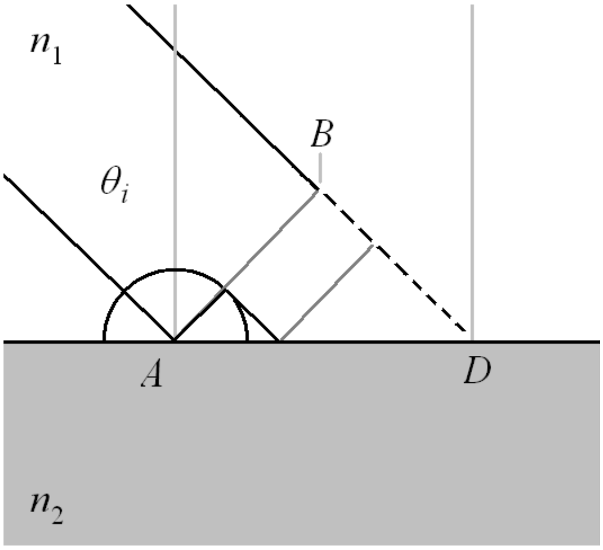
When the Principle of Superposition is explicitly added to Huygens' Principle, it is becomes the **Huygens-Fresnel Principle**.



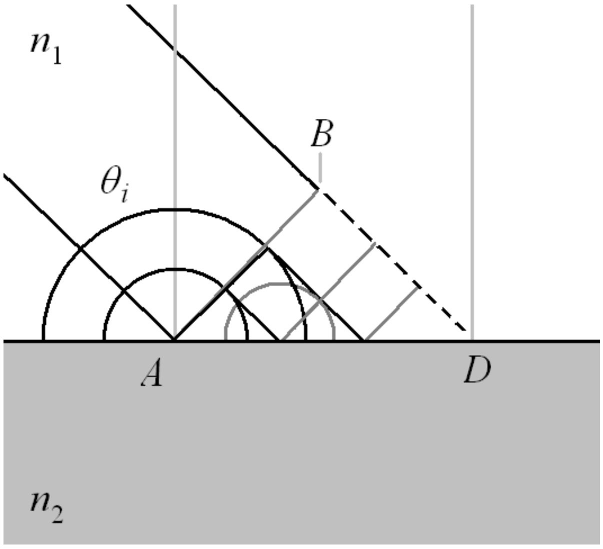
Huygens' Principle - reflection



