

# Experimental Methods

PY2108

## Practical Session 3: *Resonant series RLC circuits*

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### Contents

<b>1</b>	<b>Learning objectives</b>	<b>2</b>
<b>2</b>	<b>Pre-lab preparation</b>	<b>2</b>
2.1	Theory . . . . .	2
2.1.1	Complex inductive impedance . . . . .	2
2.1.2	Frequency response of an RLC circuit . . . . .	3
2.2	Plotting the transfer functions . . . . .	4
<b>3</b>	<b>Methods</b>	<b>4</b>
3.1	Soldering and de-soldering practice . . . . .	4
3.2	Constructing the <i>RLC</i> circuit . . . . .	4
3.3	Setting up the function generator and oscilloscope . . . . .	4
3.4	Voltage measurements . . . . .	4
<b>4</b>	<b>Post-lab</b>	<b>5</b>
4.1	Error analysis . . . . .	5
4.2	Report requirements . . . . .	5

## 1 Learning objectives

The objectives of this practical section are to

1. Understand the behaviour of resistors, inductors and capacitors in electronic circuits
2. Understand the dynamics of  $RLC$  circuits and appreciate their use to select out a particular frequency
3. Learn the basics of soldering and de-soldering for circuit construction
4. Consolidate experience of measuring frequency response

## 2 Pre-lab preparation

### 2.1 Theory

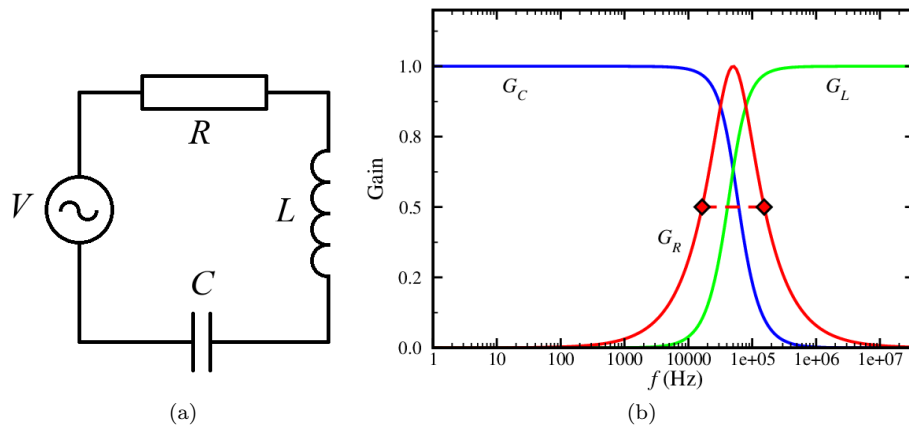


Figure 1: (a) Series  $RLC$  circuit diagram. (b) The frequency response of an  $RLC$  circuit with  $R = 50 \Omega$ ,  $L = 100 \mu\text{H}$  and  $C = 100 \text{ nF}$ . The diamonds mark the frequencies on the  $G_R$  curve giving the full-width-half-maximum (FWHM).

#### 2.1.1 Complex inductive impedance

For a time-dependent current  $I$ , the complex impedance of an inductor  $Z_L$  may be found from

$$V_L = L \frac{dI}{dt} = I Z_L. \quad (1)$$

If  $I$  is given by

$$I = I_0 e^{i\omega t}, \quad (2)$$

then we find

$$Z_L = i\omega L. \quad (3)$$

### 2.1.2 Frequency response of an RLC circuit

Treating the circuit of Fig. 1 (a) as a voltage divider with complex impedances, the transfer functions for each element may be found as

$$H_R = \frac{1}{1 + i[\omega L/R - 1/(\omega RC)]}, \quad (4)$$

$$H_L = i\frac{\omega L}{R}H_R \quad (5)$$

and

$$H_C = -\frac{i}{\omega RC}H_R \quad (6)$$

(recall that the impedance of a capacitor is  $1/(i\omega C)$ ).

Similarly, the gains for each element are

$$G_R = \frac{1}{\left\{1 + [\omega L/R - 1/(\omega RC)]^2\right\}^{1/2}}, \quad (7)$$

$$G_L = \frac{\omega L}{R}G_R \quad (8)$$

and

$$G_C = \frac{1}{\omega RC}G_R. \quad (9)$$

An example of the frequency dependence on these gains is shown in Fig. ?? (b). We see that the voltage over the resistor peaks around a particular frequency  $f_0 = \omega_0/(2\pi)$ . This is the *resonant frequency* of the circuit. Inspecting Eq. (7), we see that the  $G_R$  will go through a maximum when the condition

$$\frac{\omega_0 L}{R} - \frac{1}{\omega_0 RC} = 0 \quad (10)$$

is met. This yields a value of

$$\omega_0 = \frac{1}{(LC)^{1/2}} \quad (11)$$

for the resonant angular frequency.

