
Physics PY4118

Physics of Semiconductor Devices

The Diode – Part A

The depletion region

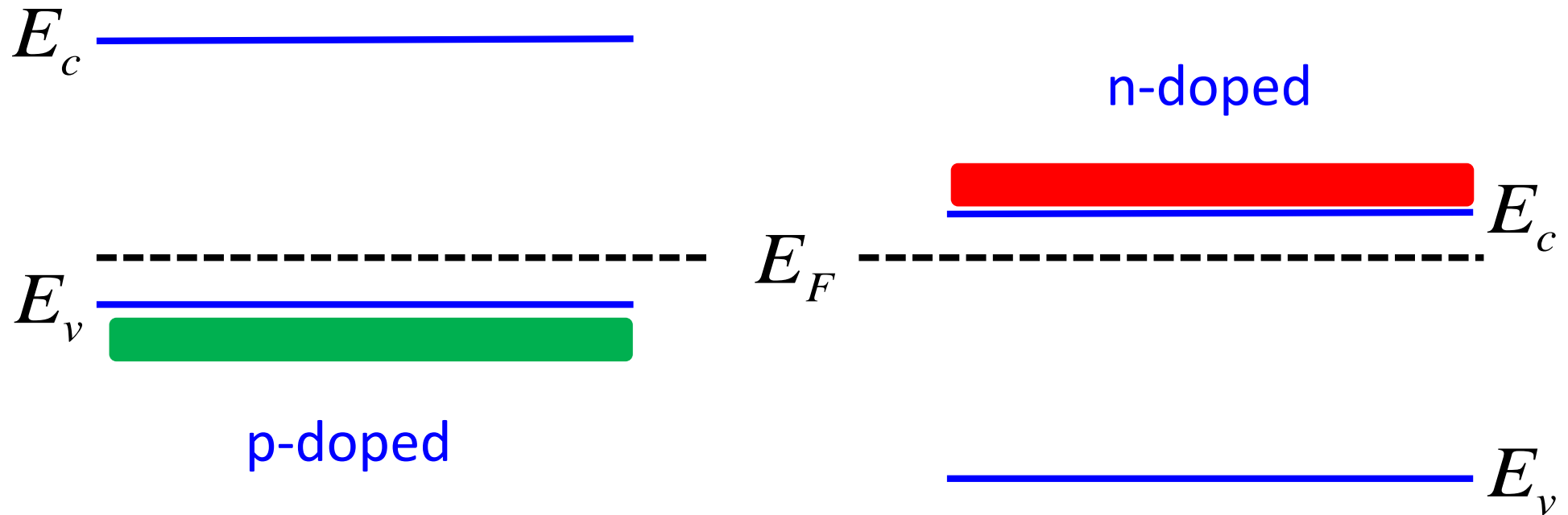
Focus



We can now start to analyse devices properly!