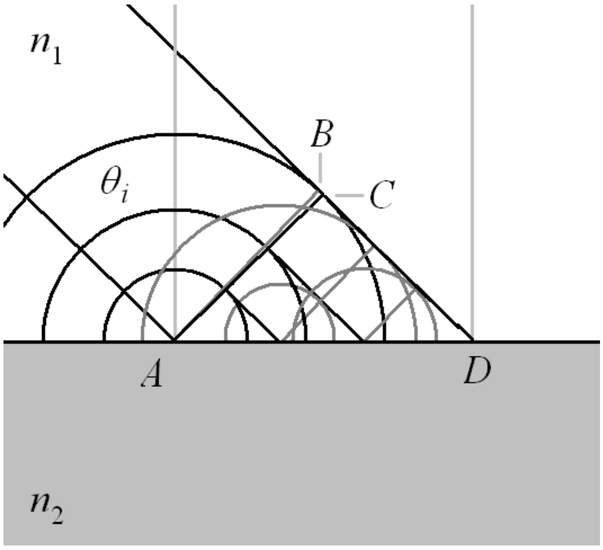
Huygens' Principle - reflection



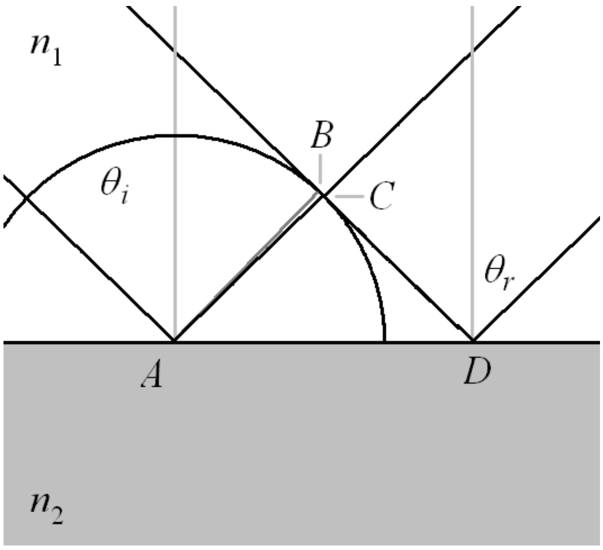
Huygens' Principle - reflection



Huygens' Principle - reflection



Huygens' Principle - reflection



Huygens' Principle - reflection

In a homogeneous medium

