Physics PY4118
Physics of Semiconductor Devices

The Diode – Part B

Forward bias

Carrier density under bias (1)

How large is the current density?

The ***net*** minority current remains very small, even under bias:

$$J_n = \mu_n \left(qn\mathcal{E} + kT\frac{dn}{dx} \right) \sim 0$$

No bias: $$\int_{n_{p0}}^{n} \frac{dn}{n} = \frac{q}{kT}\int_{V_p}^{V_n} dV = \frac{q}{kT}\int_{0}^{V_{bi}} dV$$

Under bias: $$\int_{n(-x_p)}^{n(x_n)} \frac{dn}{n} = \frac{q}{kT}\int_{V_p}^{V_n} dV = \frac{q}{kT}\int_{0}^{V_{bi}-V_A} dV$$

Carrier density under bias (2)

Thus: $$V_{bi} = \frac{kT}{q}\ln\left(\frac{p_p}{p_{n0}}\right) = \frac{kT}{q}\ln\left(\frac{n_n}{n_{p0}}\right)$$

From the built in voltage, under zero bias: $$\frac{p_p}{p_{n0}} = e^{\frac{qV_{bi}}{kT}} = \frac{n_n}{n_{p0}} \qquad \textbf{(1)}$$

Under bias: $$\frac{p_p(-x_p)}{p_n(x_n)} = e^{\frac{q(V_{bi}-V_A)}{kT}} = \frac{n_n(x_n)}{n_p(-x_p)} \qquad \textbf{(2)}$$

Carrier density under bias (3)

Divide **(1)** by **(2)**:

$$\frac{\cancel{p_p} p_n(x_n)}{p_{n0} \cancel{p_p}(-x_p)} = e^{\frac{qV_A}{kT}} = \frac{\cancel{n_n} n_p(-x_p)}{n_{p0} \cancel{n_n}(x_n)}$$

We have already pointed out that the number of injected carriers does not affect the majority carrier population.

$$p_p(-x_p) \Rightarrow p_p \qquad = n_n(x_n) \Rightarrow n_n$$

Carrier density under bias (4)

So: $$\frac{p_n(x_n)}{p_{n0}} = e^{\frac{qV_A}{kT}} = \frac{n_p(-x_p)}{n_{p0}}$$

And: $$p_n(x_n) = p_{n0}e^{\frac{qV_A}{kT}}, \quad n_p(-x_p) = n_{p0}e^{\frac{qV_A}{kT}}$$

But, there is another simpler way to calculate this!

Another way to get there…

$$n = n_i e^{\frac{E_{Fn}-E_i}{kT}} \qquad p = n_i e^{\frac{E_i-E_{Fp}}{kT}}$$

(Slide 8.3)

$$np = n_i^2 e^{\frac{E_{Fn}-E_{Fp}}{kT}} = n_i^2 e^{\frac{qV_A}{kT}}$$

For non-equilibrium

At the edge of the depletion region: $n_p(-x_p)p_p = n_i^2 e^{\frac{qV_A}{kT}}$

$$n_p(-x_p) = \frac{n_i^2}{p_p} e^{\frac{qV_A}{kT}} = n_{p0} e^{\frac{qV_A}{kT}}$$

Majority carriers unchanged

and equivalently for p…

Carrier density under bias (5)

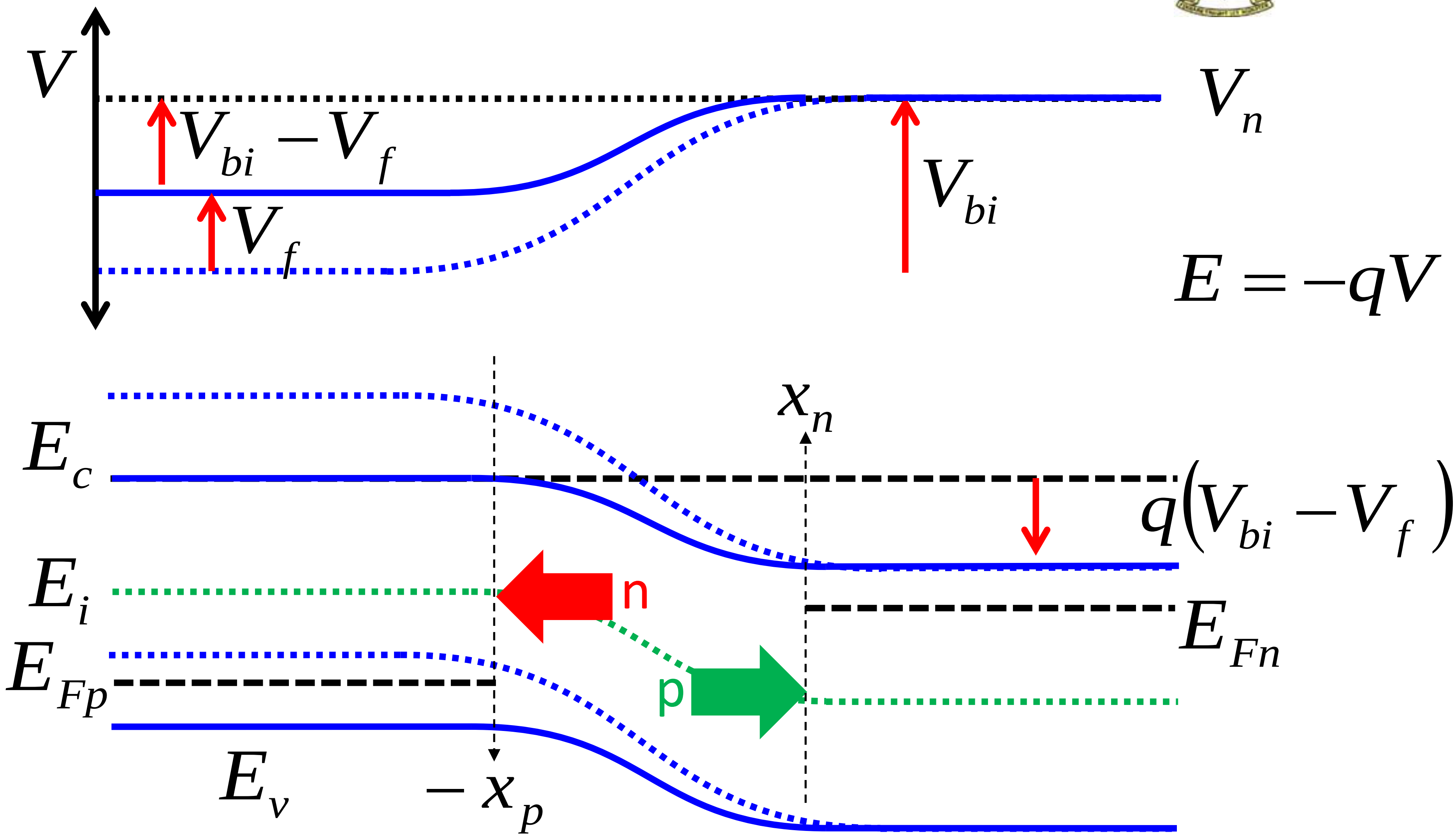
The excess carriers are therefore:

$$\Delta p_n(x_n) = p_n(x_n) - p_{n0} = p_{n0}\left(e^{\frac{qV_A}{kT}} - 1\right)$$

$$\Delta n_p(-x_p) = n_p(-x_p) - n_{p0} = n_{p0}\left(e^{\frac{qV_A}{kT}} - 1\right)$$

These excess carriers will then diffuse and recombine according to the continuity equations (Slide set 9)

Potential with forward bias



From previous notes $x = -x_p$

Steady state carrier Injection at boundary, with no Field/light:

$$\boxed{-\frac{n_p - n_{p0}}{\tau_n}} + \mu_n \mathcal{E} \cancel{\frac{dn_p}{dx}} + \mu_n n_p \cancel{\frac{d\mathcal{E}}{dx}} + D_n \frac{d^2 n_p}{dx^2} + \cancel{G} = 0$$

$$\rightarrow \frac{d^2 n_p}{dx^2} - \frac{n_p - n_{p0}}{D_n \tau_n} = 0$$

With: $L_n = \sqrt{D_n \tau_n}$

General solution: with $\Delta n_p = n_p - n_{p0}$

$$\Delta n_p(x) = A e^{-\frac{x}{L_n}} + B e^{\frac{x}{L_n}}$$

Boundary conditions

Since electrons injected at $x = -x_p$, rewrite as:

$$\Delta n_p(x) = Ae^{-\frac{x+x_p}{L_n}} + Be^{\frac{x+x_p}{L_n}}$$

Boundary conditions: $\Delta n_p(-\infty) = 0$, so $A = 0$

So: $\Delta n_p(x) = \Delta n_p(-x_p)e^{\frac{x+x_p}{L_n}}$

The minority electrons recombine toward negative x.