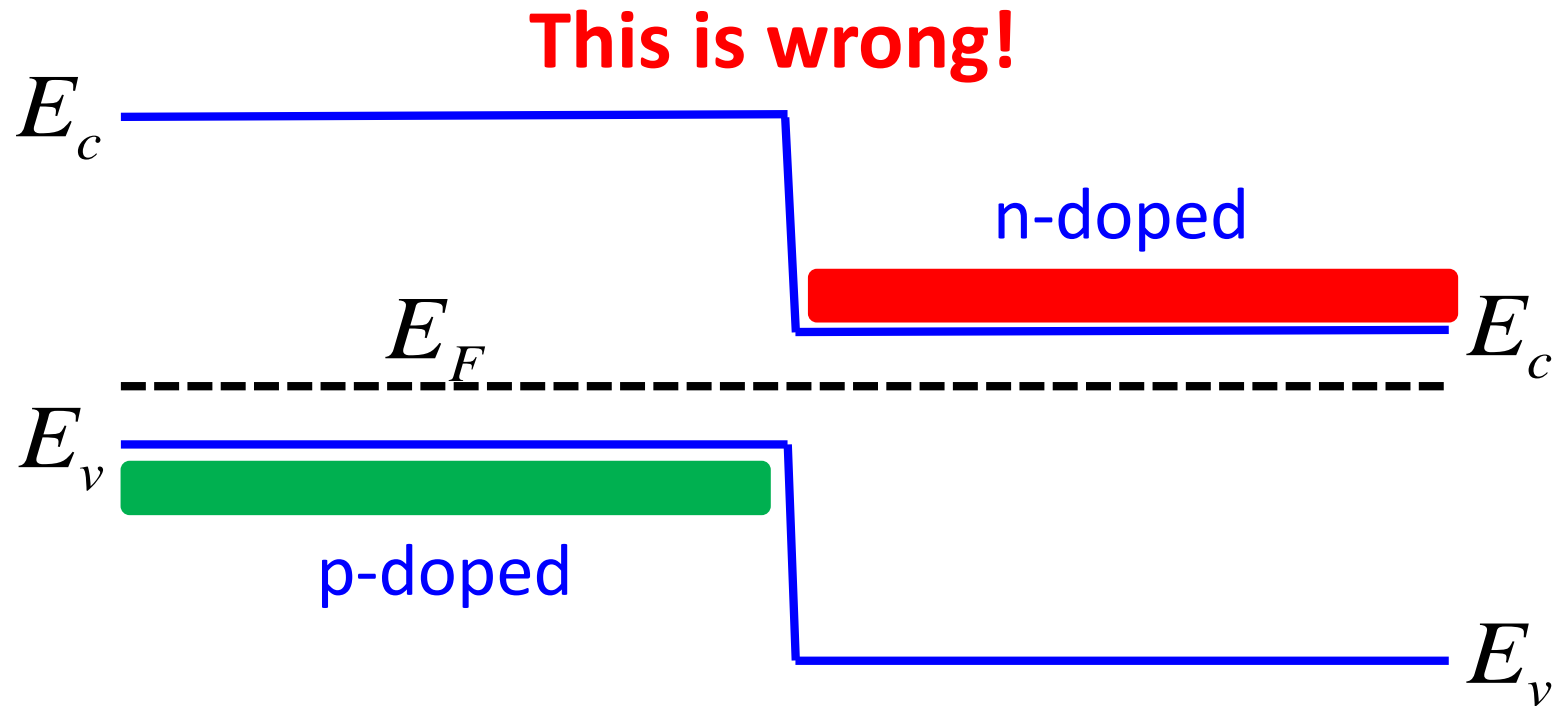
The first device is the diode, or the PN junction

PN Junction (1)



If there is no net current flow, the Fermi energy must be constant. How do the parts come together?

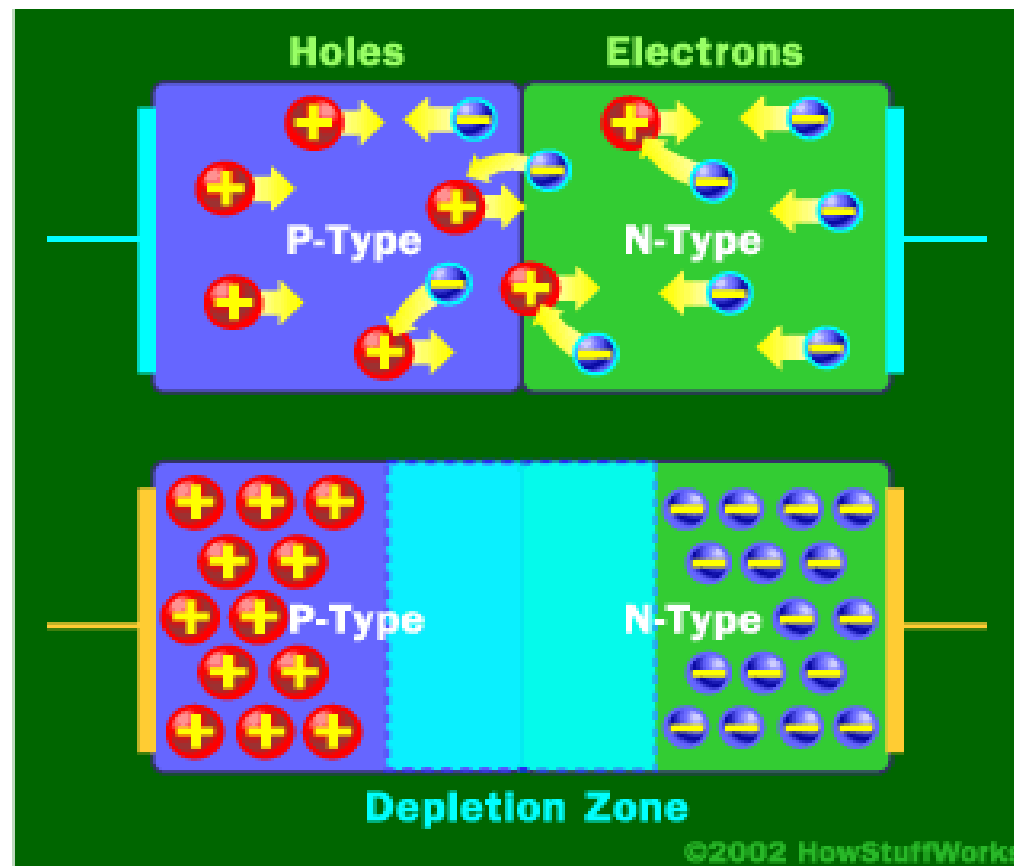
PN Junction (2)