$$AC = BD$$

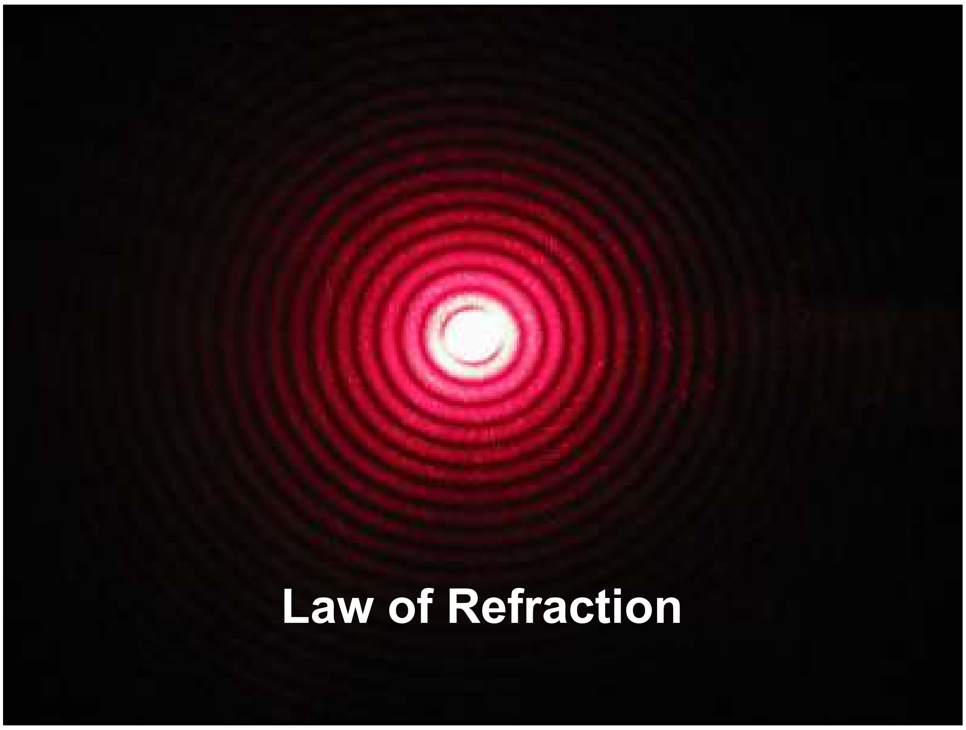
$$BD = AD\sin\theta_i$$

$$AC = AD\sin\theta_r$$

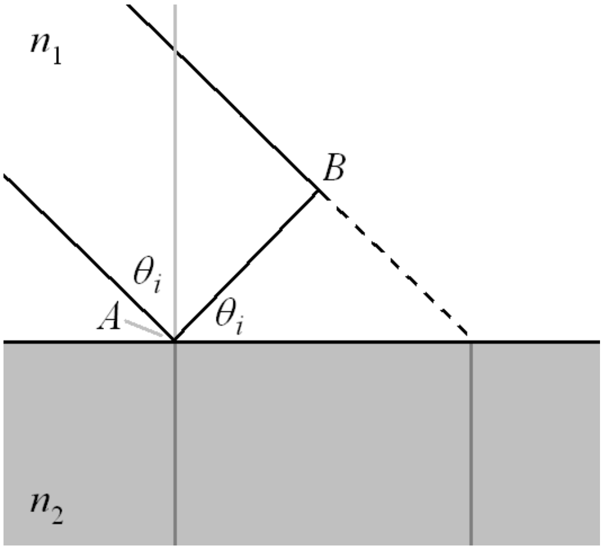
So

$$\theta_i = \theta_r$$

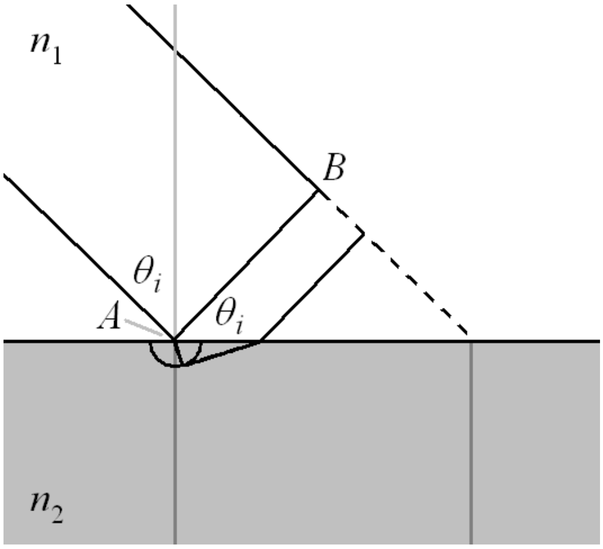
(Details given in the notes in §**4.5.2**)