And: L_n is the diffusion length

Carrier density under bias (6)

Similarly for hole diffusion and recombination at $x = x_n$

$$p_n(x) = p_{n0} + \Delta p_n(x_n) e^{-\frac{x-x_n}{L_p}}$$

With: $L_p = \sqrt{D_p \tau_p}$

Thus:

$$\Delta p_n(x) = \Delta p_n(x_n) e^{-\frac{x-x_n}{L_p}} = p_{n0}\left(e^{\frac{qV_A}{kT}} - 1\right) e^{-\frac{x-x_n}{L_p}}$$

$$\Delta n_p(x) = \Delta n_p(-x_p) e^{\frac{x+x_p}{L_n}} = n_{p0}\left(e^{\frac{qV_A}{kT}} - 1\right) e^{\frac{x+x_p}{L_n}}$$

Current though diode (1)

The current flow can then be calculated from the diffusion of carriers at the edge of the depletion region:

$$J_p = -qD_p \frac{dp_n}{dx} \qquad J_n = qD_n \frac{dn_p}{dx}$$

$$I_p = -qAD_p \frac{dp_n}{dx} \qquad I_n = qAD_n \frac{dn_p}{dx}$$

Need to calculate: $\frac{dp_n}{dx}$ and: $\frac{dn_p}{dx}$

Using:

$$\Delta p_n(x) = \Delta p_n(x_n) e^{-\frac{x-x_n}{L_p}} \qquad \Delta n_p(x) = \Delta n_p(-x_p) e^{\frac{x+x_p}{L_n}}$$

Current though diode (2)

$$\frac{dp_n}{dx} = \frac{d}{dx}\left(\Delta p_n(x_n) e^{-\frac{x-x_n}{L_p}}\right) = -\frac{\Delta p_n(x_n)}{L_p} e^{-\frac{x-x_n}{L_p}}$$

$$\frac{dn_p}{dx} = \frac{d}{dx}\left(\Delta n_p(-x_p) e^{\frac{x+x_p}{L_p}}\right) = \frac{\Delta n_p(-x_p)}{L_n} e^{\frac{x+x_p}{L_p}}$$

Thus:

$$\frac{dp_n(x_n)}{dx} = -\frac{\Delta p_n(x_n)}{L_p} \qquad \frac{dn_p(-x_p)}{dx} = \frac{\Delta n_p(-x_p)}{L_n}$$

So:

$$I_p = qAD_p \frac{\Delta p_n(x_n)}{L_p} \qquad I_n = qAD_n \frac{\Delta n_p(-x_p)}{L_n}$$

Current though diode (3)

So finally, we have:

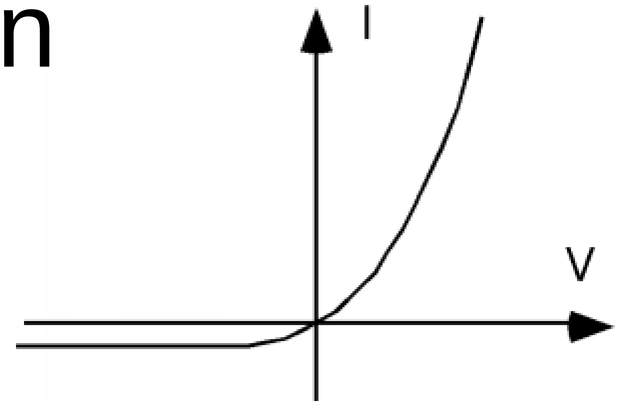
$$I_p = \frac{qAD_p}{L_p} p_{n0}\left(e^{\frac{qV_A}{kT}} - 1\right) \qquad I_n = \frac{qAD_n}{L_n} n_{p0}\left(e^{\frac{qV_A}{kT}} - 1\right)$$

For the "Ideal Diode" we assume the no combination takes place in the depletion region. This means that all the current flow is as a result of the minority carrier diffusion currents: $I = I_p + I_n$

$$I = I_0\left(e^{\frac{qV_A}{kT}} - 1\right)$$ where: $$I_0 = qA\left(\frac{D_p p_{n0}}{L_p} + \frac{D_n n_{p0}}{L_n}\right)$$

The diode equation

$$I = I_0\left(e^{\frac{qV}{kT}} - 1\right)$$



Forward bias: exponential increase

Reverse bias: saturation at: $I = -I_0$ ← **Saturation current**

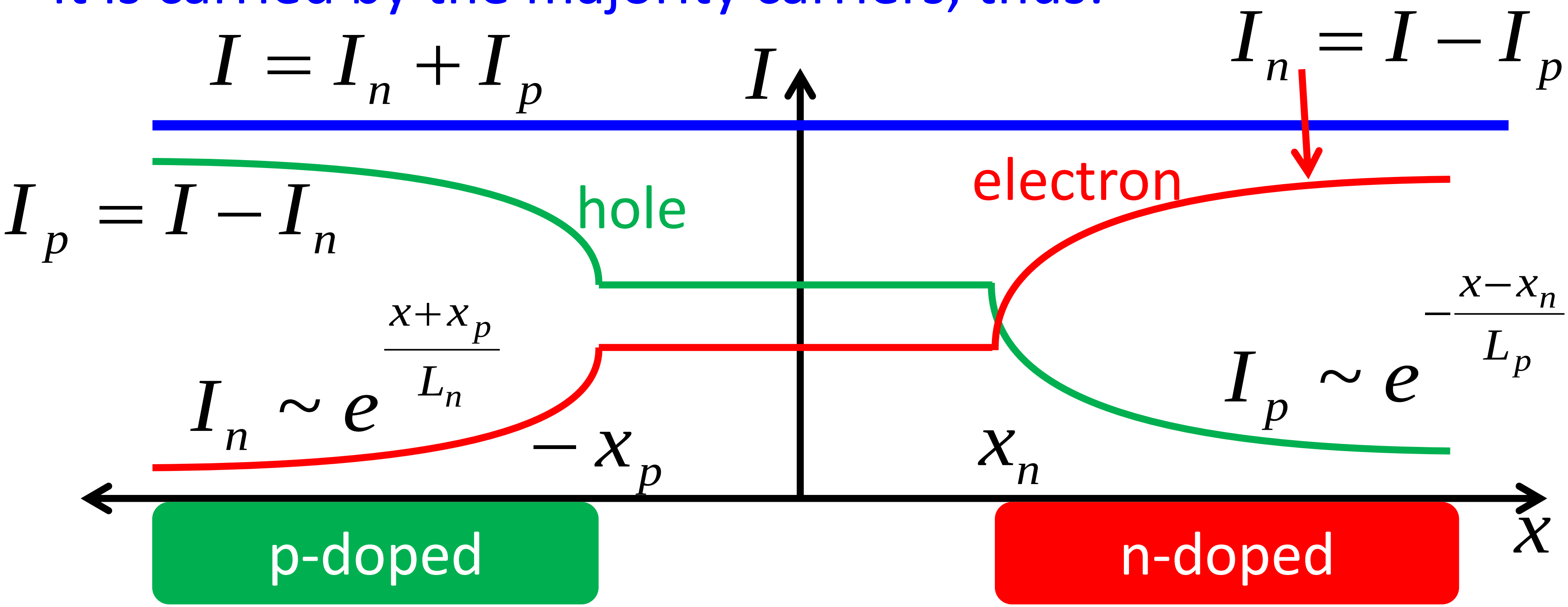
Now the previous calculation assumed that there was enough space in the diode for the exponential decrease:

$$L_{diode} >> L_{n,p}$$

Current though diode (4)

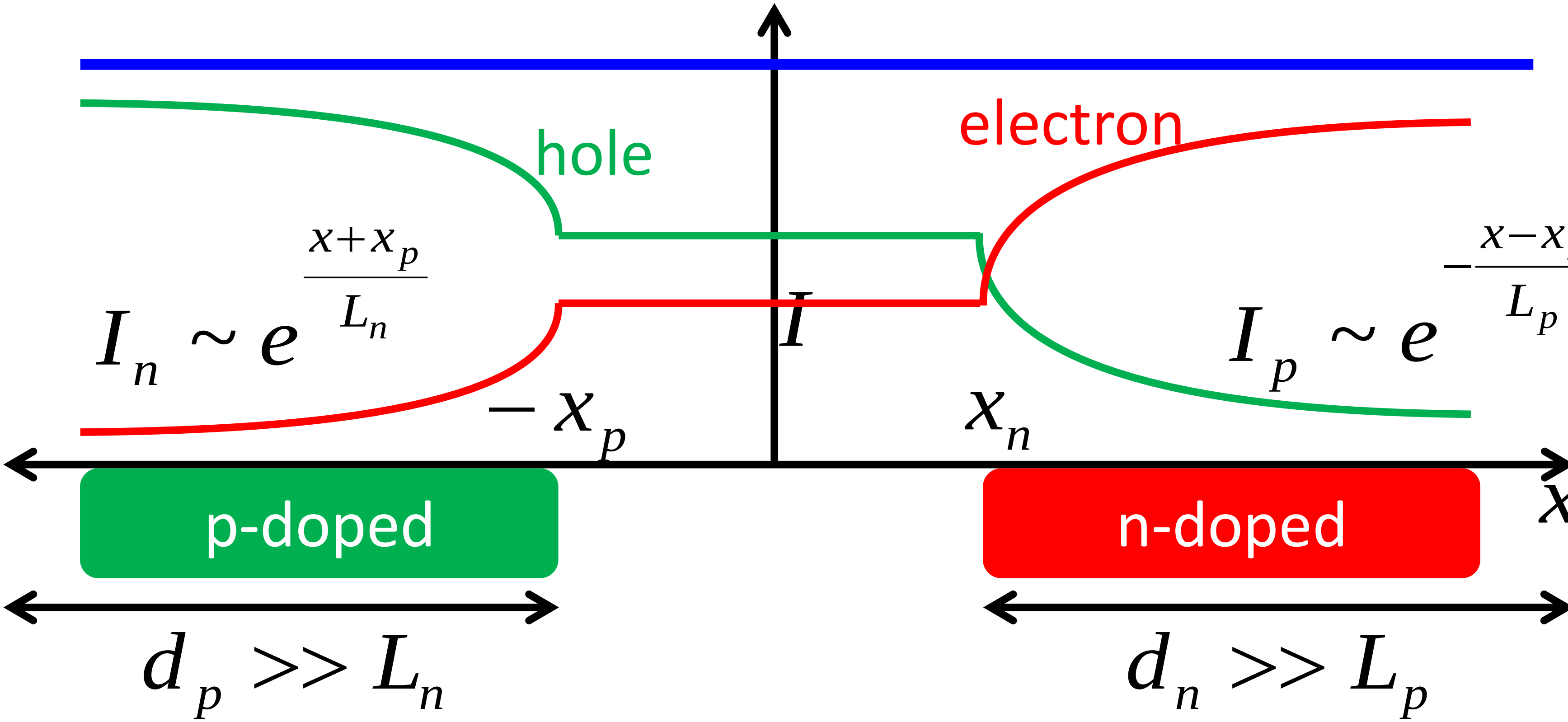
Note that we calculated the currents from the edge of the depletion region. But these excess minority carriers recombine. So what happens to the current?