What would happen in this case?

What does this band structure imply?

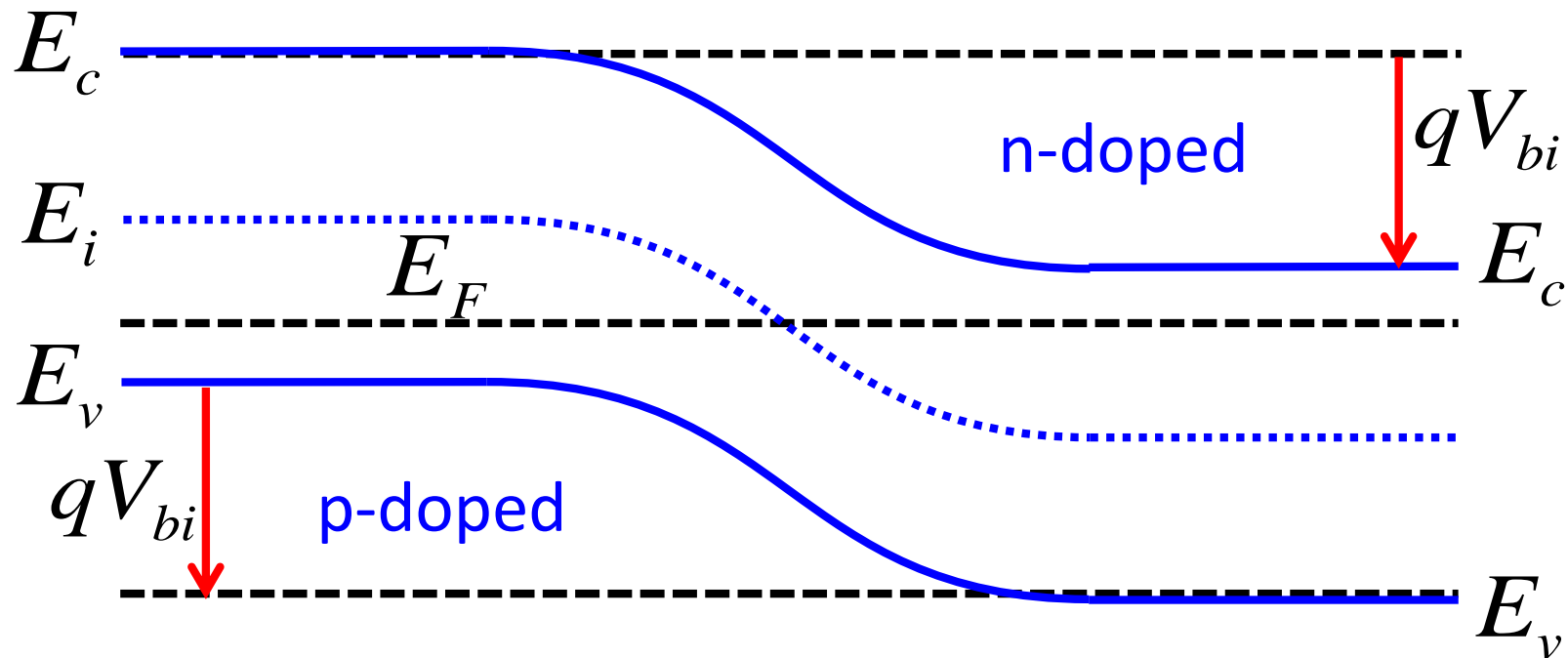
$$\rightarrow \mathcal{E} = \nabla \frac{E}{q}$$



At the junction, free electrons from the N-type material fill holes from the P-type material. This creates an insulating layer in the middle of the diode called the depletion zone.

