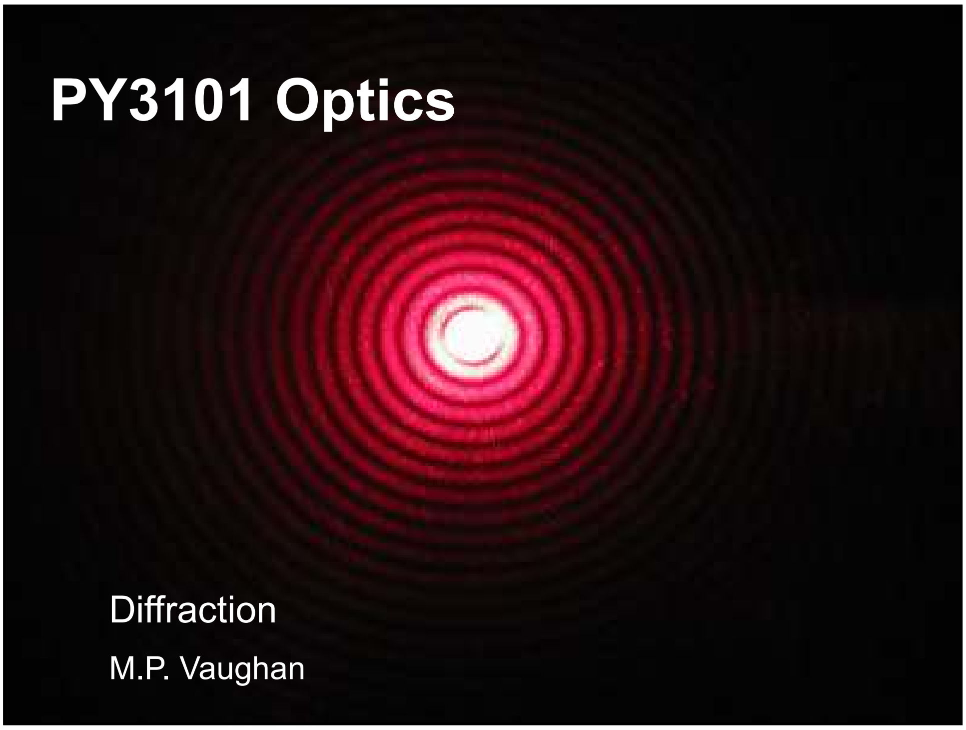
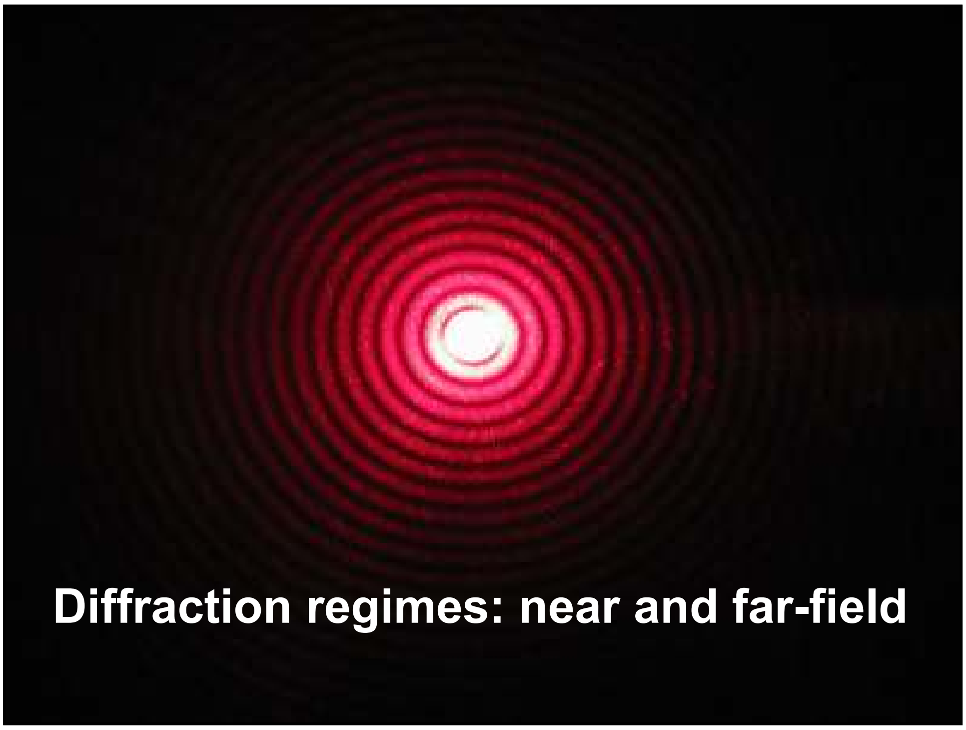


Overview

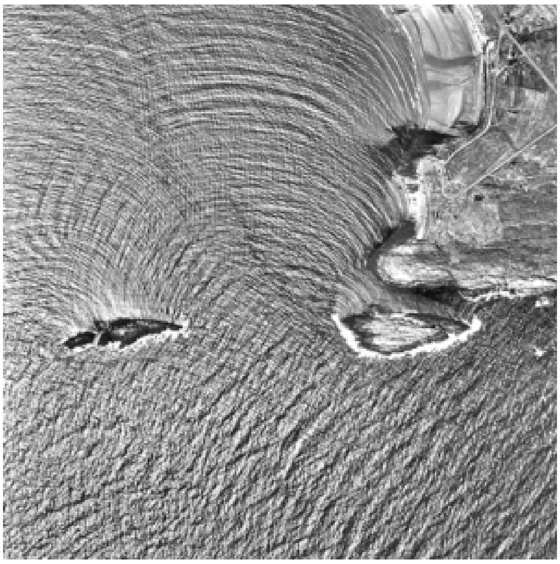
- **Near and far-field diffraction**
  - **Diffraction through a single slit**
  - **Diffraction limited imaging**
  - **Multiple slit diffraction**
  - **The diffraction condition**
- **Diffraction gratings**
  - **Reflection gratings**
  - **Monochromators**
- **Diffraction around objects**



What is diffraction?

- **Diffraction is the ‘bending’ of waves around objects or through apertures**
- **It is an *interference effect***

Diffraction – water waves



Augustin-Jean Fresnel

The *wave theory* of light

Augustin-Jean Fresnel (1788 – 1827)

Extensive work on interference and diffraction

Augustin-Jean Fresnel

Double slit diffraction

Thomas Young (1773 – 1829)

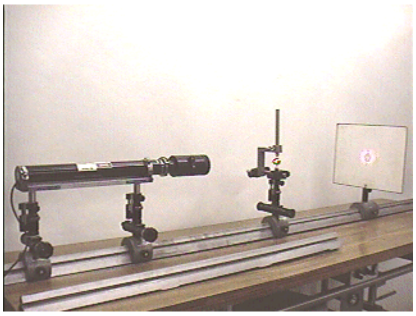
Double-slit experiment

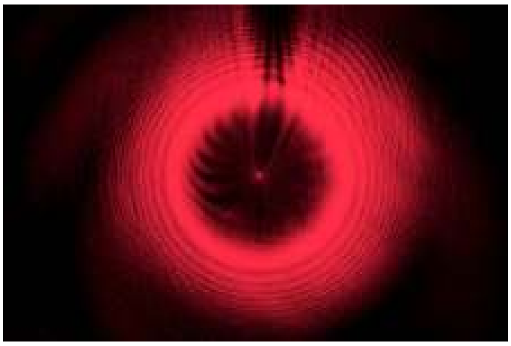
First conclusive evidence of the wave nature of light

Double slit model may be derived from the N-slit case putting $N = 2$, so not explicitly derived here.

Thomas Young

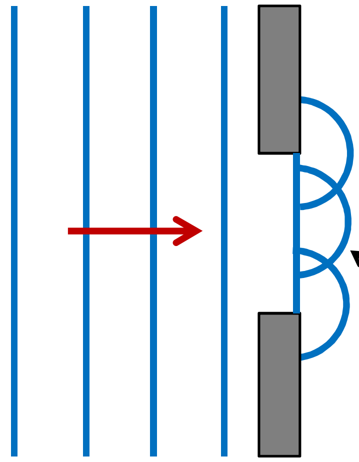
Fresnel-Poisson-Arago bright spot



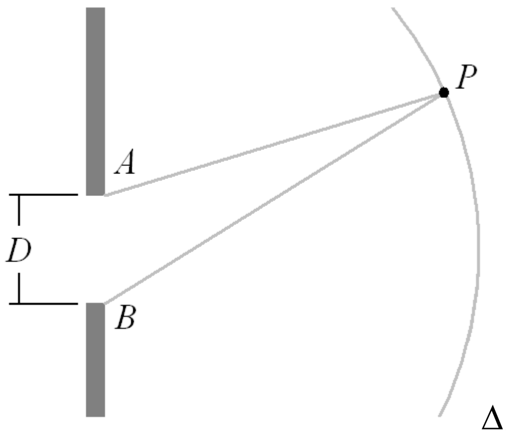


Predicted by Poisson as a **counter-example** to try to disprove Fresnel.