It is carried by the majority carriers, thus:



General Case (1)

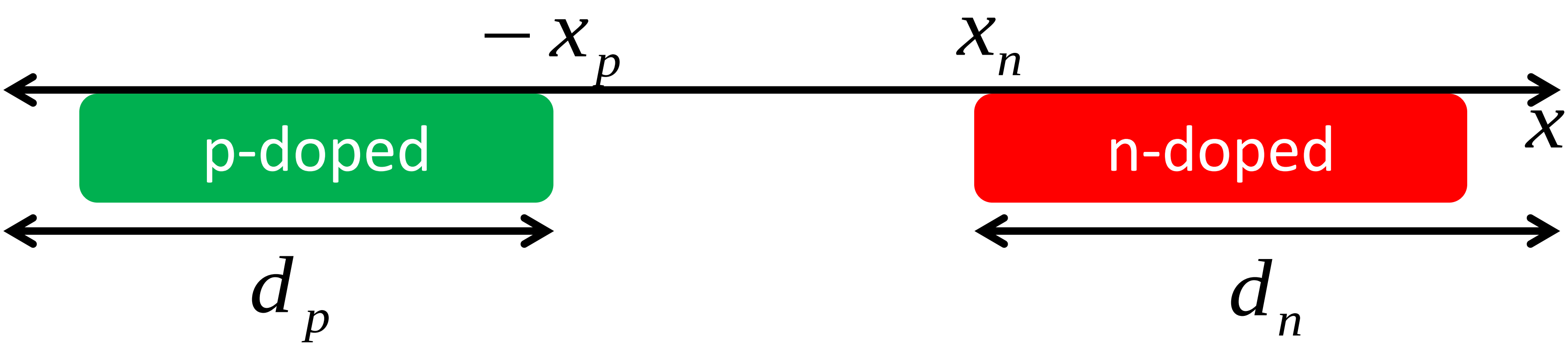
The case we just discussed…



Here the diode is very long, what if it is shorter?

General Case (2)

For a general solution…



We need to return to continuity equation

$$\Delta n_p(x) = Ae^{-\frac{x+x_p}{L_n}} + Be^{\frac{x+x_p}{L_n}}$$ from slide 11.10

General Case (3)

But, this time we have different boundary conditions:

$$\Delta n_p(-x_p) = \Delta n_p(-x_p)$$ as before, and

$$\Delta n_p(-x_p - d_p) = 0$$ Not: $\Delta n_p(-\infty) = 0$ as before

Inserting into the general solution…

$$\Delta n_p(-x_p) = A + B \rightarrow Ae^{\frac{d_p}{L_n}} + Be^{\frac{d_p}{L_n}} = \Delta n_p(-x_p)e^{\frac{d_p}{L_n}}$$

$$\Delta n_p(-x_p - d_p) = Ae^{\frac{d_p}{L_n}} + Be^{-\frac{d_p}{L_n}} = 0$$

Subtracting: $$B\left(e^{\frac{d_p}{L_n}} - e^{-\frac{d_p}{L_n}}\right) = \Delta n_p(-x_p)e^{\frac{d_p}{L_n}}$$

General Case (4)

Similarly:

$$\Delta n_p(-x_p) = A + B \rightarrow Ae^{-\frac{d_p}{L_n}} + Be^{-\frac{d_p}{L_n}} = \Delta n_p(-x_p)e^{-\frac{d_p}{L_n}}$$

$$\Delta n_p(-x_p - d_p) = Ae^{\frac{d_p}{L_n}} + Be^{-\frac{d_p}{L_n}} = 0$$

Again subtracting: $-A\left(e^{\frac{d_p}{L_n}} - e^{-\frac{d_p}{L_n}}\right) = \Delta n_p(-x_p)e^{-\frac{d_p}{L_n}}$

Thus:

$$A = -\frac{\Delta n_p(-x_p)e^{-\frac{d_p}{L_n}}}{\sinh\left(\frac{d_p}{L_n}\right)} \qquad B = \frac{\Delta n_p(-x_p)e^{\frac{d_p}{L_n}}}{\sinh\left(\frac{d_p}{L_n}\right)}$$

General Case (5)

Inserting into: $\Delta n_p(x) = Ae^{-\frac{x+x_p}{L_n}} + Be^{\frac{x+x_p}{L_n}}$

$$\Delta n_p(x) = -\frac{\Delta n_p(-x_p)e^{-\frac{d_p}{L_n}}}{\sinh\left(\frac{d_p}{L_n}\right)} e^{-\frac{x+x_p}{L_n}} + \frac{\Delta n_p(-x_p)e^{\frac{d_p}{L_n}}}{\sinh\left(\frac{d_p}{L_n}\right)} e^{\frac{x+x_p}{L_n}}$$

$$= \frac{\Delta n_p(-x_p)}{\sinh\left(\frac{d_p}{L_n}\right)}\left(e^{\frac{x+x_p+d_p}{L_n}} - e^{-\frac{x+x_p+d_p}{L_n}}\right)$$

General Case (6)

So, finally:

$$\Delta n_p(x) = \frac{n_{p0}\left(e^{\frac{qV_A}{kT}} - 1\right)\sinh\left(\frac{x + x_p + d_p}{L_n}\right)}{\sinh\left(\frac{d_p}{L_n}\right)}$$

And for the n-side

$$\Delta p_n(x) = -\frac{p_{n0}\left(e^{\frac{qV_A}{kT}} - 1\right)\sinh\left(\frac{x - x_n - d_n}{L_p}\right)}{\sinh\left(\frac{d_n}{L_p}\right)}$$

Limits to General Case (1)

For: $L_n << d_p \qquad L_n >> w_p$

$$\sinh\left(\frac{x+x_p+d_p}{L_n}\right) = e^{\frac{x+x_p+d_p}{L_n}} - e^{-\frac{x+x_p+d_p}{L_n}} \approx e^{\frac{d_p}{L_n}} e^{\frac{x+x_p}{L_n}}$$

$$\sinh\left(\frac{d_p}{L_n}\right) = e^{\frac{d_p}{L_n}} - e^{-\frac{d_p}{L_n}} \approx e^{\frac{d_p}{L_n}}$$

$$\rightarrow \Delta n_p(x) = n_{p0}\left(e^{\frac{qV_A}{kT}} - 1\right) e^{\frac{x+x_p}{L_n}}$$

Which is what we originally calculated in slide 11.11

Limits to General Case (2)

For: $L_p << d_n \qquad L_p >> w_n$

$$\sinh\left(\frac{x-x_n-d_n}{L_p}\right) = e^{\frac{x-x_n-d_n}{L_p}} - e^{-\frac{x-x_p-d_n}{L_p}} \approx -e^{\frac{d_p}{L_n}} e^{-\frac{x-x_p}{L_n}}$$

$$\sinh\left(\frac{d_n}{L_p}\right) = e^{\frac{d_n}{L_p}} - e^{-\frac{d_n}{L_p}} \approx e^{\frac{d_n}{L_p}}$$

$$\rightarrow \Delta p_n(x) = p_{n0}\left(e^{\frac{qV_A}{kT}} - 1\right) e^{-\frac{x-x_n}{L_n}}$$

Which is what we originally calculated in slide 11.11

Limits to General Case (3)

For: $L_n >> d_p \quad L_n >> w_p$

Taylor series for sinh: $\sinh(x) \underset{x\to 0}{\longrightarrow} x$

$$\Delta n_p(x) = n_{p0}\left(e^{\frac{qV_A}{kT}} - 1\right)\left(\frac{x + x_p + d_p}{d_p}\right)$$

Here there is a linear decay of the carriers

$$\Delta p_n(x) = p_{n0}\left(e^{\frac{qV_A}{kT}} - 1\right)\left(\frac{-x + x_n + d_n}{d_n}\right)$$

Current though diode (4)

We can revise our current calculation for the thin diode.

$$I_p = -qAD_p \frac{dp_n}{dx} \qquad I_n = qAD_n \frac{dn_p}{dx}$$

Thus: $$\frac{dp_n(x_n)}{dx} = -\frac{\Delta p_n(x_n)}{d_n} \qquad \frac{dn_p(-x_p)}{dx} = \frac{\Delta n_p(-x_p)}{d_p}$$

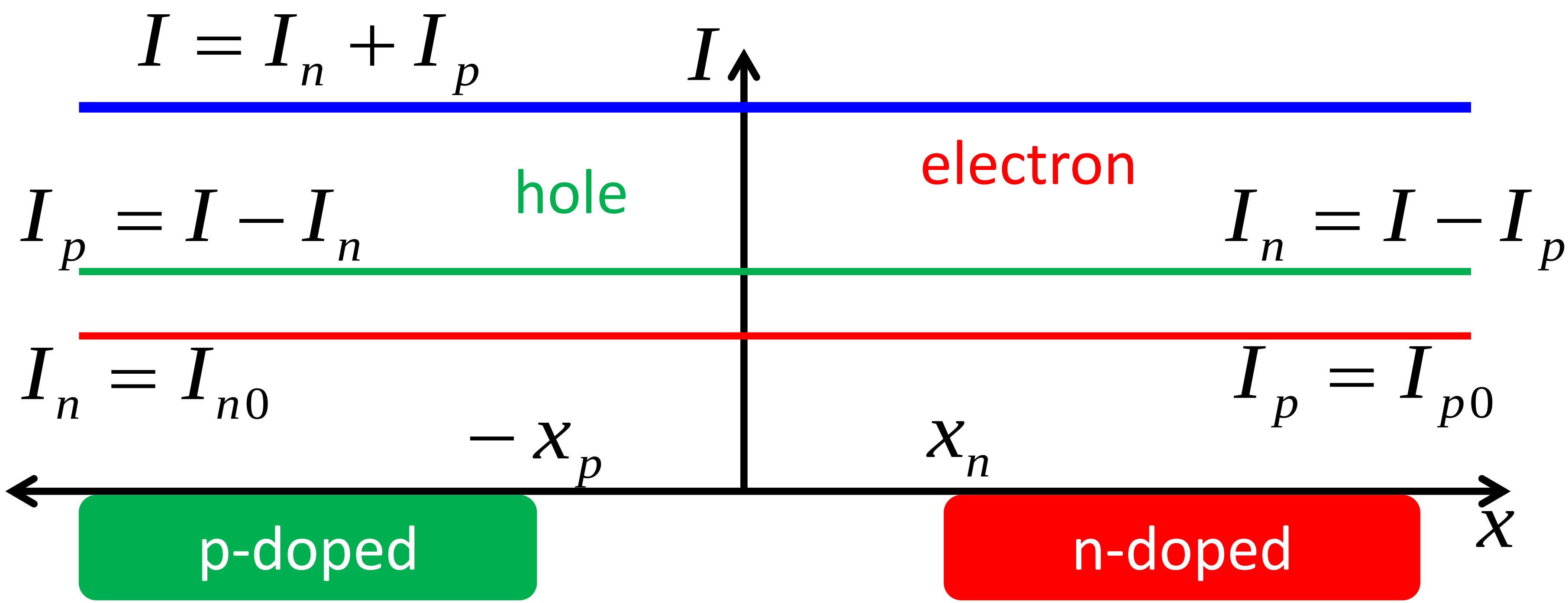
So: $$I_p = qAD_p \frac{\Delta p_n(x_n)}{d_n} \qquad I_n = qAD_n \frac{\Delta n_p(-x_p)}{d_p}$$

$$I_p = \frac{qAD_p}{d_n} p_{n0}\left(e^{\frac{qV_A}{kT}} - 1\right) \qquad I_n = \frac{qAD_n}{d_p} n_{p0}\left(e^{\frac{qV_A}{kT}} - 1\right)$$