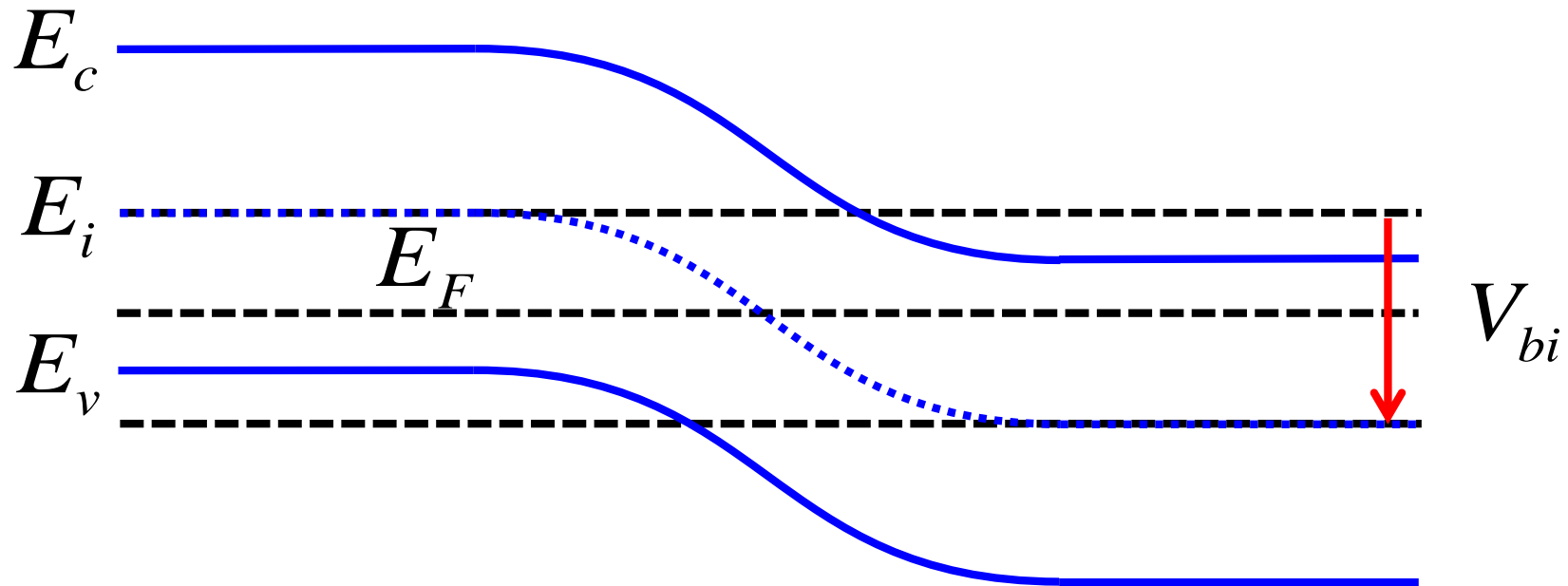
Built in voltage (1)

$$E = -qV \quad \rightarrow \quad V = -\frac{E}{q}$$



How large is: V_{bi} ?