Light passing through a narrow aperture



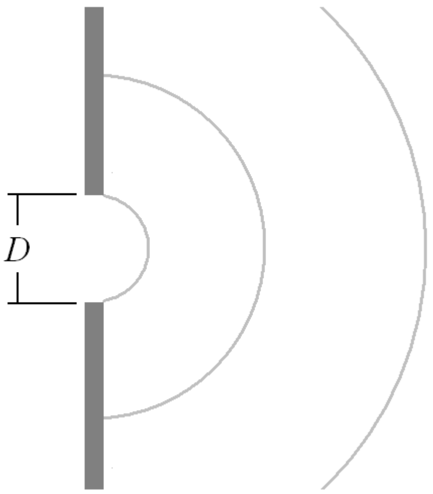
Light passing through a narrow aperture



Maximum possible path difference

$$\Delta_{max} = \left|\overrightarrow{AP} - \overrightarrow{BP}\right| = \left|\overrightarrow{AB}\right| = D.$$

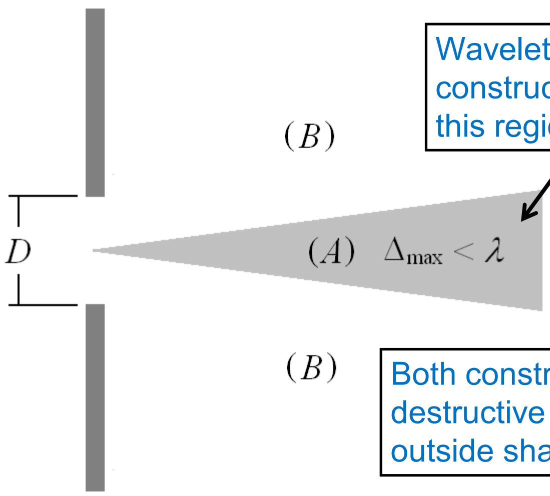
Limiting cases: $\lambda >> D$



Δ_{max} always less than λ – wavelets add constructively in all directions.

Emergent field looks like point source.

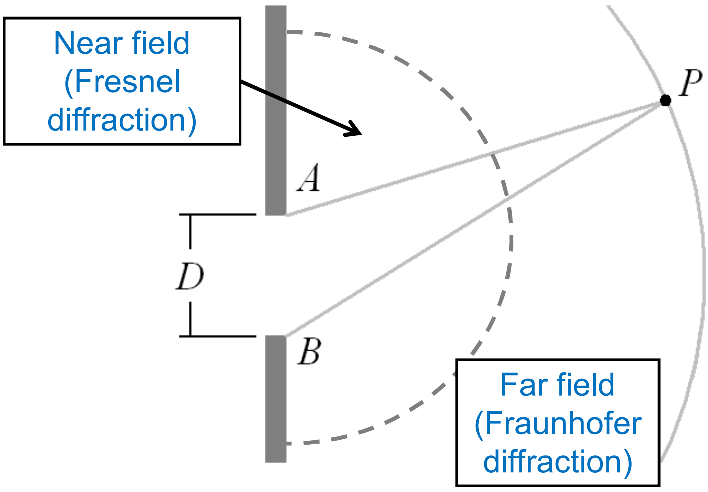
Limiting cases: $\lambda << D$



Wavelets add constructively in this region

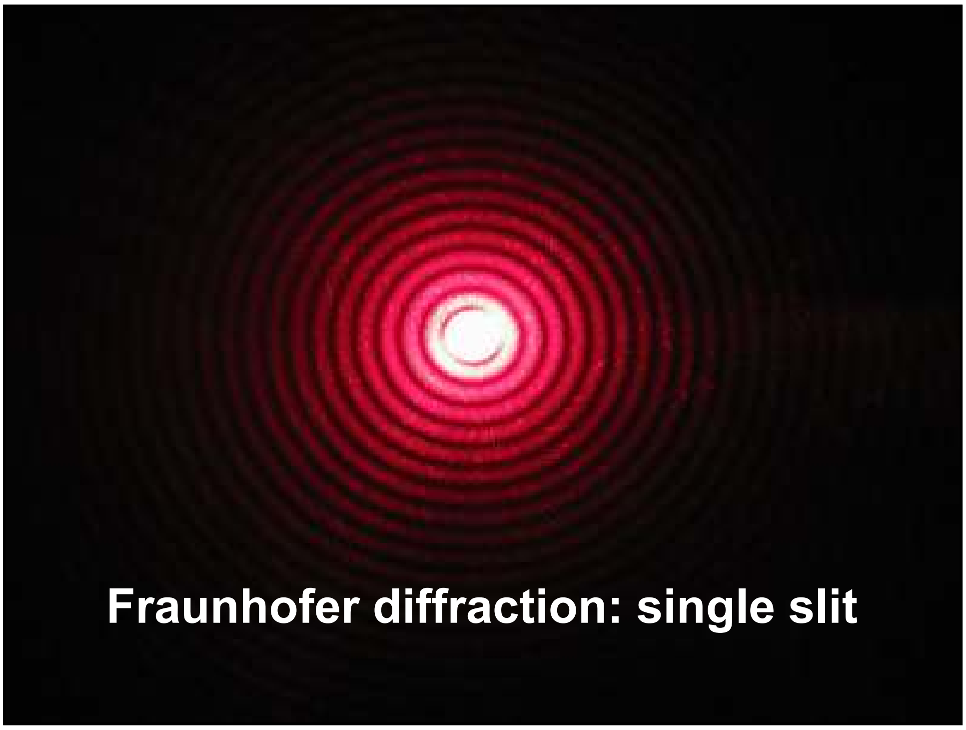
Both constructive and destructive interference outside shaded region

Fresnel and Fraunhofer diffraction

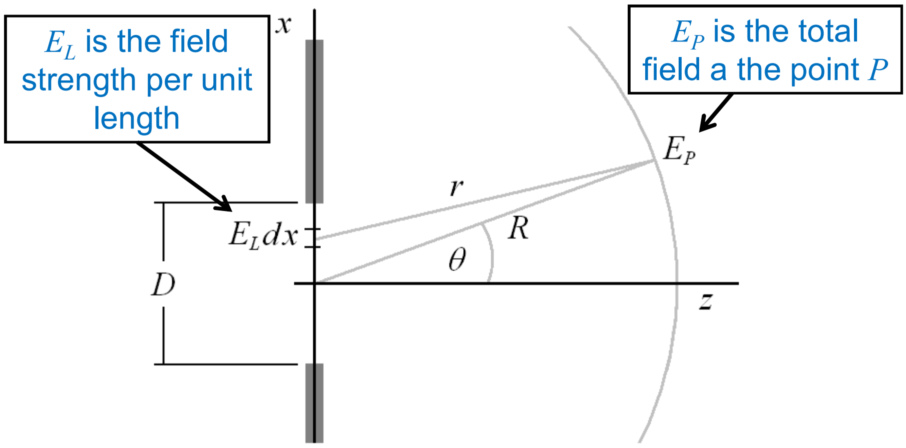


Fresnel and Fraunhofer diffraction

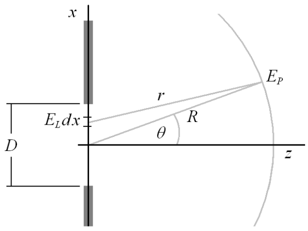
- **Fresnel diffraction**
  - **Diffraction pattern varies with increasing distance from aperture**
- **Fraunhofer diffraction**
  - **Diffraction pattern settles down to a constant profile**
  - **Applies for when the radial distance from the aperture satisfies the Fraunhofer condition (see later).**



Single slit diffraction



Single slit diffraction



Field at x

$$dE = E_L dx.$$

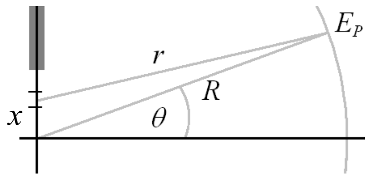
Contribution to field E_P due to dE

$$dE_P = \frac{E_L}{r(x)} \sin[\omega t - kr(x)]\, dx.$$

Total field E_P

$$E_P = \int_{-D/2}^{D/2} \frac{E_L}{r(x)} \sin[\omega t - kr(x)]\, dx.$$

Single slit diffraction



$r(x)$ is given by the **cosine rule**

$$r^2(x) = R^2 + x^2 - 2Rx\cos\left(\tfrac{\pi}{2} - \theta\right)$$

or

$$r(x) = R\left[1 + \frac{x^2}{R^2} - \frac{2x}{R}\sin\theta\right]^{1/2}.$$

To find a closed form solution, we must approximate this expression.

Taylor series expansion of *r*(*x*)

The Taylor series expansion for a function $(1+\xi)^{1/2}$ is

$$(1+\xi)^{1/2} = 1 + \frac{\xi}{2} - \frac{\xi^2}{8} + \ldots$$

Hence,

$$r(x) = R\left[1 - \frac{x}{R}\sin\theta + \frac{x^2}{2R^2}\cos^2\theta + \ldots\right]$$

and

$$kr(x) = kR - kx\sin\theta + \frac{kx^2}{2R}\cos^2\theta + \ldots.$$

The Fraunhofer condition

The third term in the expression for $kr(x)$ takes its maximum when $x \pm D/2$ and $\theta = 0$. That is

$$\frac{kx^2}{2R}\cos^2\theta \to \frac{kD^2}{8R^2} = \frac{\pi D^2}{4\lambda R^2}.$$

The condition that this term makes a negligible contribution to the phase is

$$\frac{\pi D^2}{4\lambda R^2} << \pi.$$

The Fraunhofer condition

Neglecting the factor of 4 in the denominator of the condition just found, it may be re-written as

$$\frac{D}{R} << \frac{\lambda}{D}.$$

This is the **Fraunhofer condition** for far field diffraction.