Current though diode (5)

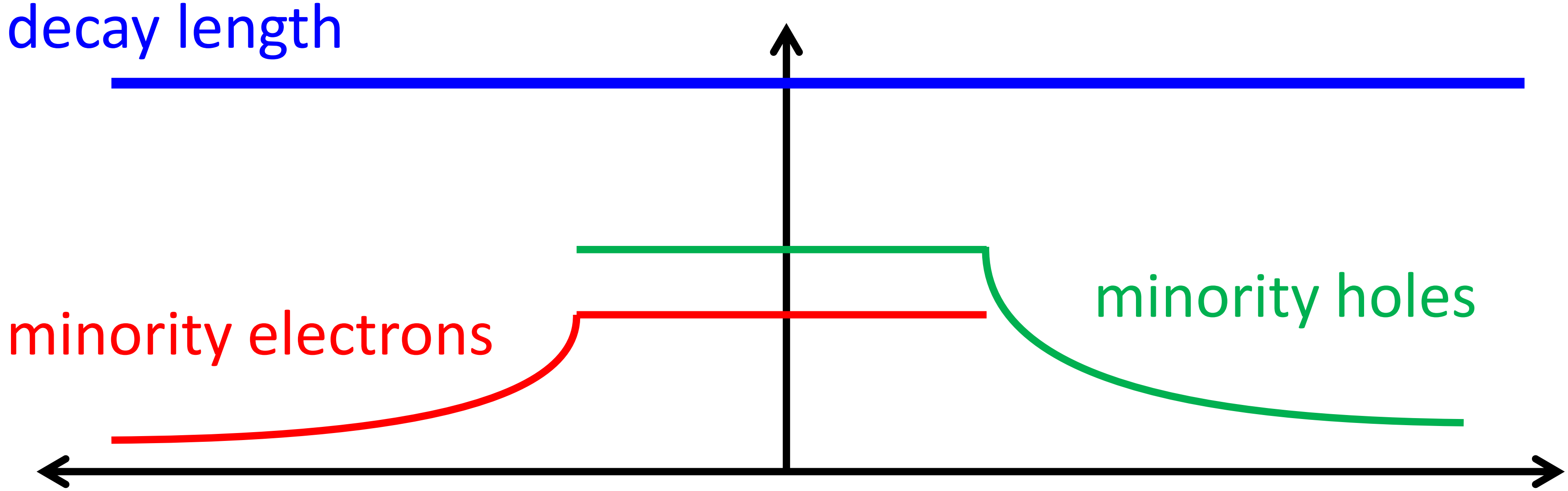
$$I = I_0\left(e^{\frac{qV_A}{kT}} - 1\right) \quad \text{where: } I_0 = qA\left(\frac{D_p p_{n0}}{d_n} + \frac{D_n n_{p0}}{d_p}\right)$$

The minority current is constant, so:



Forward Bias – Quasi Fermi (1)

Under forward bias, there was a large increase in the minority carriers in the depletion region, and over the decay length



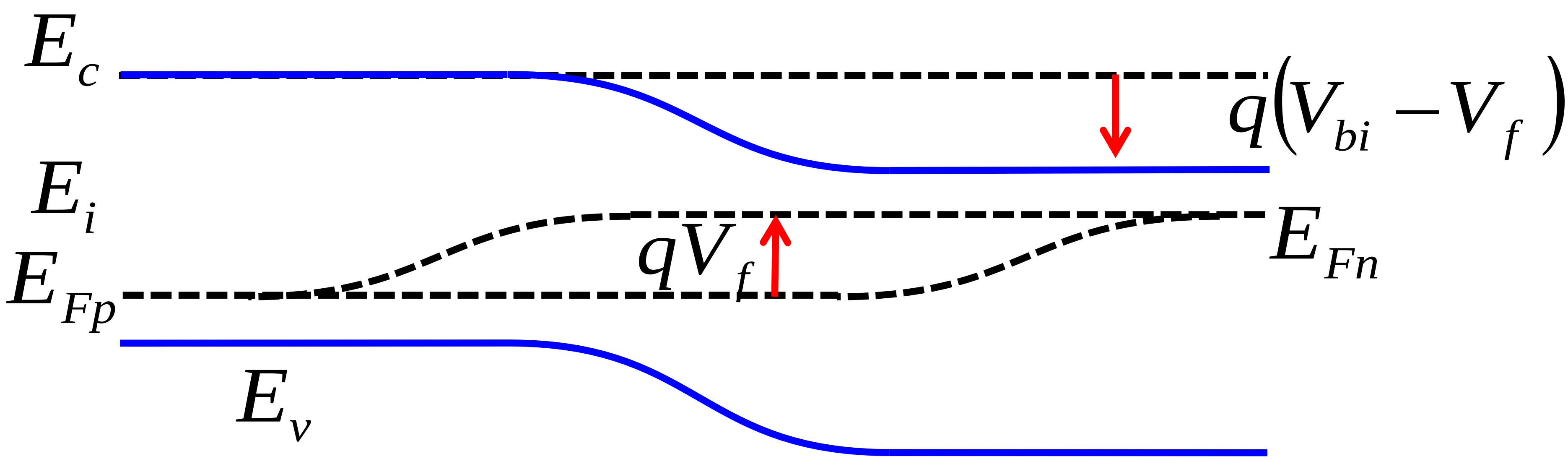
We invert the following:

$$n = n_i e^{\frac{E_{Fn} - E_i}{kT}} \qquad p = n_i e^{\frac{E_i - E_{Fp}}{kT}}$$

Forward Bias – Quasi Fermi (2)

Thus:

$$E_{Fn} = E_i + kT \ln\left(\frac{n_p(x)}{n_i}\right) \qquad E_{Fp} = E_i - kT \ln\left(\frac{p_n(x)}{n_i}\right)$$

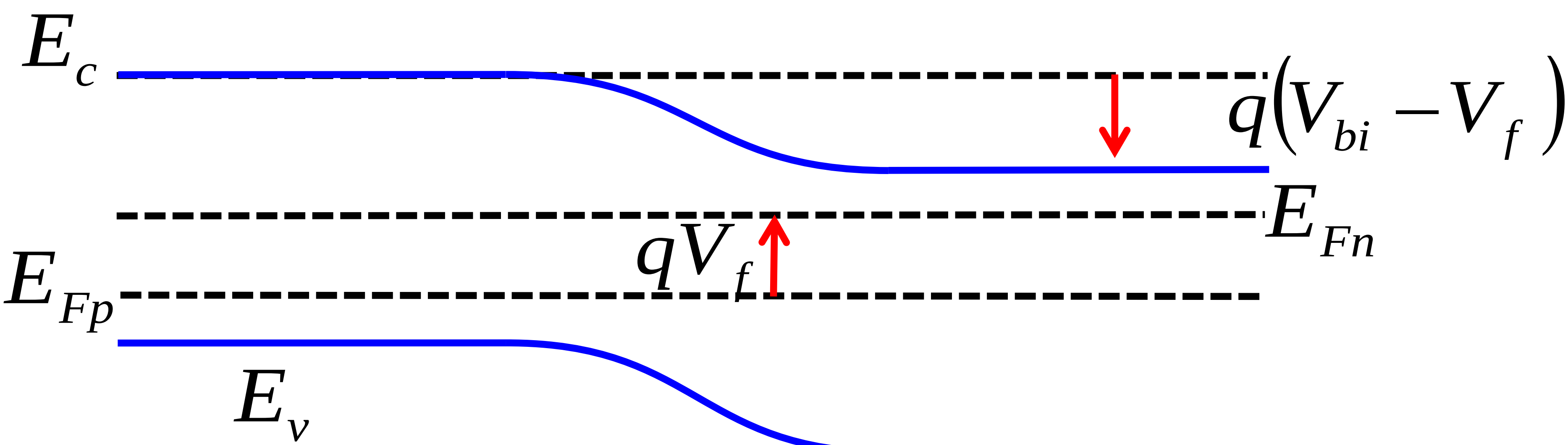


$$L << d$$

Forward Bias – Quasi Fermi (3)