The width of the peak at  $\omega_0$  may be characterised by the *full-width-half-maximum* (FWHM). This is the width of the peak at half the maximum gain. The values at which  $G_R = 1/2$  are found to be

$$\omega_{\pm} = \frac{3^{1/2}}{2}\omega_0^2\tau \left( \left[ 1 + \frac{4}{3}\frac{1}{\omega_0^2\tau^2} \right]^{1/2} \pm 1 \right), \quad (12)$$

where  $\tau = RC$  is the  $RC$  time constant. This gives for the FWHM

$$\Delta\omega = \omega_+ - \omega_- = 3^{1/2}\omega_0^2\tau = 3^{1/2}\frac{R}{L}. \quad (13)$$

A dimensionless merit factor for a series  $RLC$  may be defined as the ratio of the resonant frequency to the frequency spread. This is known as the ‘ $Q$ -factor’ ( $Q$  standing for ‘quality’) and may be defined as

$$Q \equiv \frac{\omega_0}{\Delta\omega} = \frac{1}{3^{1/2}\omega_0\tau}. \quad (14)$$

## 2.2 Plotting the transfer functions

In a spreadsheet package, plot the gain functions  $G_R$ ,  $G_L$  and  $G_C$ . Ensure that it is easy to adjust the parameters  $R$ ,  $L$  and  $C$ . Also calculate  $f_0 = \omega_0/(2\pi)$  and  $\Delta f = \Delta\omega/(2\pi)$  from the expressions given and check that these values are consistent with your graphs. You should also calculate the  $Q$ -factor.

As for your previous work on the frequency response of the series  $RC$  circuit, the frequency axis should be logarithmic (base 10). Furthermore, it will again be advantageous to set up your spreadsheet ready to accept measured data.

It is important that you bring this spreadsheet to the lab with you.

# 3 Methods

## 3.1 Soldering and de-soldering practice

Initially, you will be supplied with copper strip board and some resistors to practice soldering. Your demonstrator will show you how to solder and de-solder components. Once you are comfortable producing good solder joints, you may move on to the next task.

## 3.2 Constructing the $RLC$ circuit

For this experiment, you will be using a  $56\ \Omega$  resistor, a  $220\ \mu\text{H}$  inductor and a  $0.1\ \mu\text{F}$  capacitor. On a copper strip board, construct the circuit shown in Fig. 1 (a), soldering the components into place. You should also solder in wires to measure the voltage over the resistor. The AC voltage source will be supplied by a function generator.

## 3.3 Setting up the function generator and oscilloscope

The function generator should be set to produce a sinusoidal output. You will need to attach a ‘T’ connector to the output of the generator to allow two coaxial cables to be connected to it. One of these will drive the  $RLC$  circuit, the other should be connected to channel 1 of the oscilloscope. A second coaxial cable should be connected between the terminals over the resistor and channel 2 of the scope. Ensure that AC coupling and the dual trace mode have been selected.

## 3.4 Voltage measurements

The frequency points chosen for these measurements should span a reasonable window around the range where  $G_R$  goes from zero, through the maximum and back down to zero. The output of the generator should be adjusted to a reasonable value: a peak-to-peak voltage of 1 V is suggested.

## 4 Post-lab

### 4.1 Error analysis

Using the analytical tools in the supplementary manual ‘Error Analysis’ (available on blackboard), perform an error analysis on your data and calculated values. You should show the working of this in your report. Hence, add error bars to your data points (or argue that the errors would be too small to be noticed if this is the case).

### 4.2 Report requirements

You should type up your report. Preferably, equations should be typeset using suitable software (e.g. Equation Editor in Word, using Scientific Word or Latex). The report should have the following structure:

**Title** This should have the main heading ‘PY2108 Experimental Methods: Session (3)’ and then an appropriate sub-heading of your choice.

**Author** Your name and student number.

**Abstract** A very brief description of the report.

**Introduction** A short introduction. Possible applications of the methods employed could be included here.

5 %

**Theory** This should include any theory that you actually use to either make testable predictions (e.g. plot theoretical graphs) or analyse the data. Try to keep this as brief as possible when describing experimental work.

10 %

**Method** A description of the experimental setup and how you took the data. You should also include your error analysis in this section.

35 %

**Results / Discussion** Presentation of the results. For this experiment, this should be graphs of the transfer functions with the data plotted over the top of the theoretical curves. Ensure that you clearly label all axes (with units where appropriate) and data sets. Also ensure that there is a figure caption. You may include discussion of any points of especial interest encountered in the lab in this section.

This section should include a comparison of your measured resonant frequency and FWHM to your calculated values. You should also include your calculation of the  $Q$ -factor.

35 %

**Discussion / Conclusions** Brief conclusions summarising the report. You may also include some discussion of the experiment and possible applications in this section.

5 %

A further 10 % will be given for presentation, including the correct structure and grammar.