Far field approximations

Assuming that the Fraunhofer condition is valid, the third term in the expression for $kr(x)$ may be neglected and we have

$$kr(x) \approx kR - kx\sin\theta.$$

The $1/r(x)$ factor appearing in the integral for E_P is less sensitive to changes in $r(x)$ than the phase and we may simply put

$$\frac{1}{r(x)} \approx \frac{1}{R}.$$

Integrating over *x*

Using these approximations, the expression for the total field E_P becomes

$$E_P = \frac{E_L}{R}\int_{-D/2}^{D/2} \sin\left[\omega t - kR + kx\sin\theta\right] dx.$$

To perform this integral, we note that

$$\sin\left[\omega t - kR + kx\sin\theta\right] = \mathrm{Im}\left\{e^{i(\omega t - kR + kx\sin\theta)}\right\}.$$

The total field E_P

Integrating over the x-dependent part

$$\int_{-D/2}^{D/2} e^{ikx\sin\theta}\, dx = \left[\frac{e^{ikx\sin\theta}}{ik\sin\theta}\right]_{-D/2}^{D/2} = D\frac{\sin\beta}{\beta},$$

where

$$\beta = \frac{kD}{2}\sin\theta.$$

Hence, the total field E_P is

$$E_P = \frac{E_L D}{R}\frac{\sin\beta}{\beta}\sin(\omega t - kR).$$

Intensity profile for a single slit

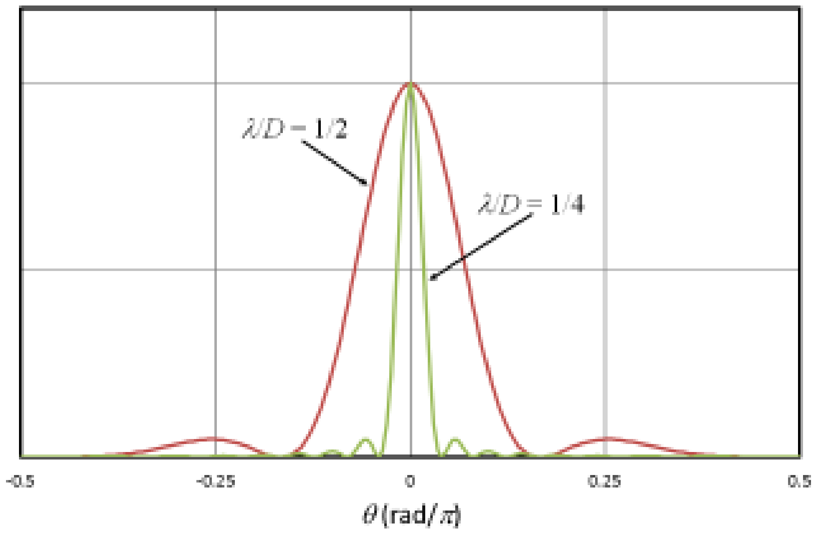
Averaging E_P over time gives

$$\langle E_P \rangle = \frac{E_L D}{2R} \frac{\sin \beta}{\beta}.$$

The squared modulus of this will be proportional to the intensity, i.e.

$$I(\theta) = I(0) \left| \frac{\sin \beta}{\beta} \right|^2.$$

Intensity profile for a single slit



Intensity profile for a single slit

The zeros of the peaks occur at values of

$$\beta = \frac{kD}{2}\sin\theta = m\pi,$$

where m is an integer. Hence, the first zeros around the central peak are given by

$$\sin\theta = \frac{\lambda}{D}.$$

Note that this result is only valid for $\lambda < D$. In other cases, there are no zeros from $-\pi$ to π.

Diffraction from a circular aperture

The analysis of Fraunhofer diffraction from a circular slit follows similar lines to that of a single slit. The result for the intensity is

$$I(\theta) = I(0)\left[\frac{2J_1(\sin\beta_c)}{\beta_c}\right]^2,$$

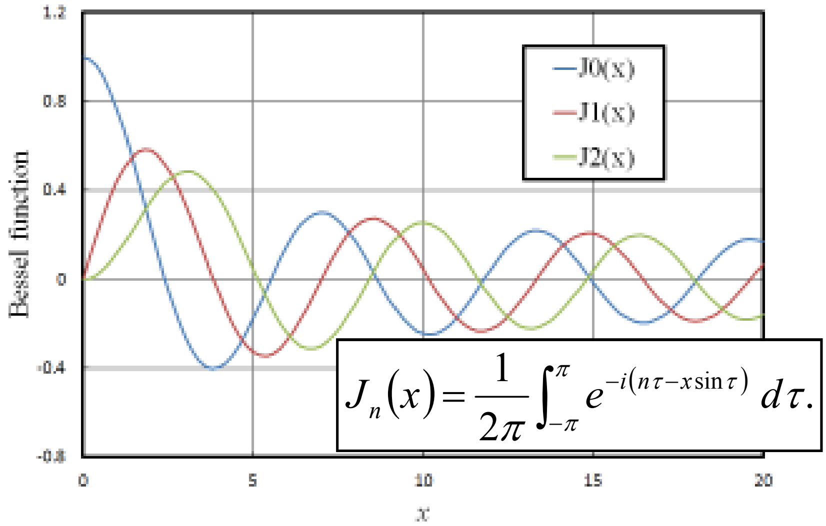
where

$$\beta_c = kD\sin(\theta/2),$$

D is the diameter of the aperture and J_1 is the **first order Bessel function**.

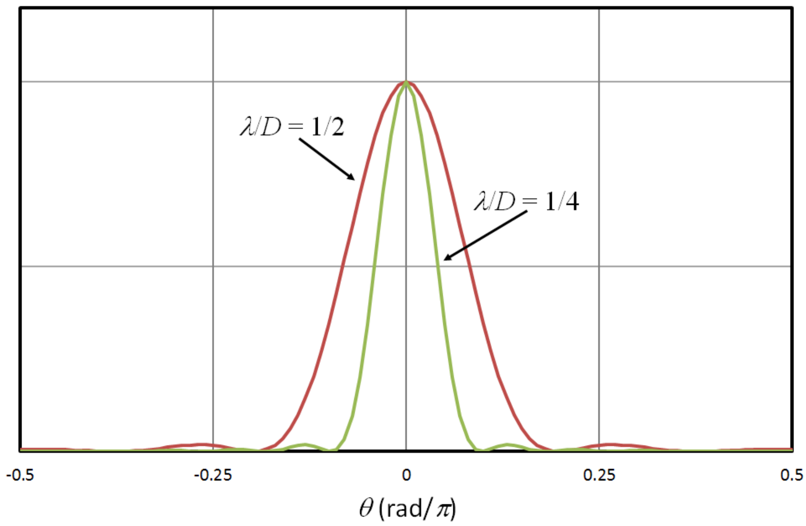
Coláiste na hOllscoile Corcaigh, Éire
University College Cork, Ireland
ROINN NA FISICE
Department of Physics
PY3101 Optics

Bessel functions

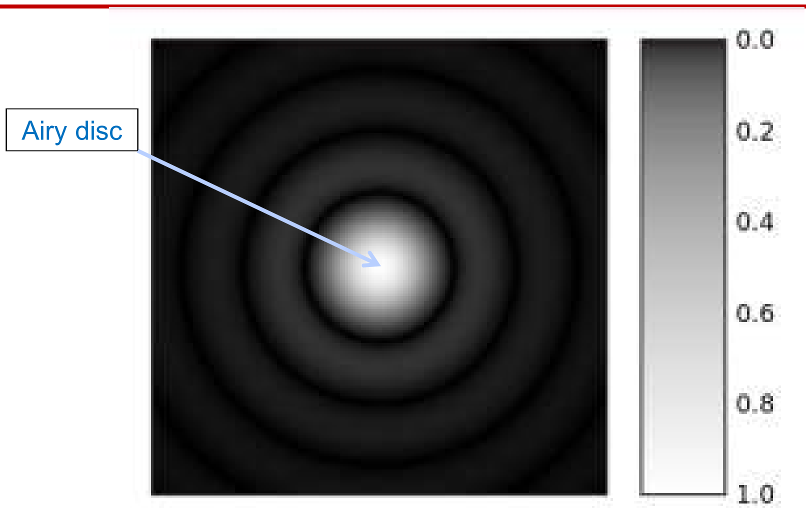


$$J_n(x) = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{-i(n\tau - x\sin\tau)}\, d\tau.$$

Intensity profile for a circular aperture

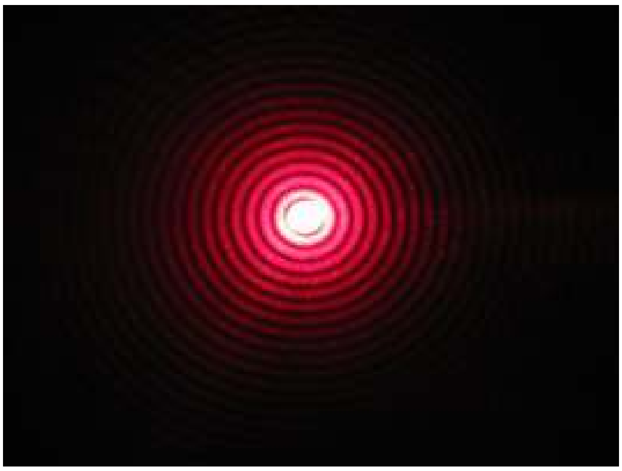


The Airy disc



Airy pattern by **Sakurambo**. A computer-generated image of an Airy disk.
URL: http://en.wikipedia.org/wiki/File:Airy-pattern.svg

The Airy disc



Laser Interference by **Petrov Victor**. Diffraction of red laser beam by a circular aperture.
URL: http://en.wikipedia.org/wiki/File:Laser_Interference.JPG

The Rayleigh criterion

The first zero of the intensity profile for diffraction from a circular aperture occurs at

$$\sin\theta \approx 1.22\frac{\lambda}{D}.$$

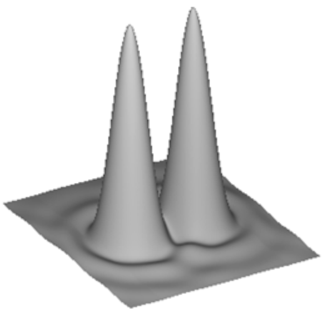
This represents the **minimum angular separation that two points can be so that they may be separately resolved**.