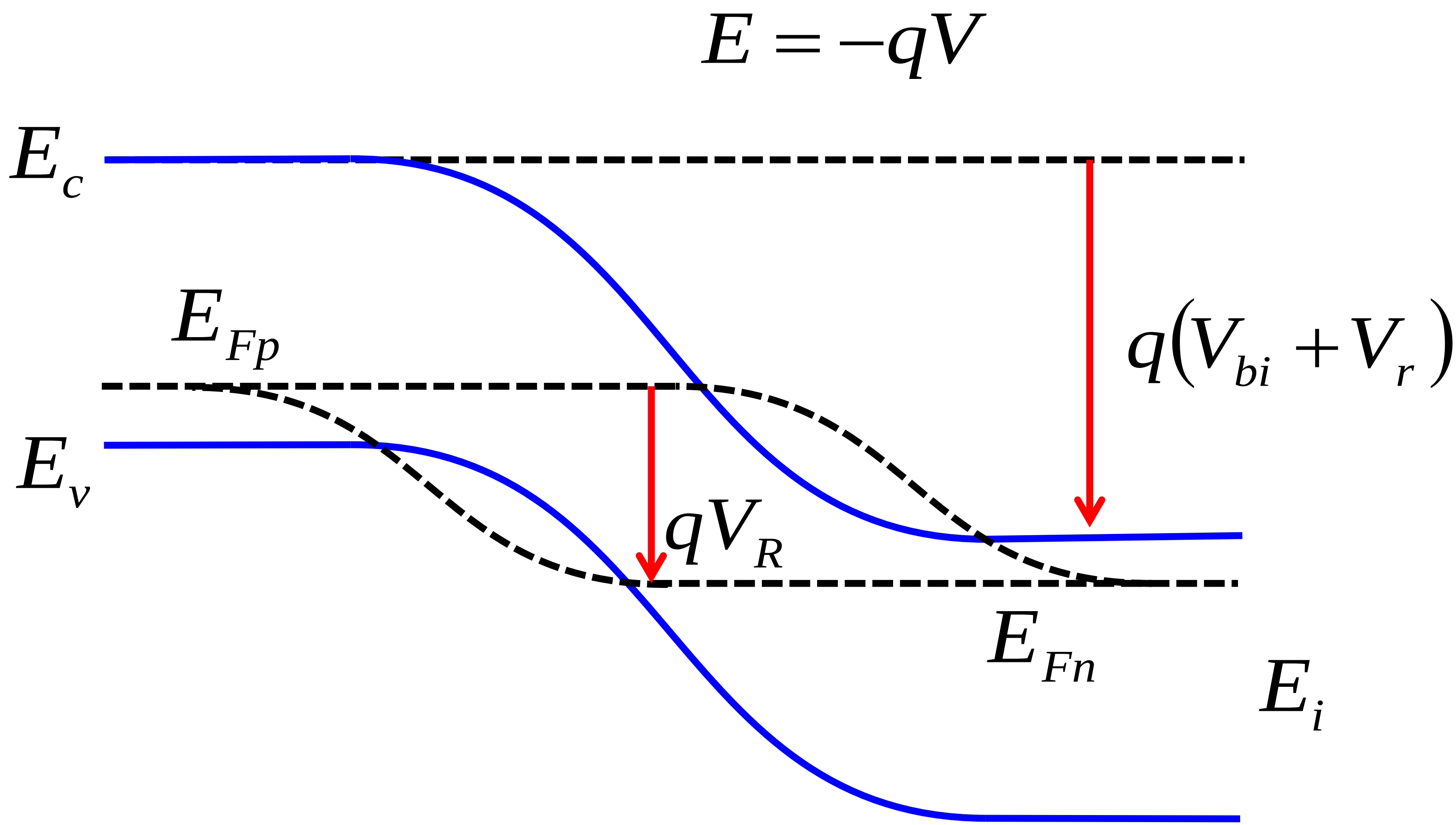
Thus:

$$E_{Fn} = E_i + kT\ln\left(\frac{n_p(x)}{n_i}\right) \qquad E_{Fp} = E_i - kT\ln\left(\frac{p_n(x)}{n_i}\right)$$



$$L >> d$$

Reverse Bias – Quasi Fermi (1)



Reverse Bias (1)

Under Reverse bias: $V_{bi} \rightarrow V_{bi} + V_R$

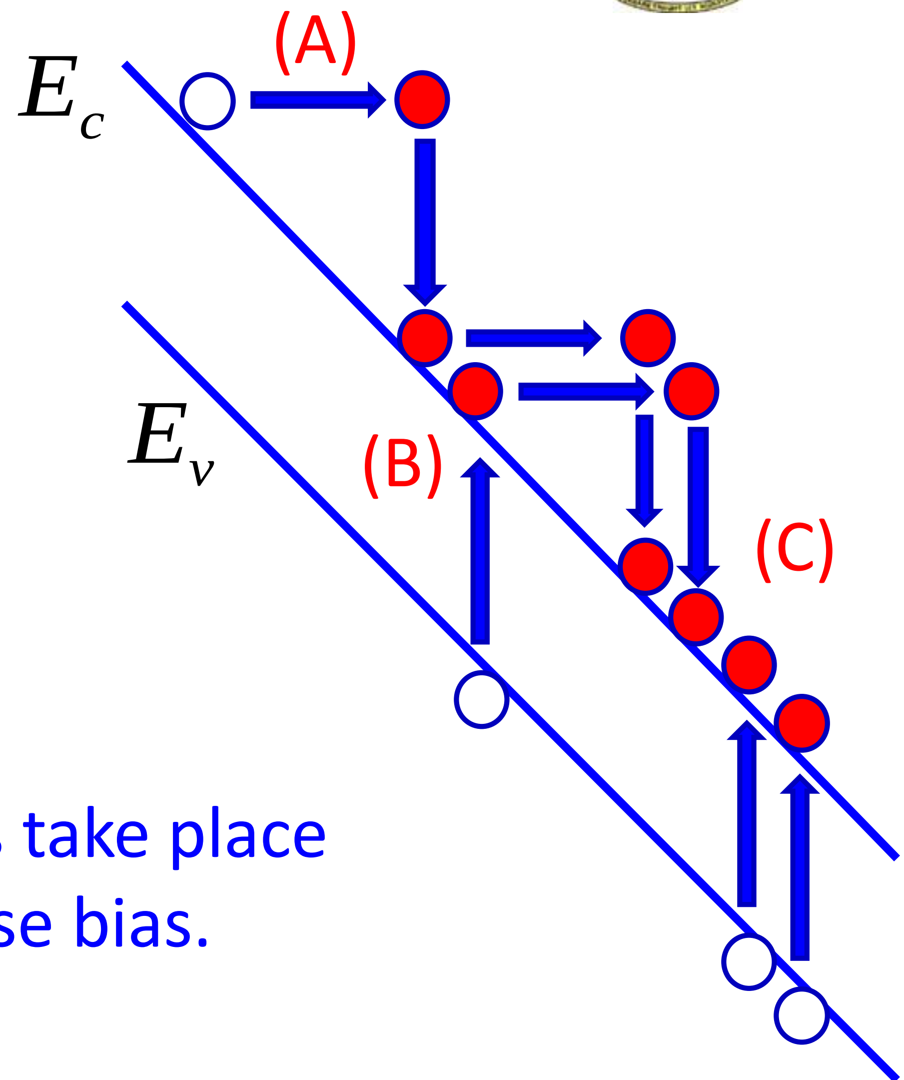
$$W = \sqrt{\frac{2\varepsilon\left(V_{bi} + V_R\right)}{q} \frac{N_D + N_A}{N_D N_A}} \propto \sqrt{V_{bi} + V_R}$$

The capacitance of the diode (parallel plate): $C = \frac{\varepsilon A}{d} \rightarrow \frac{\varepsilon A}{W}$

$$C = A\sqrt{\frac{\varepsilon q}{2\left(V_{bi} + V_R\right)} \frac{N_D N_A}{N_D + N_A}}$$

The capacitance gets smaller with increasing bias

Remember Impact Ionisation?



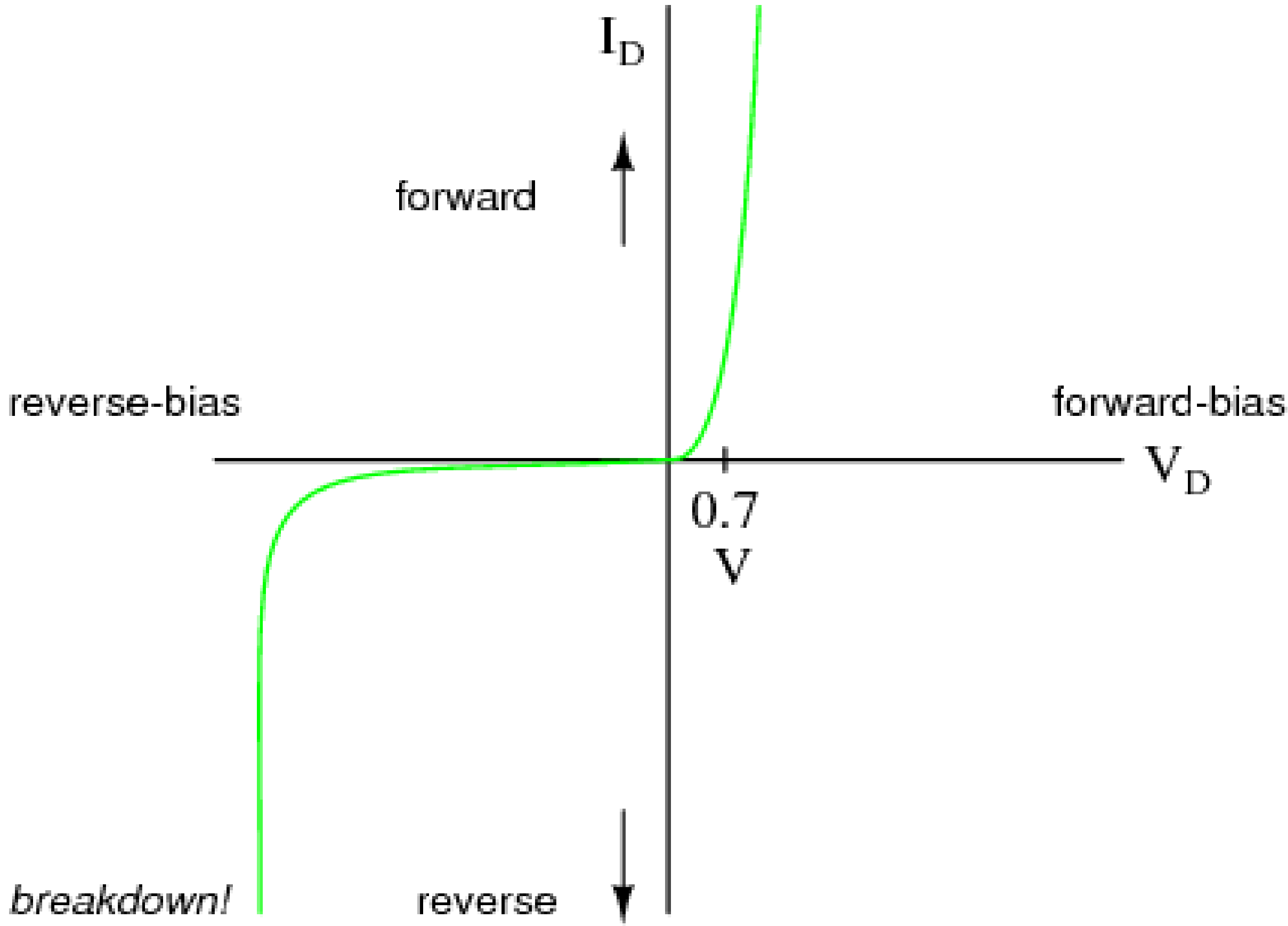
This is also referred to as Avalanche multiplication!