Built in voltage (2)



$$qV_{bi} = (E_i - E_F)_p + (E_F - E_i)_n$$

In slide set 6: $n_n = n_i e^{\frac{E_F - E_i}{kT}}$ $p_p = n_i e^{\frac{E_i - E_F}{kT}}$

Built in voltage (3)



$$(E_F - E_i)_n = kT \ln\left(\frac{n_n}{n_i}\right) \quad (E_i - E_F)_p = kT \ln\left(\frac{p_p}{n_i}\right)$$

$$qV_{bi} = kT \ln\left(\frac{p_p}{n_i}\right) + kT \ln\left(\frac{n_n}{n_i}\right) = kT \ln\left(\frac{p_p n_n}{n_i^2}\right)$$

And since: $p_p n_{p0} = p_{n0} n_n = n_i^2$ (0 – i.e. no bias)

$$V_{bi} = \frac{kT}{q} \ln\left(\frac{p_p}{p_{n0}}\right) = \frac{kT}{q} \ln\left(\frac{n_n}{n_{p0}}\right)$$

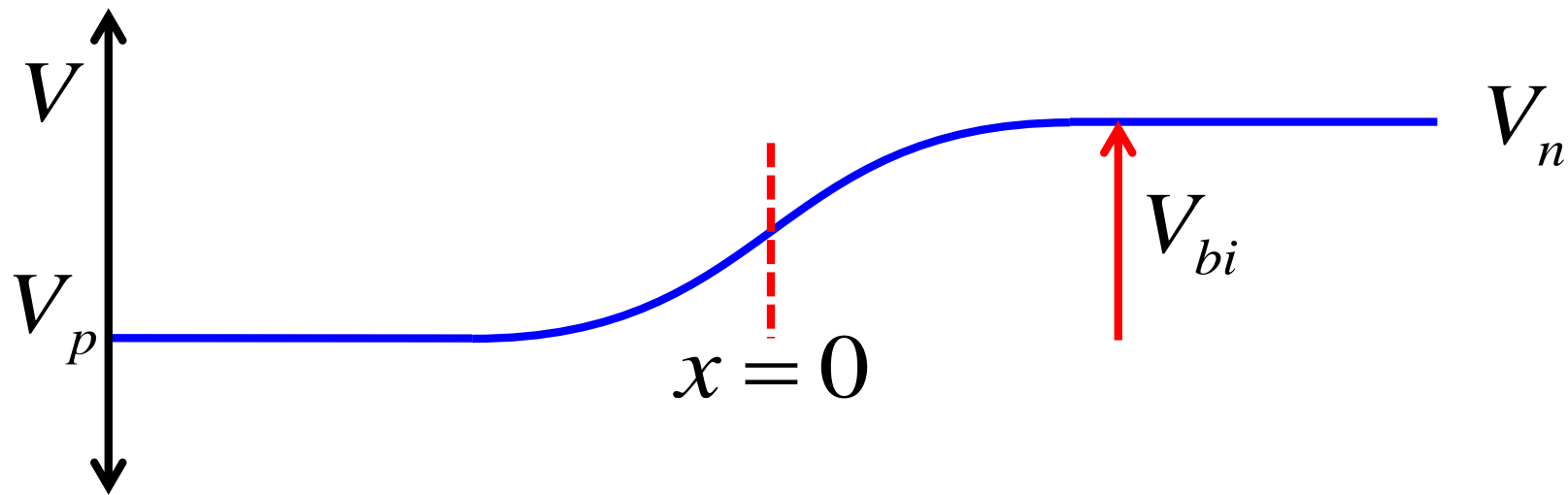
Built in voltage (4)

And, for completeness, since: $p_p = N_A$

$$n_n = N_D$$

$$V_{bi} = \frac{kT}{q} \ln \left(\frac{N_A N_D}{n_i^2} \right)$$

$$E = -qV$$



Built in voltage (5)



The built in voltage can be calculated another way as well.
Using Slide 7.20:

$$J_n = \mu_n \left(qn\mathcal{E} + kT \frac{dn}{dx} \right) \quad J_p = \mu_p \left(qp\mathcal{E} - kT \frac{dp}{dx} \right)$$

The current density must be zero when unbiased, thus:

$$qn\mathcal{E} + kT \frac{dn}{dx} = 0 \quad qp\mathcal{E} - kT \frac{dp}{dx} = 0$$

Remember that: $V = -\int \mathcal{E} dx$

Built in voltage (6)