Using the small angle approximation, this becomes

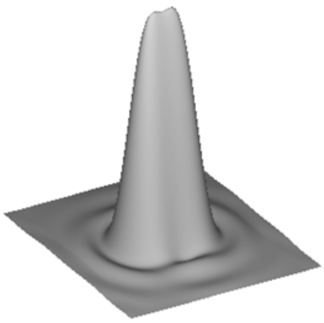
$$\theta \approx 1.22\frac{\lambda}{D}.$$

This is known as the **Rayleigh criterion**.

Diffraction limited imaging

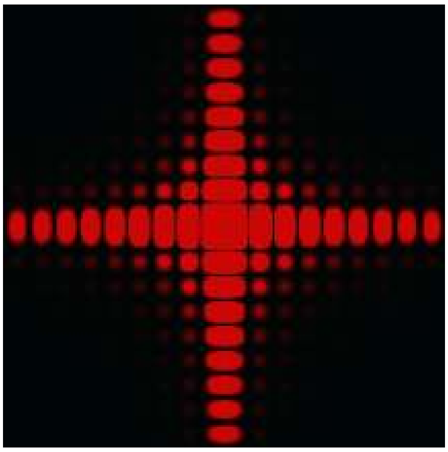


Intensity profiles for two resolvable distant point sources.

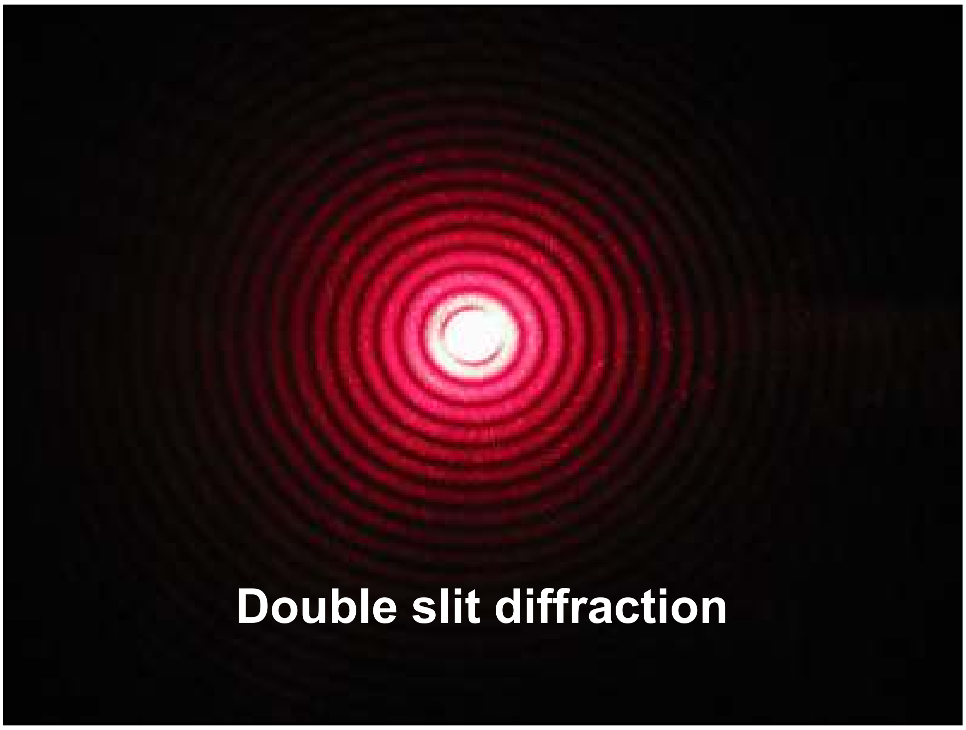


Merged intensity profiles for unresolvable distant point sources.

Diffraction from a square aperture

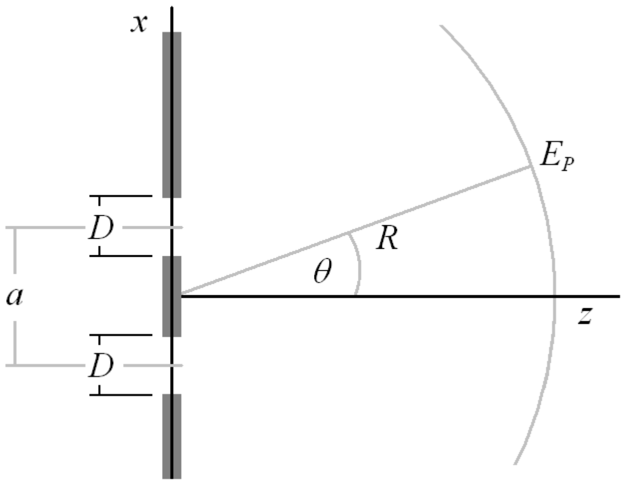


Square diffraction **(public domain).** A computer simulation of the intensity pattern formed by a laser of 663 nm wavelength incident on a square aperture of 20 by 20 micrometer, visible on a screen placed 1 meter from the aperture. In reality the image measures 30 by 30 cm.
URL: http://en.wikipedia.org/wiki/File:Square_diffraction.jpg

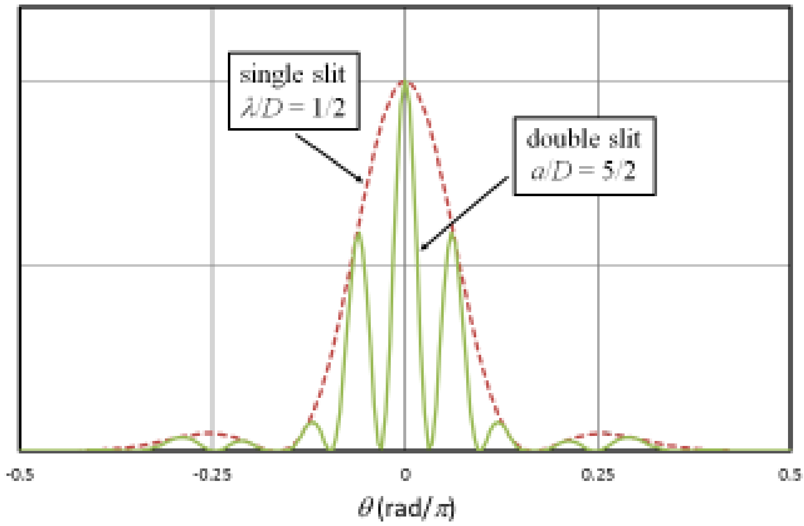


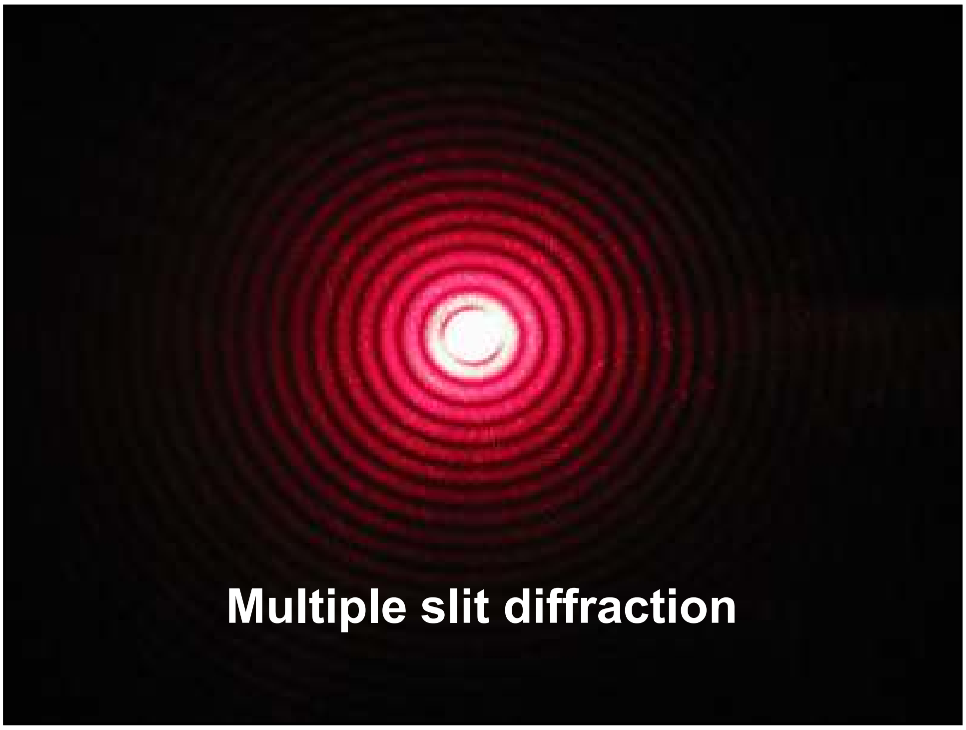
Double slit diffraction

Double slit diffraction

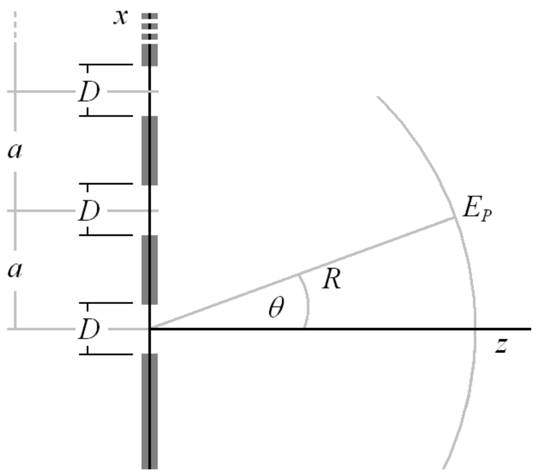


Intensity profile for a double slit





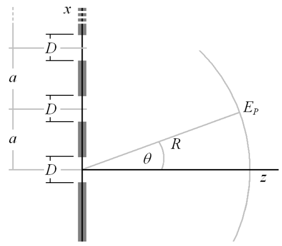
Multiple slit diffraction (transmission)



Assume incident wavevector on left-hand-side is parallel to z.