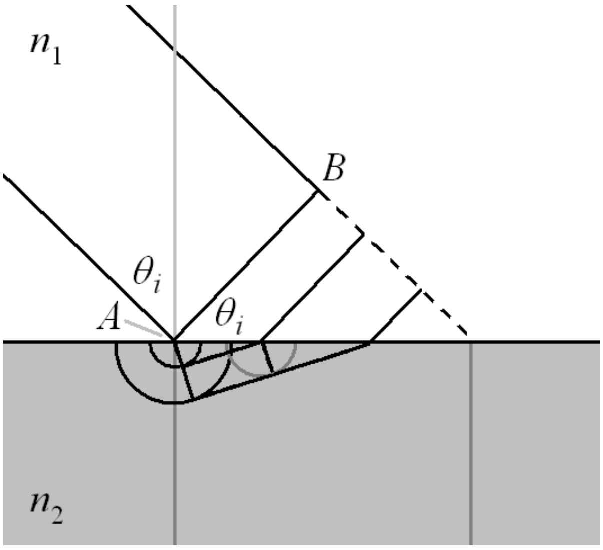
Huygens’ Principle - refraction



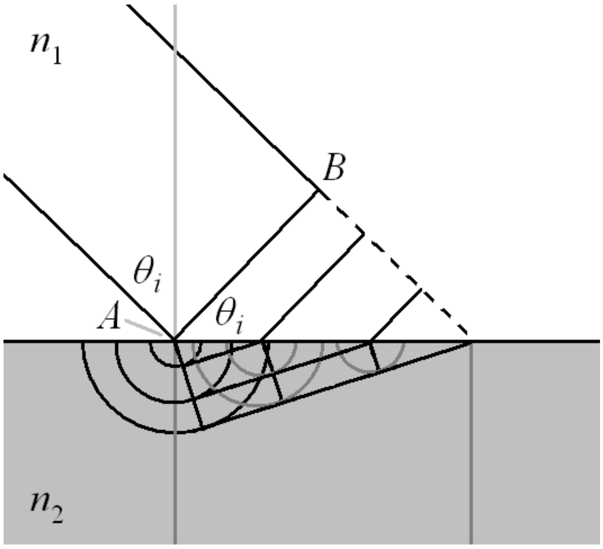
Huygens’ Principle - refraction



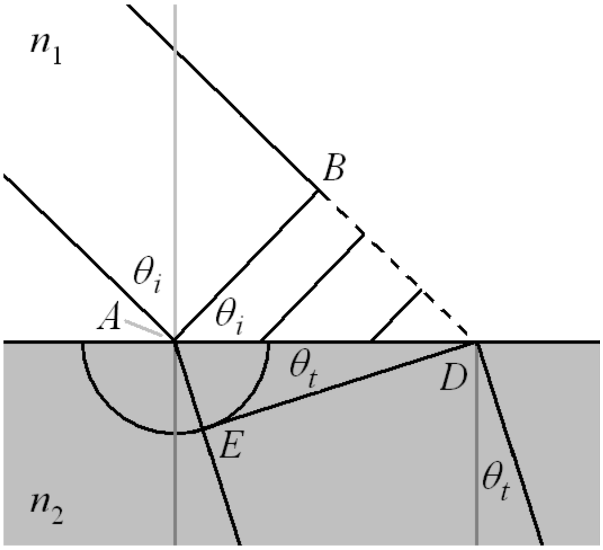
Huygens' Principle - refraction



Huygens' Principle - refraction



Huygens' Principle - refraction



Huygens' Principle - refraction