It happens when an electron accelerates so much that:

$$\frac{1}{2}mv^2 > E_g$$

When sufficient of these events take place the diode breaks down in reverse bias.

Reverse Bias Breakdown (1)



Reverse Bias Breakdown (2)

For a diode we already calculated the peak Electric Field:

$$\mathcal{E}(0) = -q\frac{N_D}{\varepsilon}x_n = -q\frac{N_A}{\varepsilon}x_p$$

where:

$$W = x_n + x_p = \sqrt{\frac{2\varepsilon(V_{bi}+V_R)}{q}\frac{N_D+N_A}{N_D N_A}}$$

Thus, for: $n >> p$

$$W \cong x_p = \sqrt{\frac{2\varepsilon(V_{bi}+V_R)}{q}\frac{1}{N_A}}$$

$n << p$

$$W \cong x_n = \sqrt{\frac{2\varepsilon(V_{bi}+V_R)}{q}\frac{1}{N_D}}$$

Reverse Bias Breakdown (3)

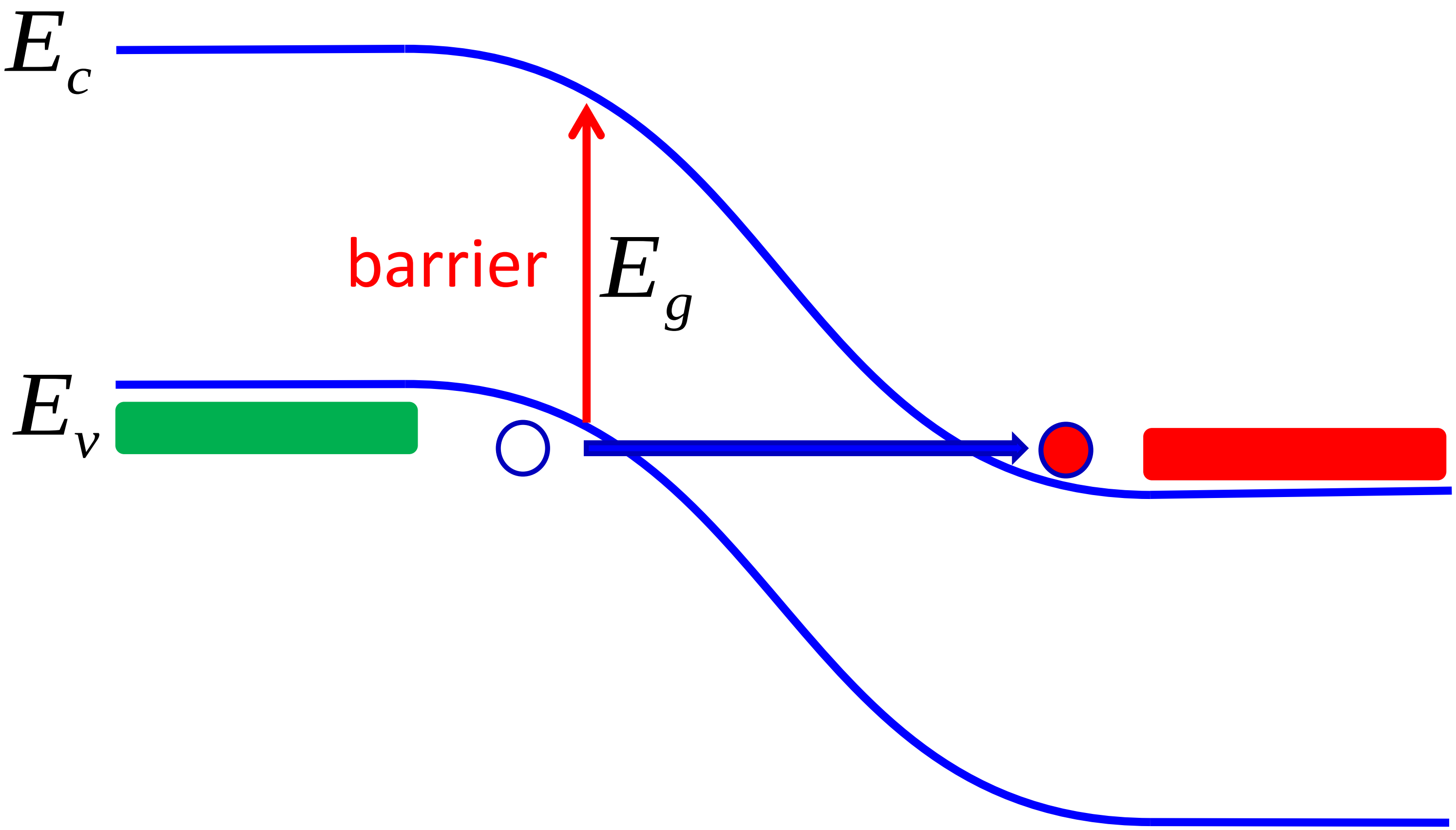
Inserting these into our equation for Electric Field:

$$\mathcal{E}(0) = -q\frac{N_A}{\varepsilon}\sqrt{\frac{2\varepsilon\left(V_{bi}+V_R\right)}{q}\frac{1}{N_A}} \quad \text{for} \quad n >> p$$

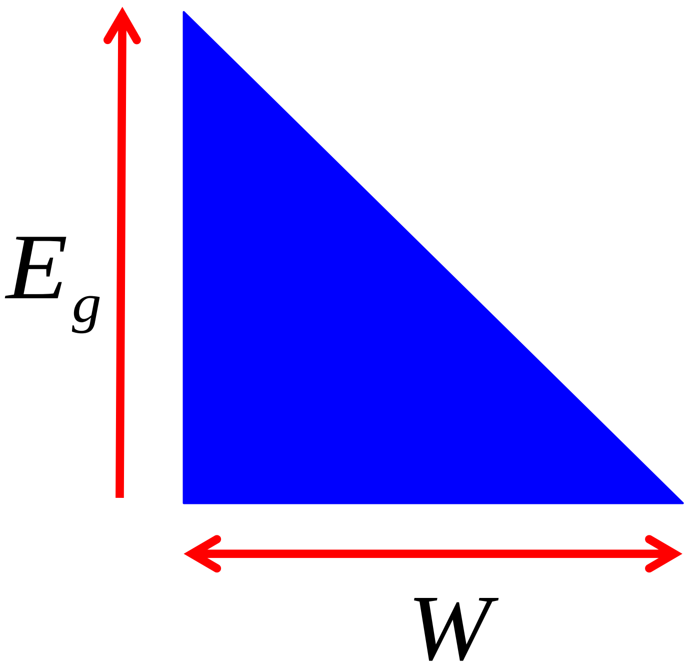
$$\frac{\varepsilon\mathcal{E}^2(0)}{qN_A} = 2\left(V_{bi}+V_R\right) \Rightarrow V_{BD} = \frac{\varepsilon\mathcal{E}^2_{critical}}{2qN_A} - V_{bi}$$

And for $n << p$ $\quad V_{BD} = \dfrac{\varepsilon\mathcal{E}^2_{critical}}{2qN_D} - V_{bi}$

Reverse Bias – Tunnelling (1)



Reverse Bias – Tunnelling (2)



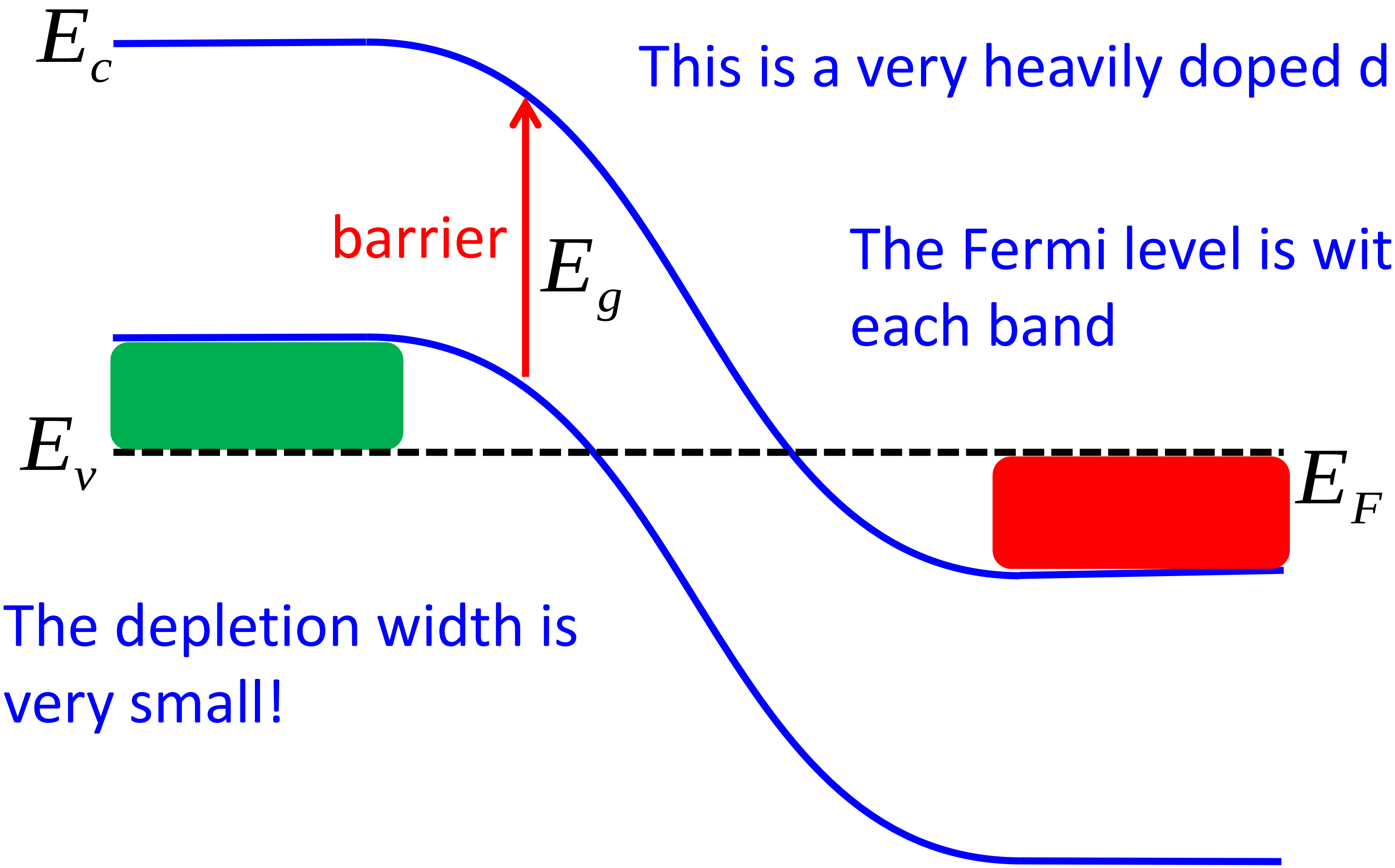
Using the WKB approximation, the QM tunnelling can be calculated as:

$$T = \exp\left(-\frac{4}{3\hbar q \mathcal{E}} \sqrt{2m} \left(E_g\right)^{\frac{3}{2}} \right)$$

Electric field

So for a given semiconductor the tunnelling increases with increasing electric field, which increases with decreasing depletion width or increasing voltage.

Tunnel Diode (1)

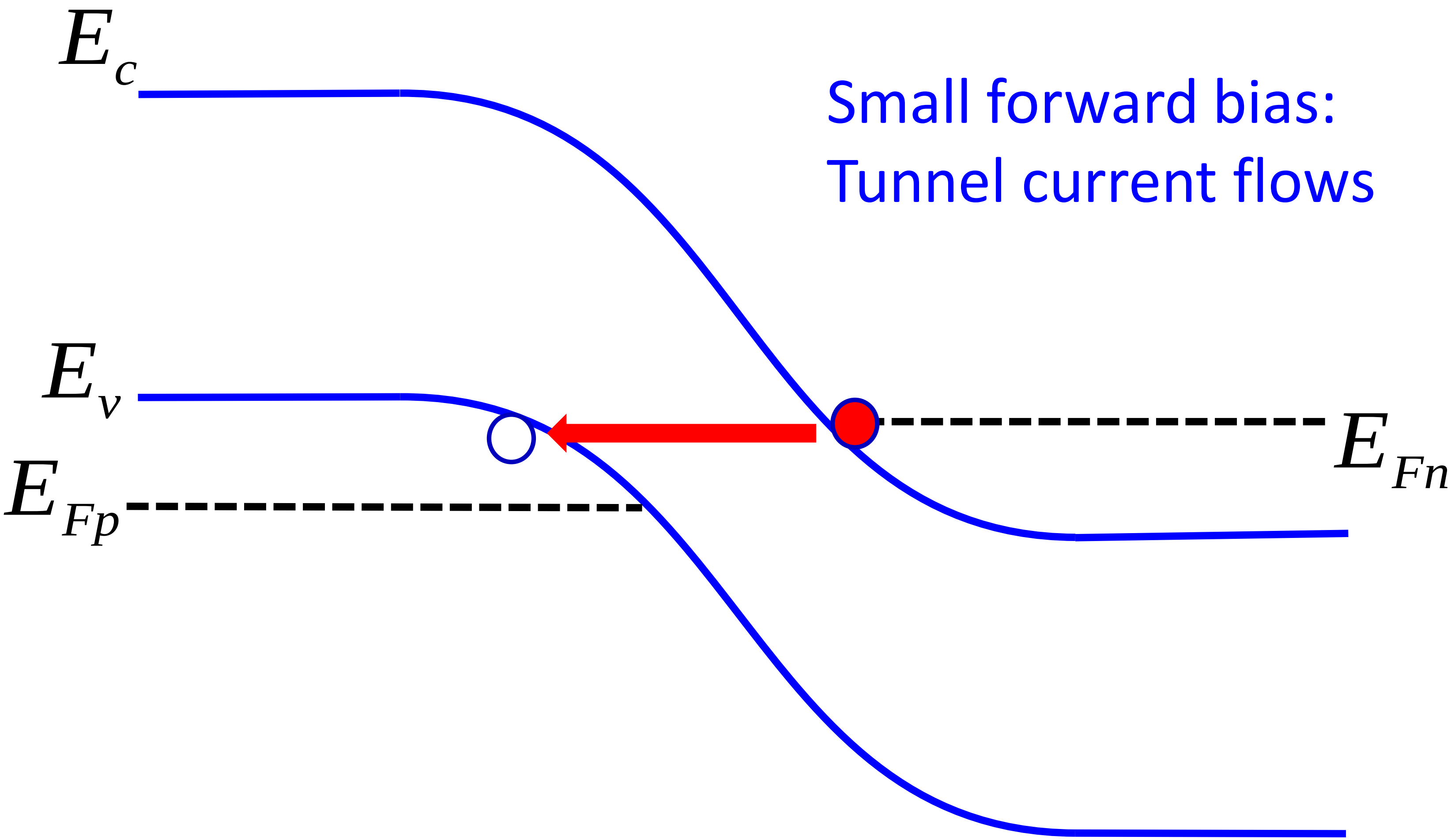


This is a very heavily doped diode

The Fermi level is within each band

The depletion width is very small!

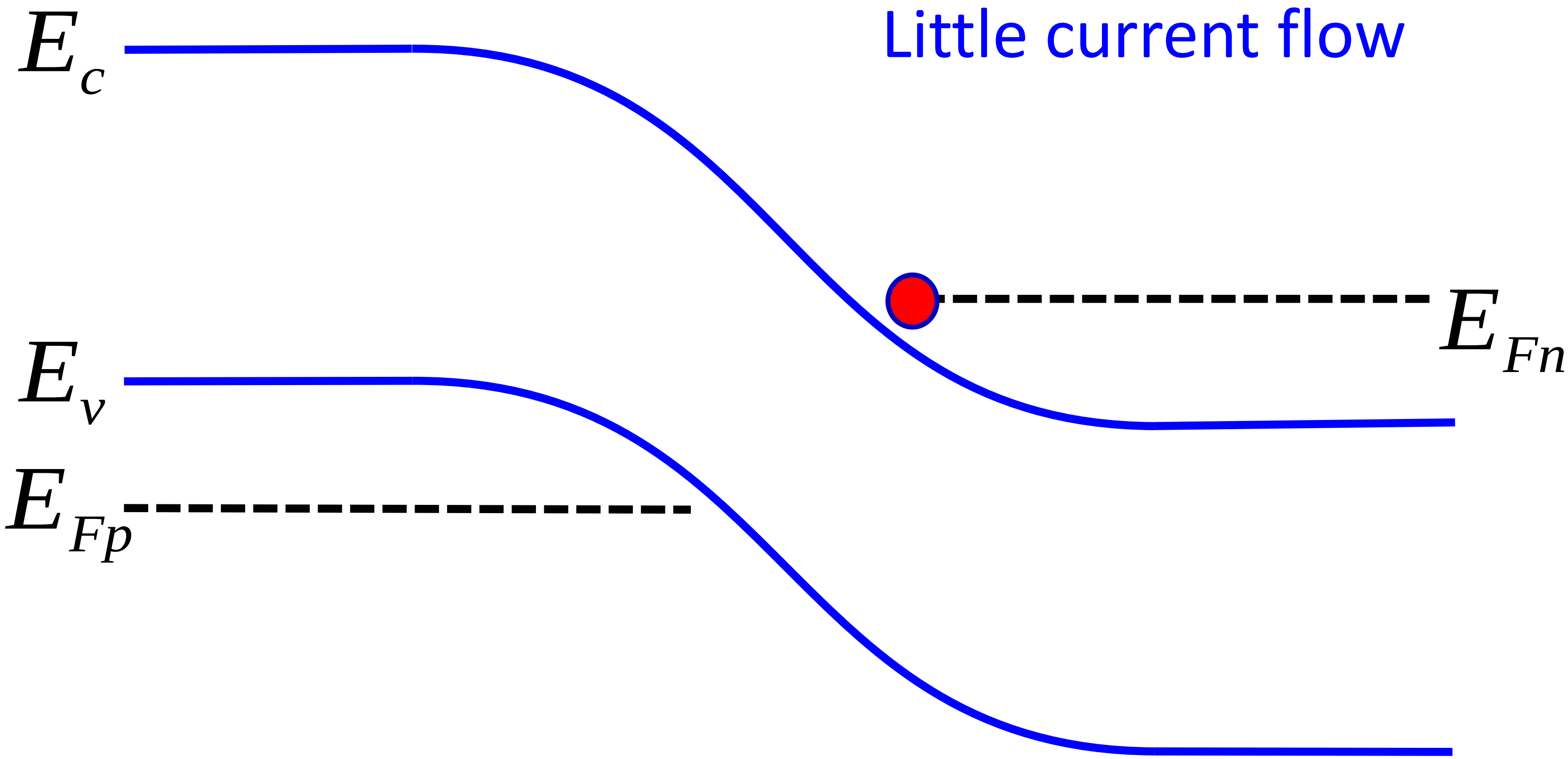
Tunnel Diode (2)