Multiple slit diffraction

Again, we make use of the earlier approximations for far field diffraction.



For N slits, the total contribution of the field E_P is

$$E_P = \sum_{n=0}^{N-1} \int_{na-D/2}^{na+/2} \frac{E_L}{R} \sin\left[\omega t - kR + kx\sin\theta\right] dx.$$

Multiple slit diffraction

Focussing on the x-dependent part of the integral and factorising as before, we find

$$\sum_{n=0}^{N-1} \left[\frac{e^{ikx\sin\theta}}{ik\sin\theta} \right]_{na-D/2}^{na+/2} = \sum_{n=0}^{N-1} e^{i2n\alpha} \left(D\frac{\sin\beta}{\beta} \right),$$

where

$$\alpha = \frac{ka}{2}\sin\theta.$$

The new factor is a geometric progression with common factor $e^{i2\alpha}$

$$S_N = \sum_{n=0}^{N-1} e^{i2n\alpha}.$$

Multiple slit diffraction

Multiplying S_N by $e^{i2\alpha}$

$$S_N e^{i2\alpha} = \sum_{n=1}^{N} e^{i2n\alpha},$$

so

$$S_N\left(1-e^{i2\alpha}\right) = 1-e^{i2N\alpha},$$

which gives

$$S_N = e^{i(N-1)\alpha}\frac{\sin N\alpha}{\sin\alpha}.$$

Intensity profile for multiple slits

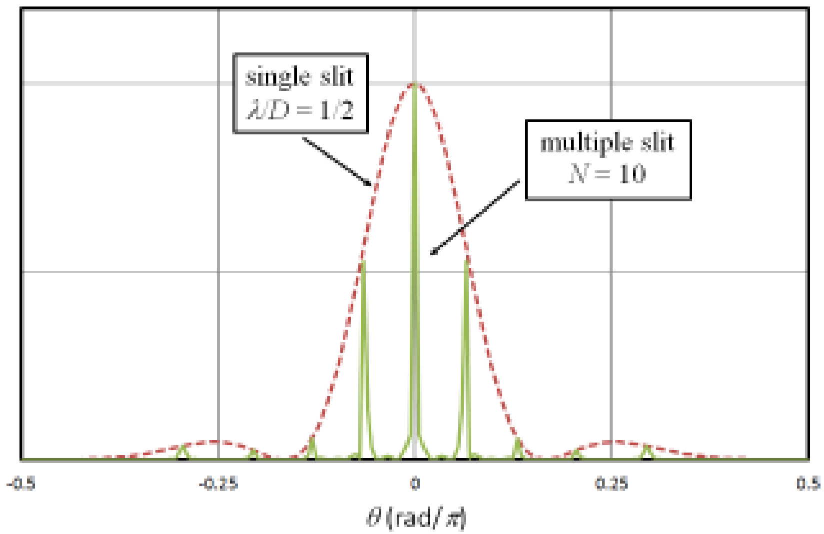
The phase factor may be dropped from this expression. Note also that since

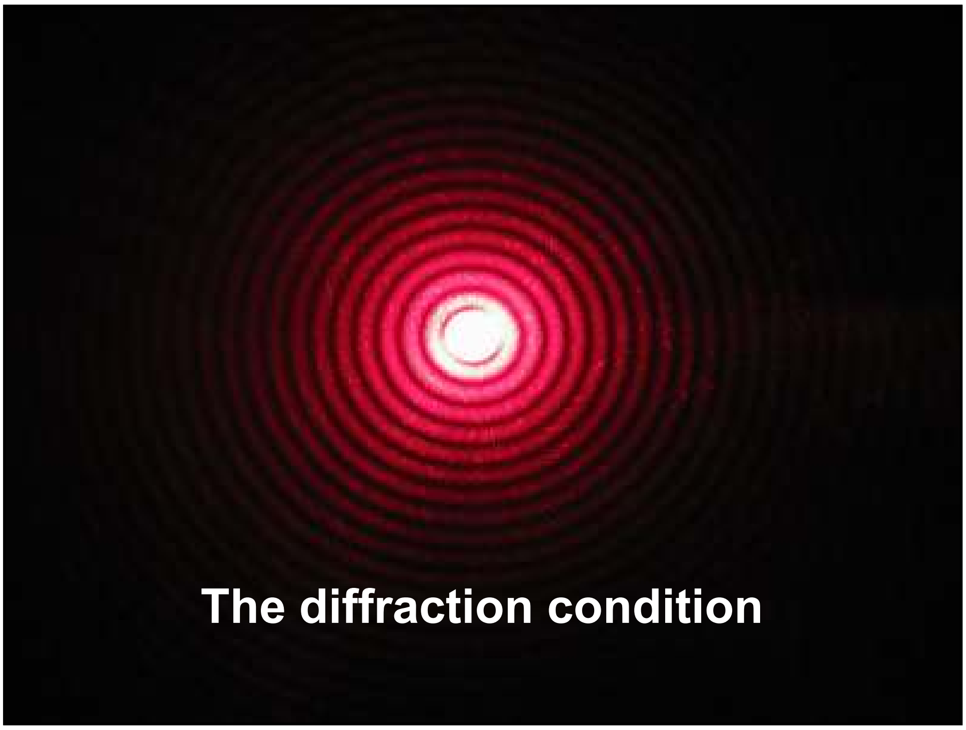
$$\lim_{\alpha\to 0}\frac{\sin N\alpha}{\sin\alpha} = N,$$

it is useful to incorporate a normalising factor $1/N$ into this ratio. Hence, the intensity takes the form

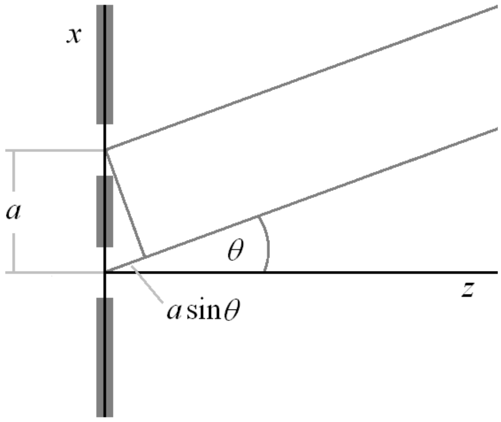
$$I(\theta) = I(0)\left(\frac{\sin N\alpha}{N\sin\alpha}\right)^2\left(\frac{\sin\beta}{\beta}\right)^2.$$

Intensity profile for multiple slits





Diffraction condition



Note that the condition for constructive interference is

$$a \sin\theta = m\lambda.$$

Diffraction condition

We can re-write this as

$$\frac{ka}{2}\sin\theta = \pi m.$$

But this is just

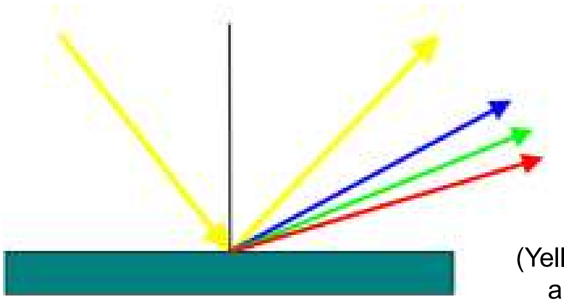
$$\alpha = \pi m,$$

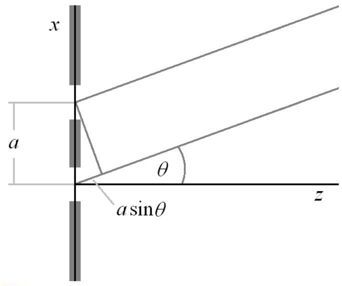
which gives the condition for the local maxima of the intensity

$$I(\theta) = I(0)\left(\frac{\sin N\alpha}{N\sin\alpha}\right)^2\left(\frac{\sin\beta}{\beta}\right)^2.$$

Diffraction – wavelength dependence

Red (longer wavelength) light is diffracted to a greater extent than blue (shorter wavelength).





$$a \sin\theta = m\lambda.$$

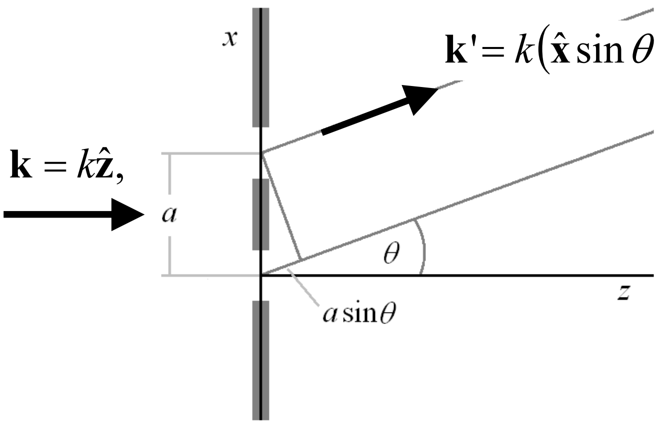
(Yellow arrow is incident light and specular reflection)