If the time the wavefront takes to propagate from B to D is T, then

$$AE = \frac{cT}{n_2} = AD \sin\theta_t$$

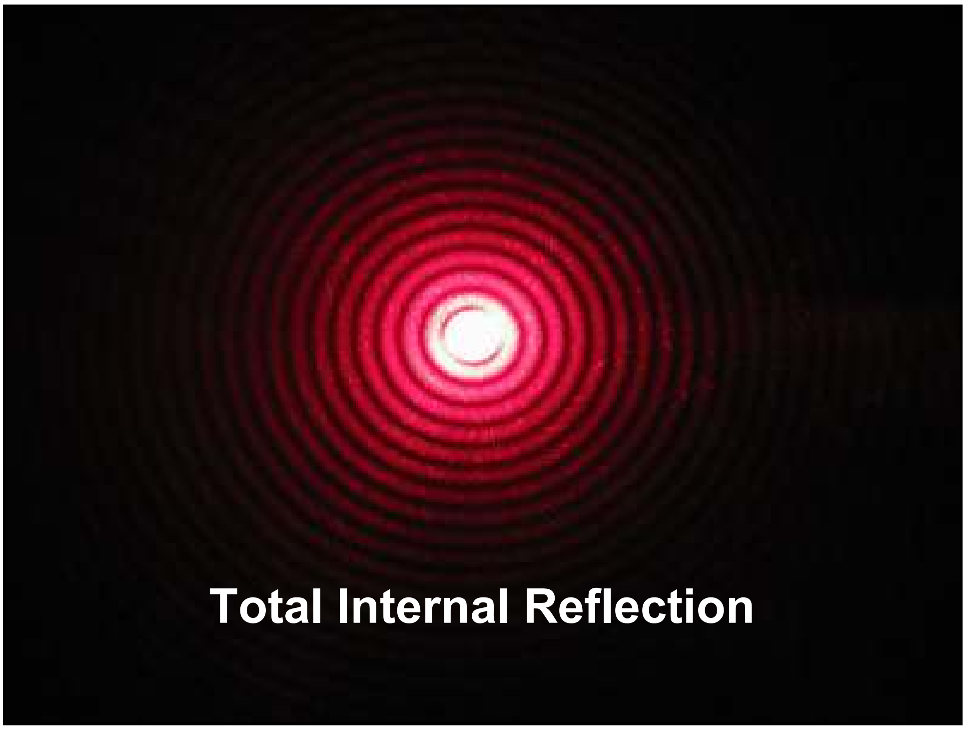
and

$$BD = \frac{cT}{n_1} = AD \sin\theta_i$$

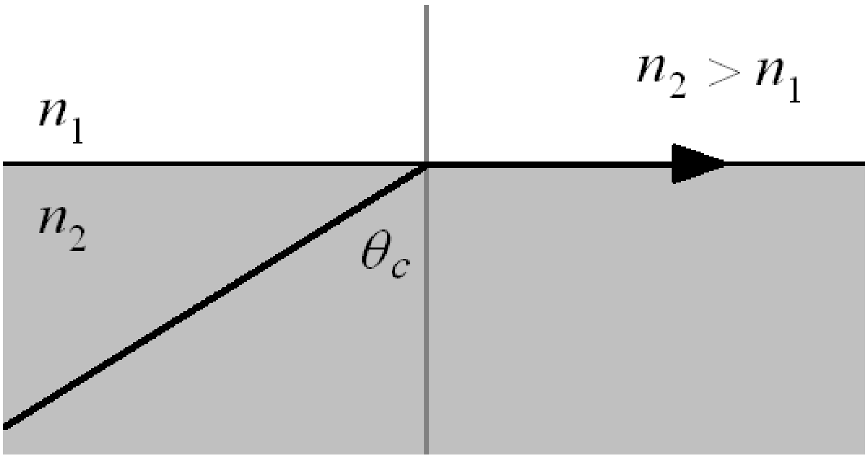
So

$$n_1 \sin\theta_i = n_2 \sin\theta_t \qquad \text{(Snell's Law)}$$

(Details given in the notes in §**4.5.3**)