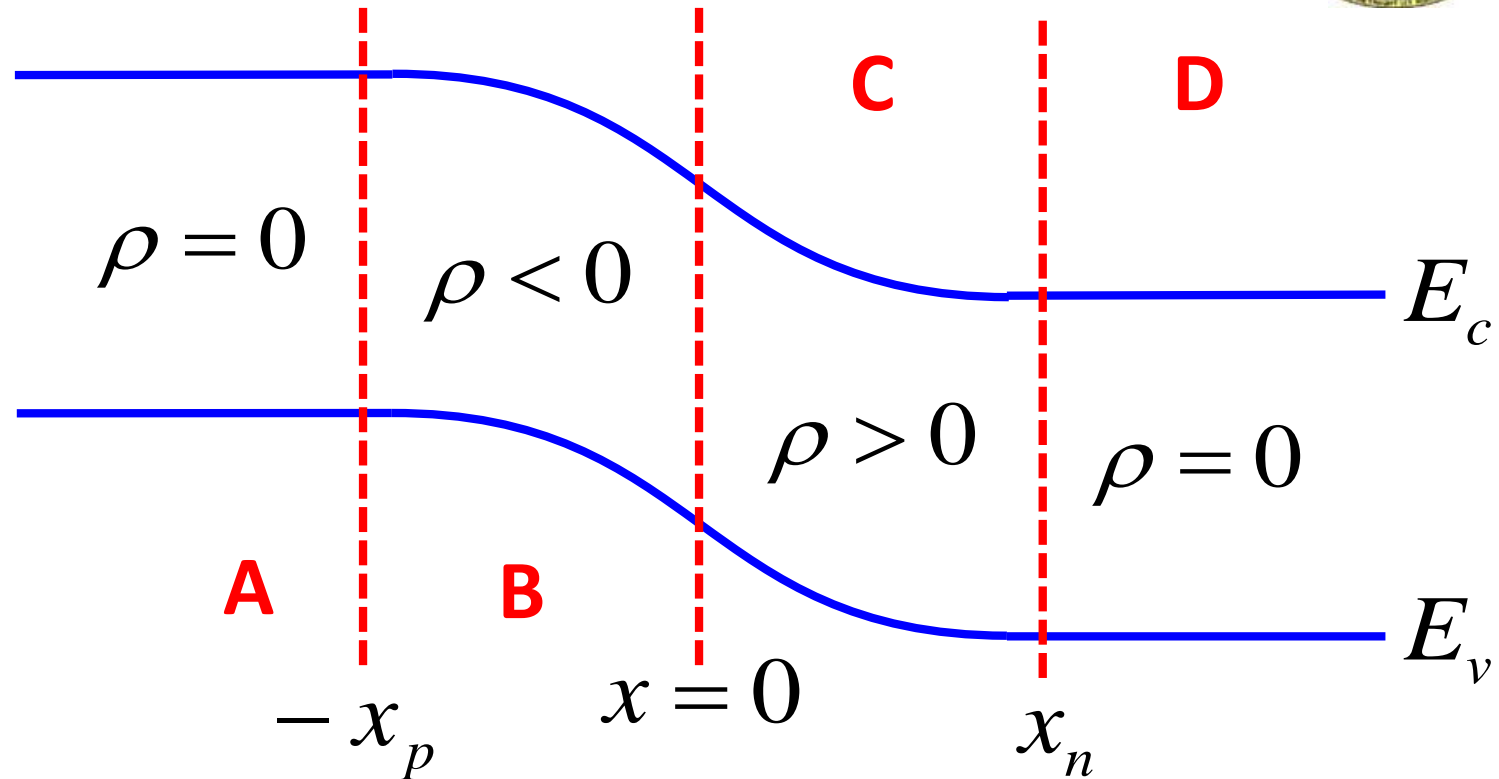
$$\frac{dn}{n} = -\frac{q}{kT} \mathcal{E} dx = \frac{q}{kT} dV \quad \frac{dp}{p} = \frac{q}{kT} \mathcal{E} dx = -\frac{q}{kT} dV$$

Integrate over depletion region:

$$\int_{n_{p0}}^n \frac{dn}{n} = \frac{q}{kT} \int_{V_p}^{V_n} dV \quad \rightarrow \quad \ln\left(\frac{n}{n_{p0}}\right) = \frac{q}{kT} (V_n - V_p)$$

$$V_{bi} = V_n - V_p = \frac{kT}{q} \ln\left(\frac{n}{n_{p0}}\right) = \frac{kT}{q} \ln\left(\frac{p}{p_{n0}}\right)$$