Diffraction condition

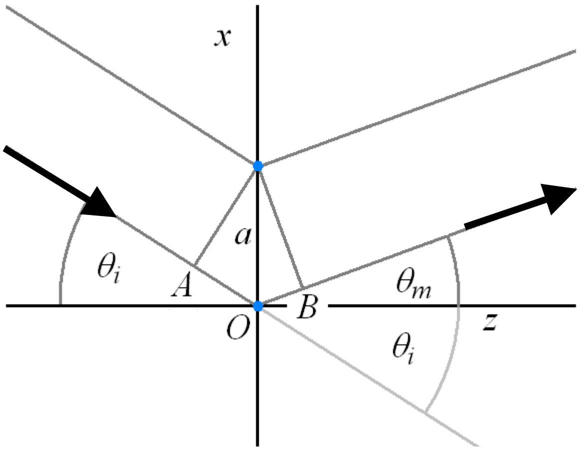
Incident and diffracted wavevectors:

$$\mathbf{k}' = k(\hat{\mathbf{x}}\sin\theta + \hat{\mathbf{z}}\cos\theta).$$

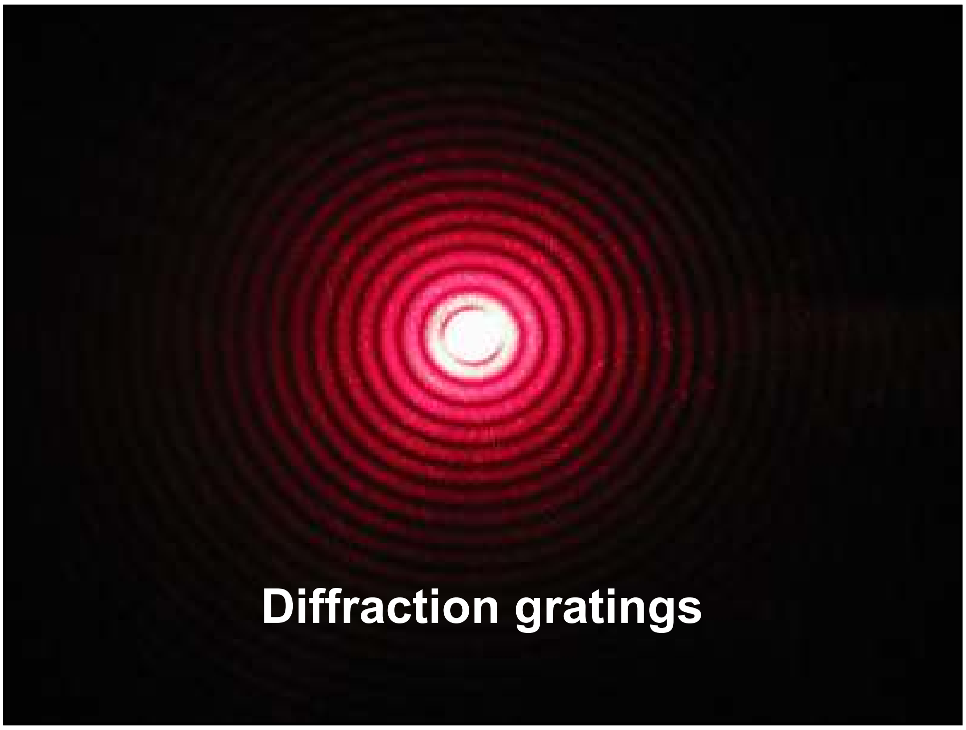
$$\mathbf{k} = k\hat{\mathbf{z}},$$



Diffraction condition: off-axis incidence



$$AO = a\sin\theta_i,\ \ OB = a\sin\theta_m.$$



The grating equation

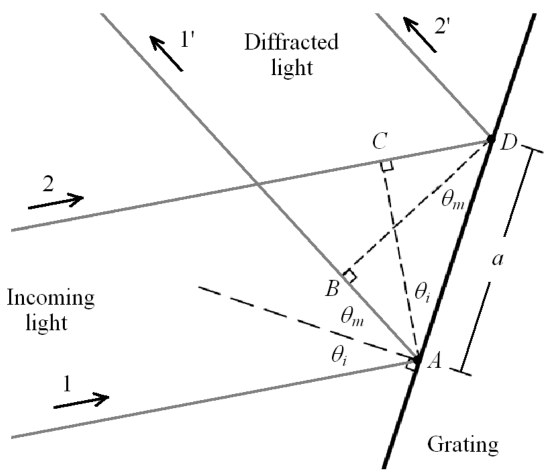
For off-axis transmission, the diffraction condition is now

$$a\left(\sin\theta_m + \sin\theta_i\right) = m\lambda.$$

This reduces to the case of normal incidence when $\theta_i = 0$.

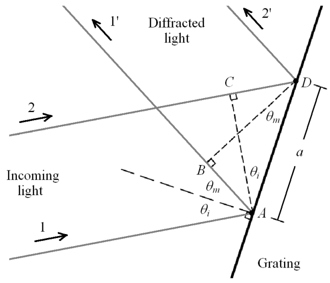
This result may be further generalised by taking the incident angle around to the front of the grating – i.e. making the grating into a **reflection grating**.

Reflection gratings



Reflection gratings

Light strikes the reflection grating at an angle θ_i. For certain angles θ_m, the **diffraction condition** will be met:



The path lengths of rays from the incident wavefront via the successive rulings of the grating and leaving at the same angle must differ only be integral multiples of the wavelength λ.

Derivation of the reflection grating equation

Incident wavefront AC

Path from A to wavefront BD

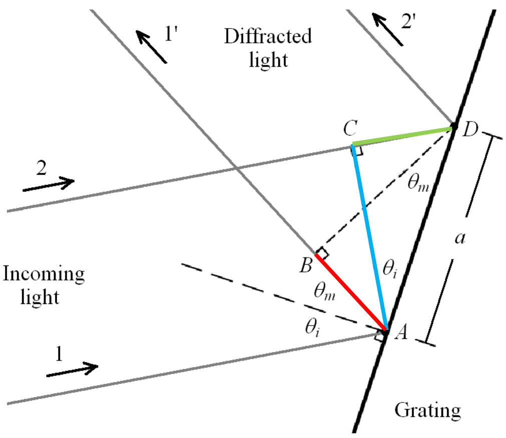
$$AB = a\sin\theta_m .$$

Path from C to wavefront BD

$$CD = a\sin\theta_i .$$

Path difference

$$AB - CD = a(\sin\theta_m - \sin\theta_i).$$



The reflection grating equation

The **diffraction condition** for a reflection grating may then be expressed mathematically as

$$m\lambda = a(\sin\theta_m - \sin\theta_i).$$

This is known as the **reflection grating equation**.

Dispersion

The **dispersion** of a grating is defined as

$$D_\theta = \frac{d\theta_m}{d\lambda}.$$

Differentiating the grating equations, we have

To be solved in the the Problem Set!

Coláiste na hOllscoile Corcaigh, Éire
University College Cork, Ireland
ROINN NA FISICE
Department of Physics
PY3101 Optics

Dispersion

Hence,

To be solved in the Problem Set!