The Critical Angle



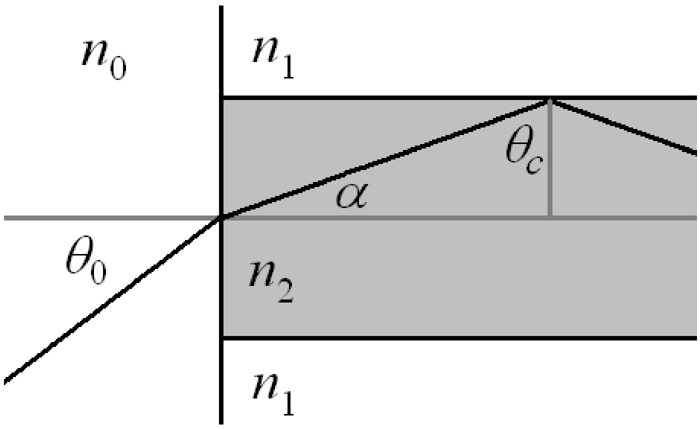
The Critical Angle

$$n_1 \sin\theta_1 = n_2 \sin\theta_C$$ Snell's Law

$$\theta_1 = \frac{\pi}{2}$$ Condition

$$\theta_C = \sin^{-1}\left(\frac{n_1}{n_2}\right)$$ Critical angle

The slab waveguide



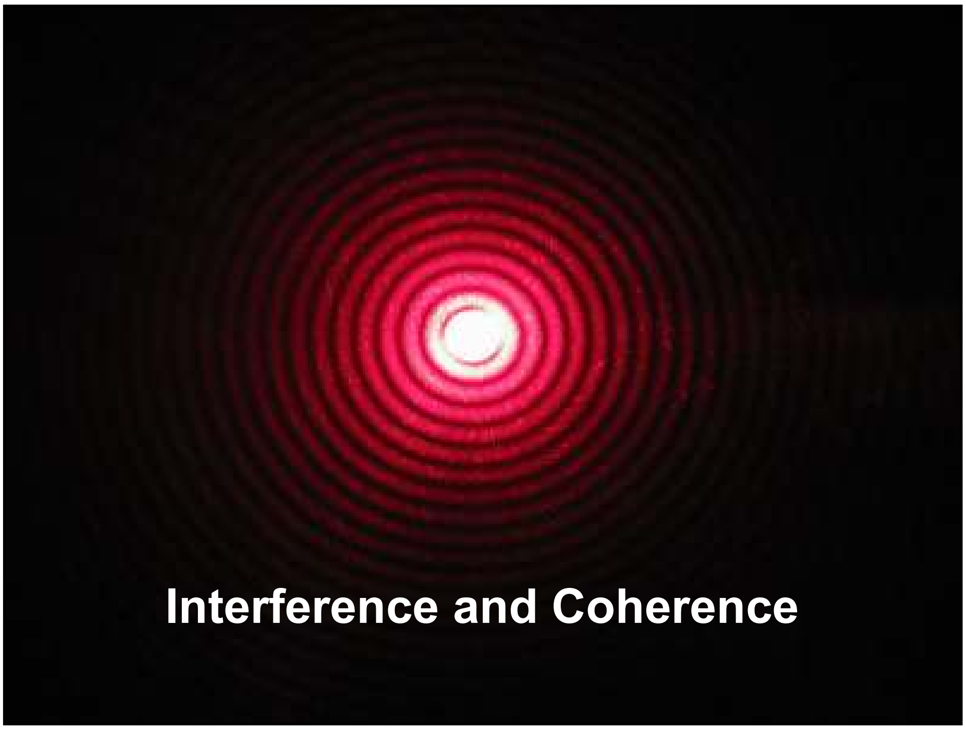
Numerical aperture (NA)

$$NA = n_0 \sin \theta_0$$

It can be shown that

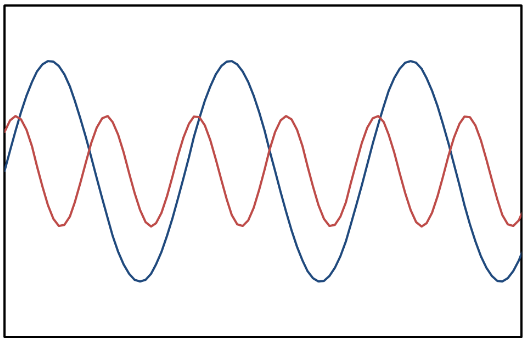
$$NA = \left(n_2^2 - n_1^2\right)^{1/2}$$

(Details given in the notes in §**4.6.2**)