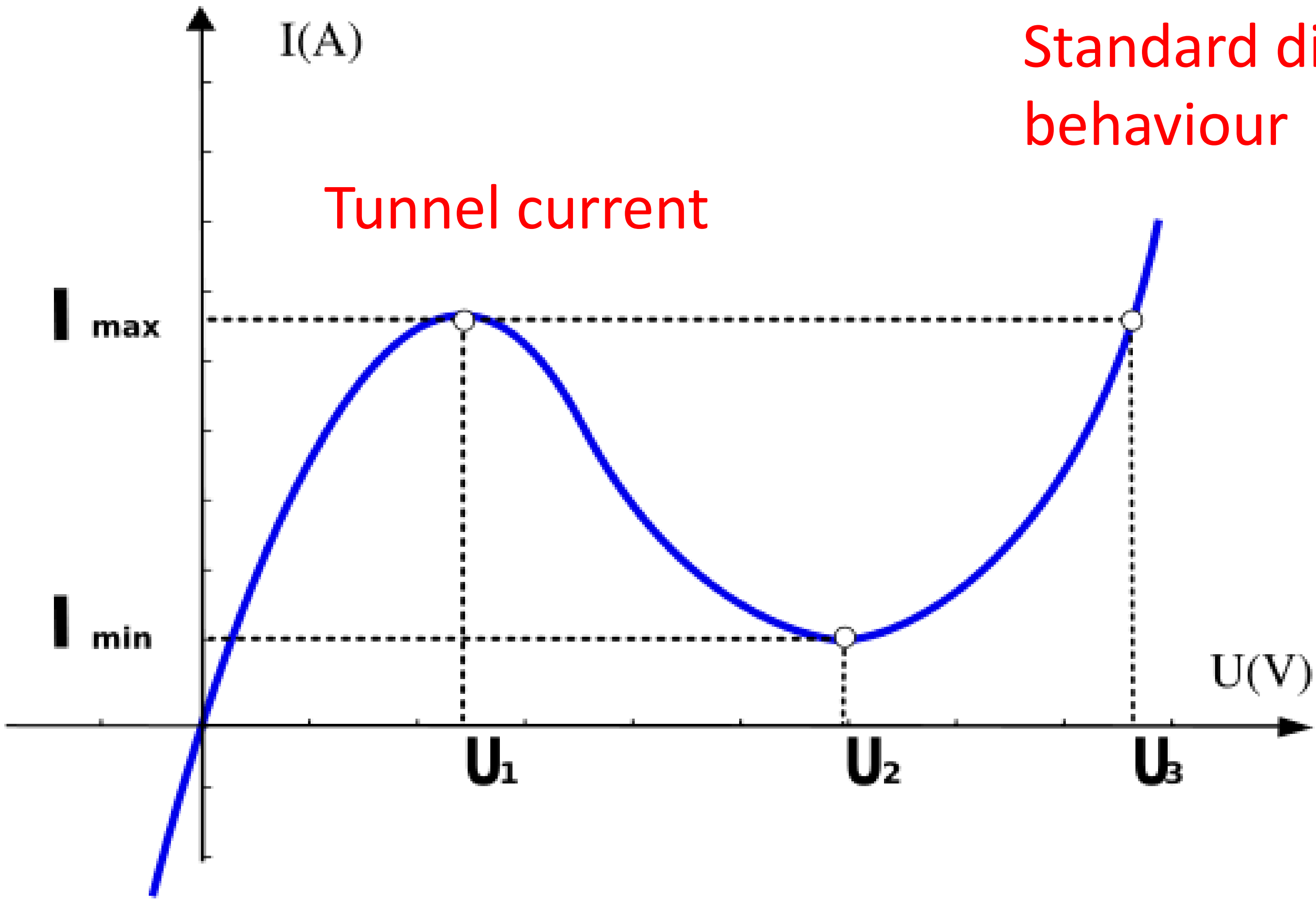
Small forward bias:
Tunnel current flows

Tunnel Diode (3)

Larger forward bias:
Little current flow

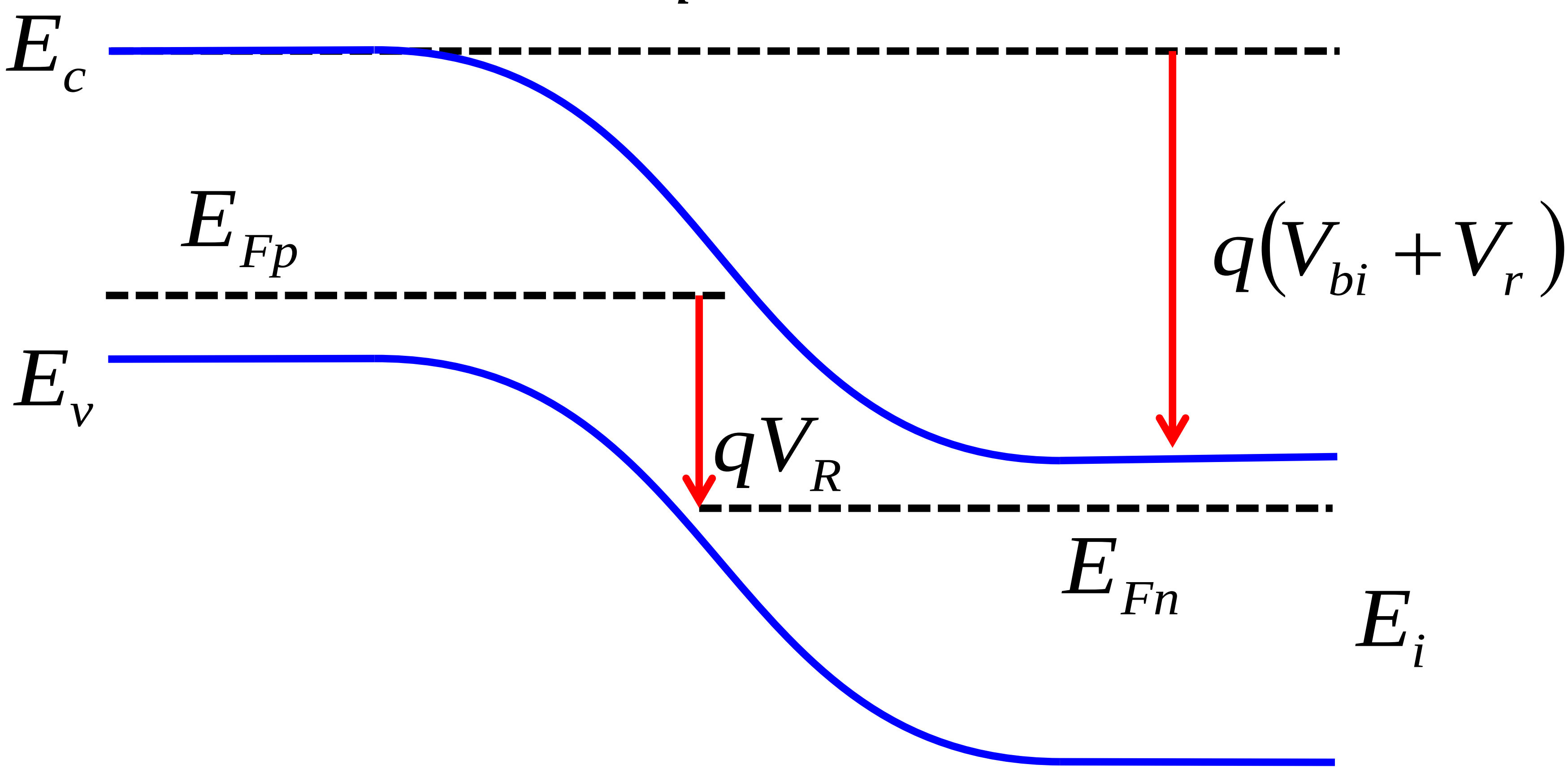


Tunnel Diode (4)



Photodiode photocurrent (1)

$$\mathcal{E} = \frac{1}{q}\frac{dE}{dx} < 0$$



Photodiode photocurrent (2)

Uniform light generation at steady state in a photodiode:

$$\cancel{\frac{dp_n}{dt}} = \cancel{-\frac{p_n - p_{n_0}}{\tau_p}} - \frac{1}{q}\frac{dJ_p}{dx} + G$$

In the depletion region assume no recombination

$$\frac{dJ_p}{dx} = qG \quad \rightarrow \quad \int_{J_p(-x_p)}^{J_p(x_n)} J_p = qG \int_{-x_p}^{x_n} dx$$

$$J_p(x_n) - J_p(-x_p) = qG(x_n + x_p)$$

$$\rightarrow J_n(x_n) - J_n(-x_p) = -qG(x_n + x_p)$$

Photodiode photocurrent (3)

But, all photogenerated electrons are swept to the n-region, and all photogenerated holes are swept to the p-region

Thus: $J_p(x_n) = J_n(-x_p) = 0$

And: $J_p(-x_p) = J_n(x_n) = -qG(x_n + x_p)$

So the total current: $J_T = J_p + J_n$

This must be constant through the device.

$$@x = -x_p : J_T = J_p(-x_p) + J_n(-x_p) = -qG(x_n + x_p)$$

$$@x = x_n : J_T = J_p(x_n) + J_n(x_n) = -qG(x_n + x_p)$$

Photodiode photocurrent (4)

The dark current was given by:

$$I = I_0\left(e^{\frac{qV_A}{kT}} - 1\right) \quad \text{where: } I_0 = qA\left(\frac{D_p p_{n0}}{L_p} + \frac{D_n n_{p0}}{L_n}\right)$$

The current from illumination is:

$$I_L = AJ_T = -qAG(x_n + x_p)$$

So, the total current is: $I = I_0\left(e^{\frac{qV_A}{kT}} - 1\right) - I_L$

Photodiode photocurrent (5)

Short circuit: $V_A = 0 \quad \rightarrow I_{SC} = -I_L$

Open circuit: $I = 0 = I_0\left(e^{\frac{qV_{OC}}{kT}} - 1\right) - I_L$

$$\frac{qV_{OC}}{kT} = \ln\left(\frac{I_L}{I_0} + 1\right) \quad \rightarrow V_{OC} = \frac{kT}{q}\ln\left(\frac{I_L}{I_0} + 1\right)$$

Large reverse bias: $I = I_0\left(e^{-\frac{qV_R}{kT}} - 1\right) - I_L \Rightarrow -(I_0 + I_L)$

$$I_L = -qAG\left(x_n + x_p\right)$$