Number of orders

Recall that

$$a\left(\sin\theta_m \pm \sin\theta_i\right) = m\lambda.$$

So the highest order m is the largest integral value of

$$\frac{a}{\lambda}\left(\sin\theta_m \pm \sin\theta_i\right).$$

Hence

$$m_{\max} < \frac{2a}{\lambda}.$$

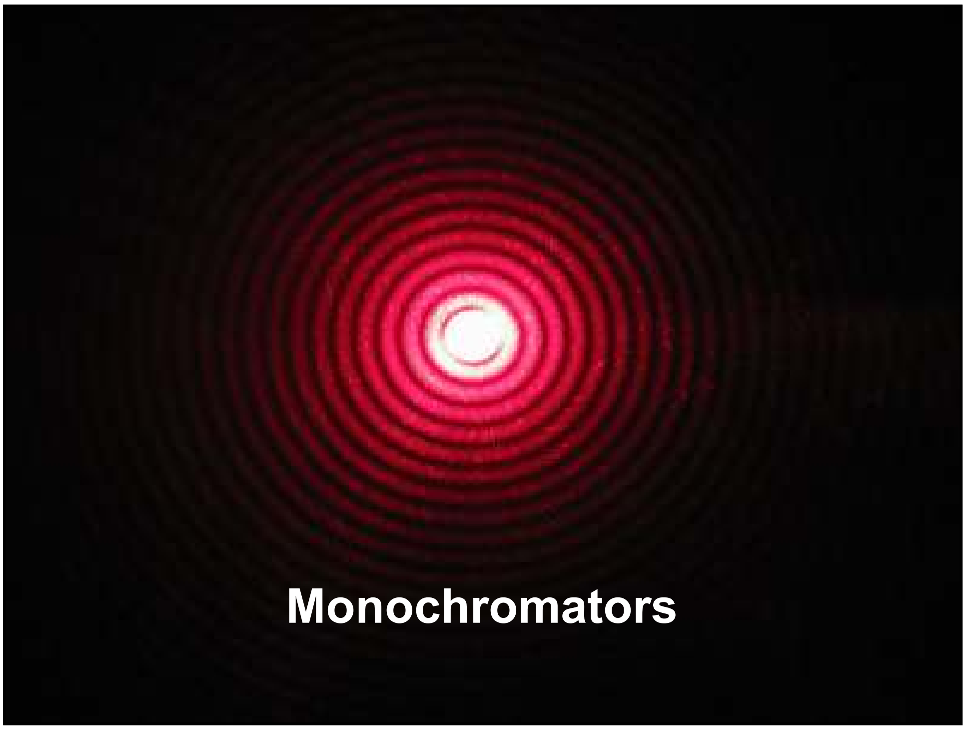
Resolving power

The **resolving power** of a grating is defined as

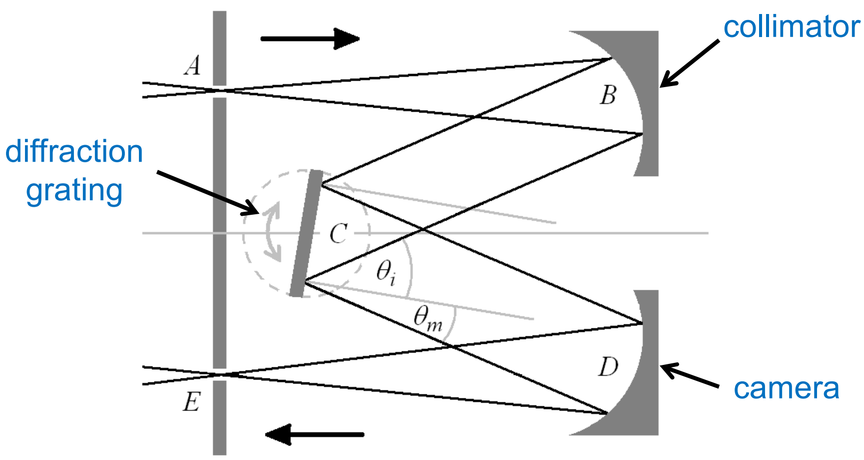
$$R = \frac{\lambda}{\Delta\lambda},$$

where, via **Rayleigh's criterion**, $\Delta\lambda$ is the minimum resolvable wavelength between the peaks of two wavelengths with midpoint λ.

Common example of a grating



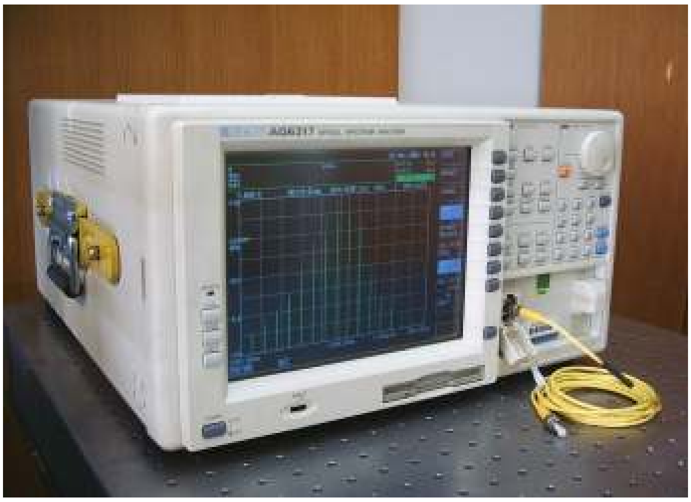
The Czerny-Turner monochromator



Czerny-Turner monochromator

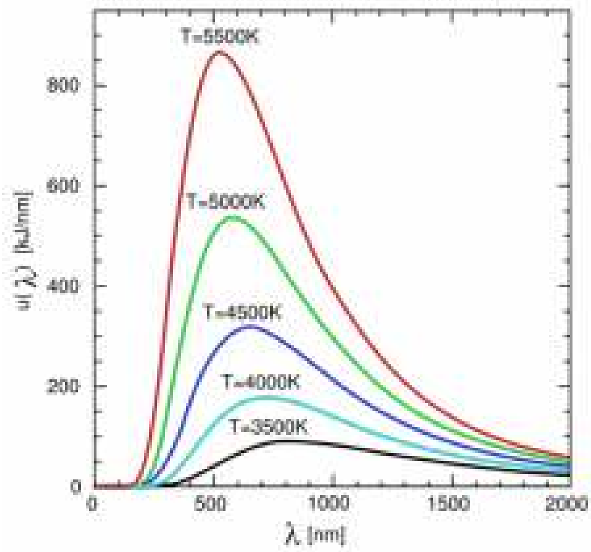
- Entrance slit
- Collimator (concave mirror)
- Diffraction grating
- Camera (concave mirror)
- Exit slit

Optical spectrum analyser (OSA)

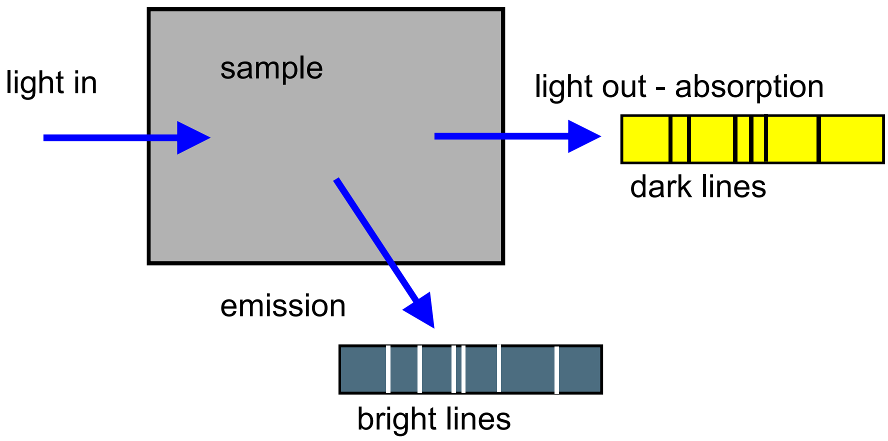


Optical spectrum analyser (OSA) based on diffraction gratings

Spectroscopy – blackbody spectra

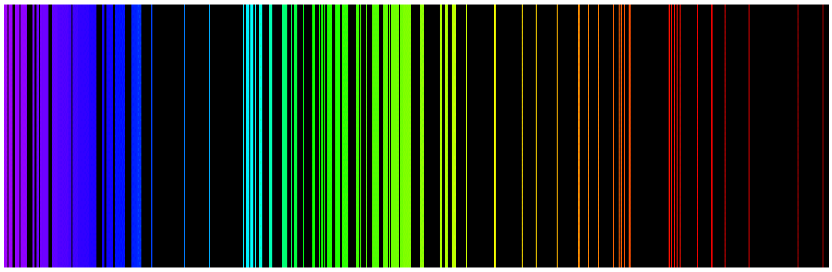


Spectroscopy – emission and absorption lines



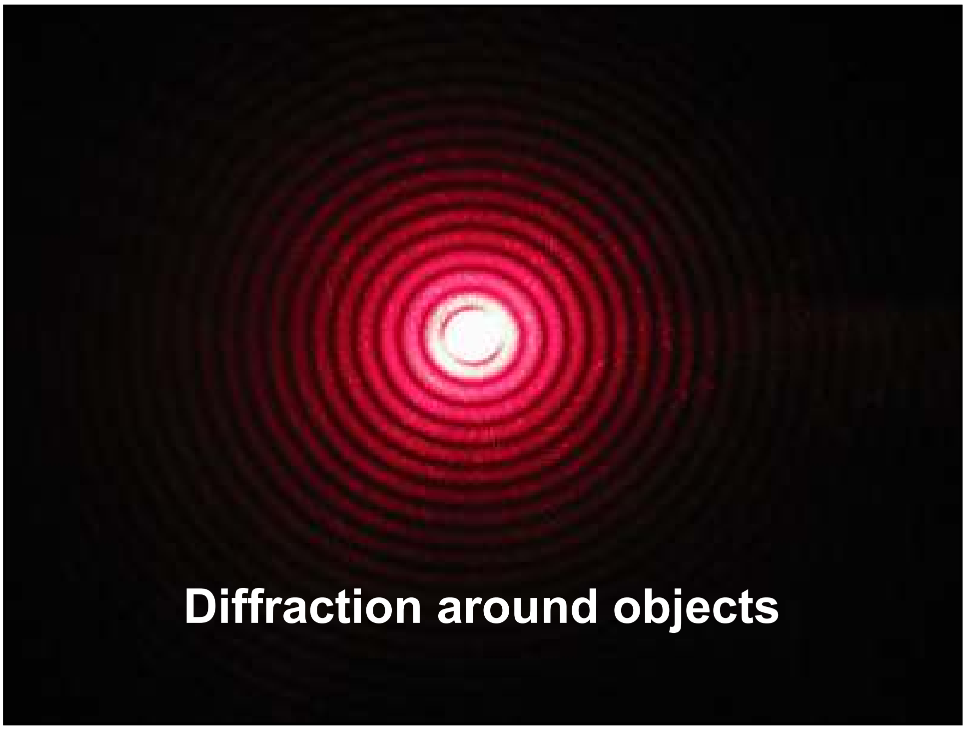
Spectroscopy – chemical analysis

Spectra provides `chemical fingerprint'