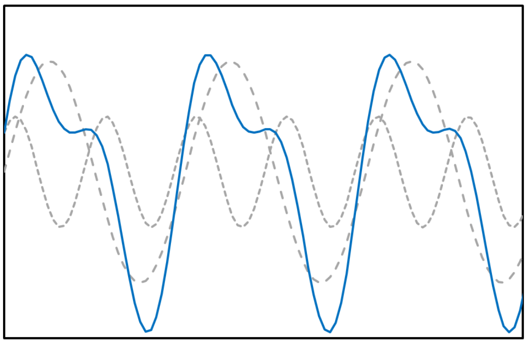
Coherence

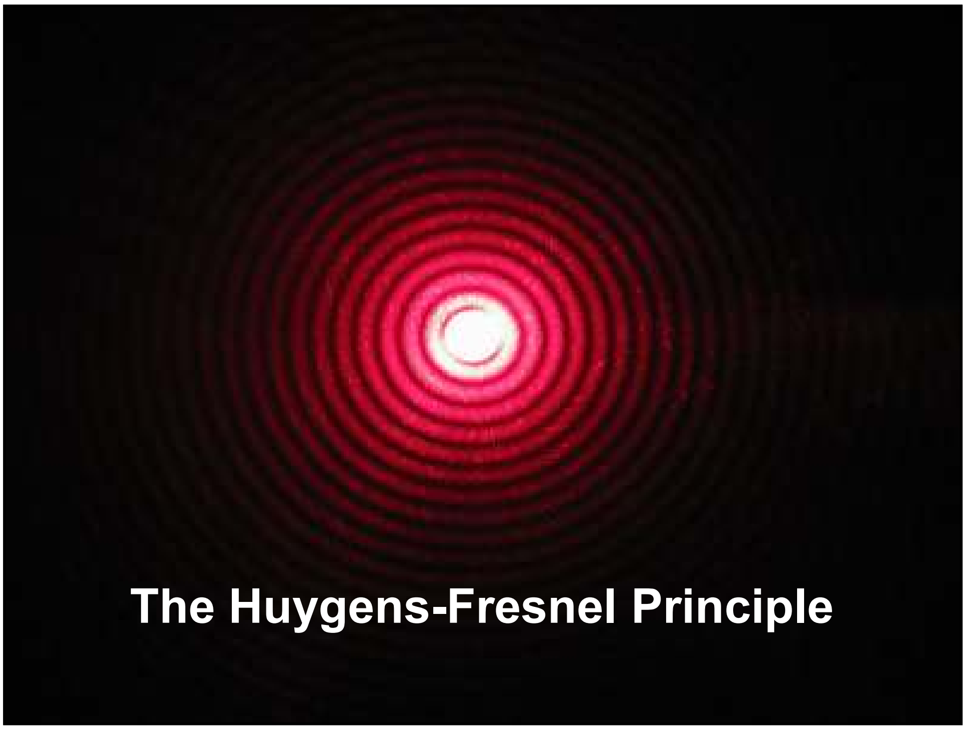
If two beams of light are **coherent** *with each other, then there is a* **fixed relation between their phases**



Interference

Two **coherent** *beams may add together via the* **Principle of Linear Superposition** *to obtain* **interference**





The Huygens-Fresnel Principle

For light of a given frequency, every point on a wavefront acts as a secondary source of spherical wavelets with the same frequency and the same initial phase.

The wavefront at a later time and position is then the linear superposition of all of these wavelets.

Rectilinear propagation

Consider a plane wave with amplitude E_0 at $z = 0$. According to the Huygens-Fresnel principle, every point on this plane acts as a source of secondary spherical wavelets. We shall take the form of these wavelets to be

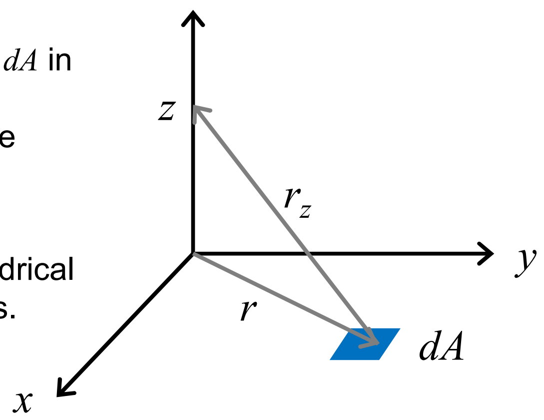
$$E(r,t) = \frac{E_0 k}{2\pi r} e^{-i\omega t} \sin kr.$$

We shall now consider the superposition of all these wavelets at a point a perpendicular distance z from the plane.

Rectilinear propagation

Surface element dA in the $z = 0$ plane contributing to the amplitude at z.

Note use of cylindrical polar coordinates.



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Rectilinear propagation

The amplitude at z along the z-axis will be

$$E(z,t) = \frac{E_0 k}{2\pi} e^{-i\omega t} \int \frac{1}{r_z} \sin kr_z \, dA,$$

where

$$r_z = \left(x^2 + y^2 + z^2\right)^{1/2}.$$

In cylindrical polar coordinates, this becomes

$$E(z,t) = E_0 k e^{-i\omega t} \int_0^\infty \frac{r}{r_z} \sin kr_z \, dr.$$

Rectilinear propagation

Changing the variable of integration, we may use

$$r_z dr_z = r dr$$

to obtain

$$E(z,t) = E_0 k e^{-i\omega t} \int_z^\infty \sin kr_z \, dr_z.$$

Hence we have

$$E(z,t) = E_0 e^{-i\omega t} \cos kz,$$

i.e. **a plane wave travelling in the z direction**.