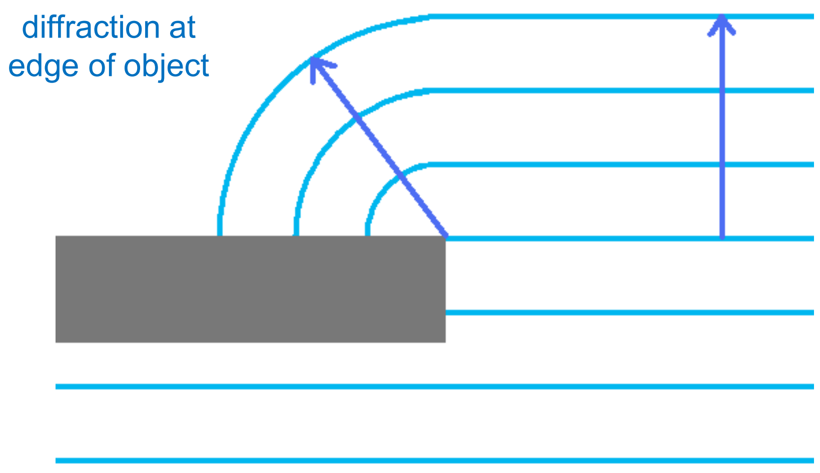


Emission spectrum of iron

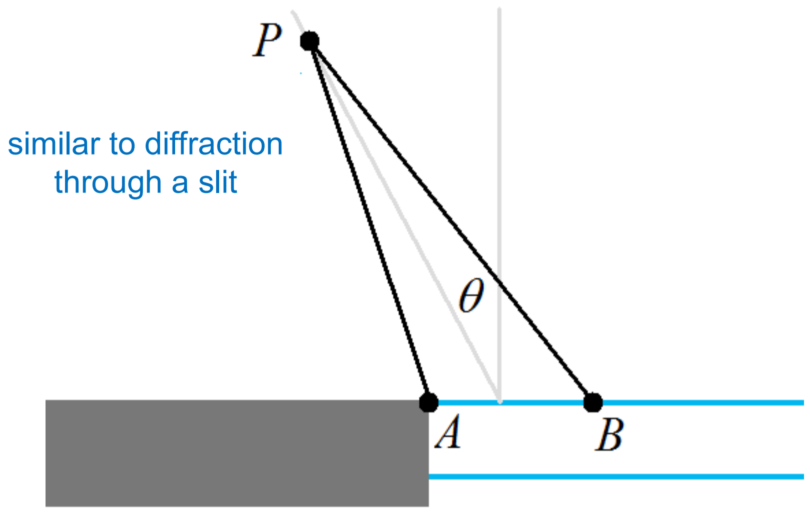
Coláiste na hOllscoile Corcaigh, Éire
University College Cork, Ireland
ROINN NA FISICE
Department of Physics



Diffraction around objects

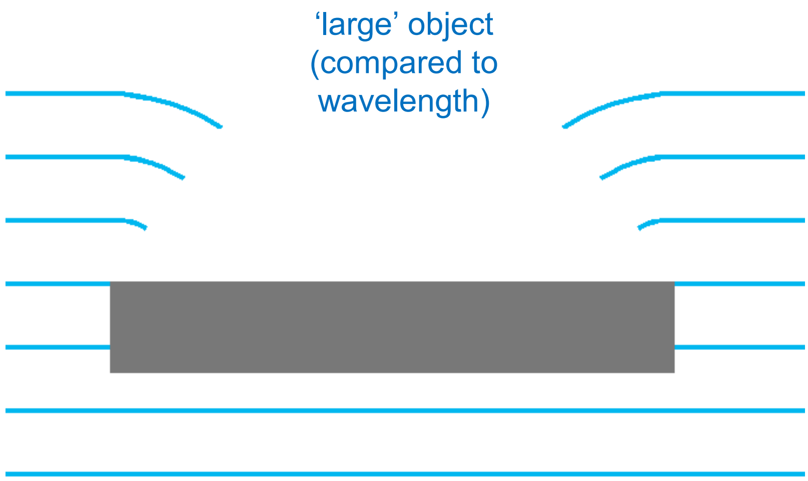


Diffraction around objects

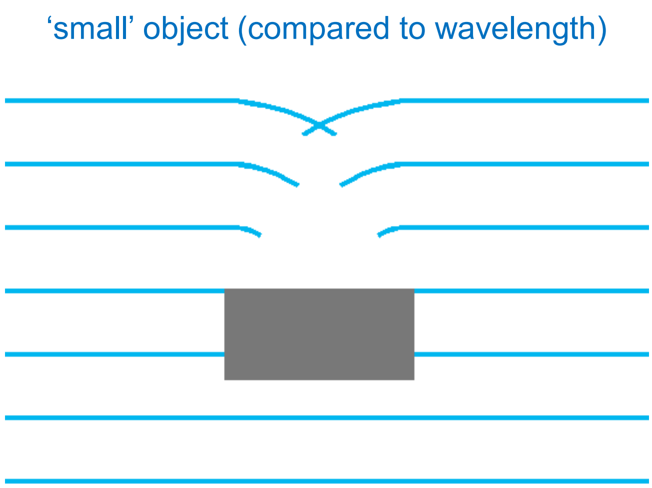


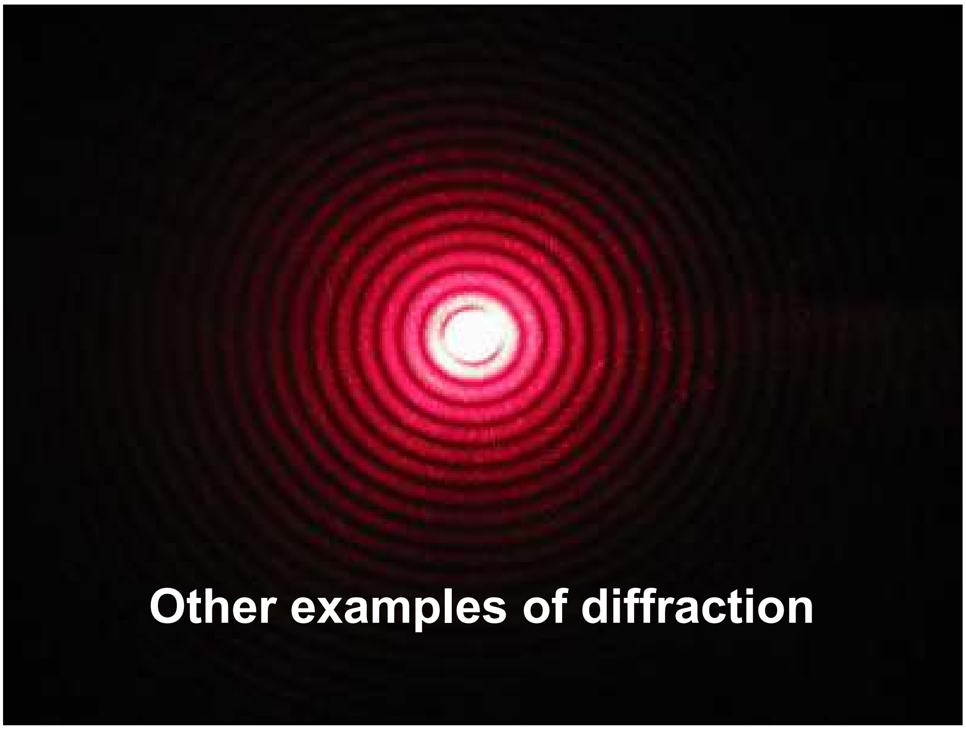
Coláiste na hOllscoile Corcaigh, Éire
University College Cork, Ireland
ROINN NA FISICE
Department of Physics
PY3101 Optics

Diffraction around objects

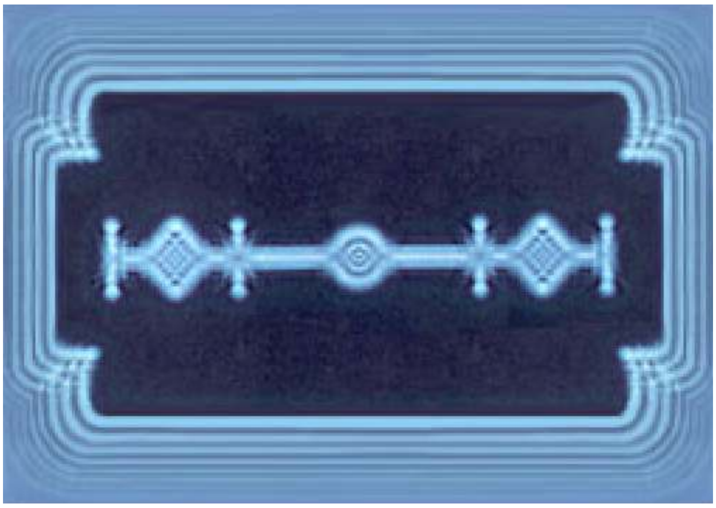


Diffraction around objects

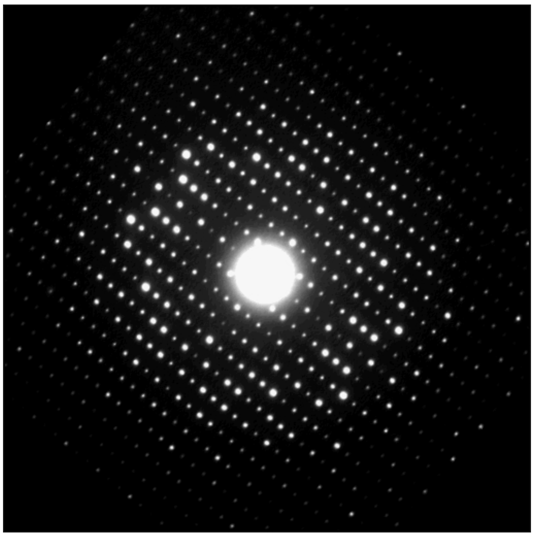




Diffraction around a razor blade



X-ray diffraction (non-optical)



(See PY3105)

Related phenomena:

Thin-Film Interference

(See Chapter 9, PY3101)