PN Junction (3)



Regions A & D – no net charge

Regions B – net negative charge (due to hole diffusion)

Regions C – net positive charge (due to electron diffusion)

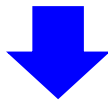
PN Junction (4)



Gauss's Law: $\nabla \cdot \mathcal{E} = \frac{\rho}{\epsilon}$

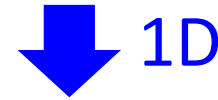


$$\frac{d}{dx} \mathcal{E}_x = \frac{\rho}{\epsilon}$$

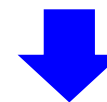


Regions A & D: $\frac{d}{dx} \mathcal{E}_x = 0$

$$\mathcal{E} = -\nabla V$$



$$\mathcal{E} = -\frac{d}{dx} V = \frac{d}{dx} \frac{E}{q}$$



$$\mathcal{E} = 0$$

$$V = -\int \mathcal{E} dx \rightarrow V = \text{const} = V_{n,p}$$

PN Junction (5)



Region B: $\frac{d}{dx} \mathcal{E} = -q \frac{N_A}{\varepsilon} \rightarrow \mathcal{E} = -q \frac{N_A}{\varepsilon} x + C_1$

But at: $x = -x_p \quad \mathcal{E} = 0 \rightarrow C_1 = -q \frac{N_A}{\varepsilon} x_p$

$$V = -\int \mathcal{E} dx \rightarrow V = q \frac{N_A}{2\varepsilon} x^2 + q \frac{N_A}{\varepsilon} x_p x + C_2$$

But at: $x = -x_p \quad V = V_p \rightarrow C_2 = V_p + q \frac{N_A}{2\varepsilon} x_p^2$

Thus at: $x = 0 \quad \mathcal{E} = -q \frac{N_A}{\varepsilon} x_p \rightarrow V = V_p + q \frac{N_A}{2\varepsilon} x_p^2$

PN Junction (6)



Region C: $\frac{d}{dx} \mathcal{E} = q \frac{N_D}{\varepsilon} \rightarrow \mathcal{E} = q \frac{N_D}{\varepsilon} x + C_3$

But at: $x = x_n \quad \mathcal{E} = 0 \rightarrow C_3 = -q \frac{N_D}{\varepsilon} x_n$

$$V = -\int \mathcal{E} dx \rightarrow V = -q \frac{N_D}{2\varepsilon} x^2 + q \frac{N_A}{\varepsilon} x_n x + C_4$$

But at: $x = x_n \quad V = V_n \rightarrow C_4 = V_n - q \frac{N_D}{2\varepsilon} x_n^2$

Thus at: $x = 0 \quad \mathcal{E} = -q \frac{N_D}{\varepsilon} x_n \rightarrow V = V_n - q \frac{N_D}{2\varepsilon} x_n^2$

PN Junction (7)



At $x = 0$ the electric field and potential are continuous

$$\mathcal{E}: \quad x_n N_D = x_p N_A$$

$$V: \quad V_n - q \frac{N_D}{2\epsilon} x_n^2 = V_p + q \frac{N_A}{2\epsilon} x_p^2$$

$$\begin{aligned} V_{bi} = V_n - V_p &= \frac{q}{2\epsilon} \left(\frac{N_D^2 x_n^2}{N_D} + \frac{N_A^2 x_p^2}{N_A} \right) \\ &= \frac{q}{2\epsilon} \frac{N_D + N_A}{N_D N_A} N_D^2 x_n^2 = \frac{q}{2\epsilon} \frac{N_D + N_A}{N_D N_A} N_A^2 x_p^2 \end{aligned}$$

Depletion Width (1)

We use this to calculate the depletion width:

$$W = x_n + x_p$$
$$x_n = \frac{1}{N_D} \sqrt{\frac{2\varepsilon V_{bi}}{q} \frac{N_D N_A}{N_D + N_A}}$$
$$x_p = \frac{1}{N_A} \sqrt{\frac{2\varepsilon V_{bi}}{q} \frac{N_D N_A}{N_D + N_A}}$$



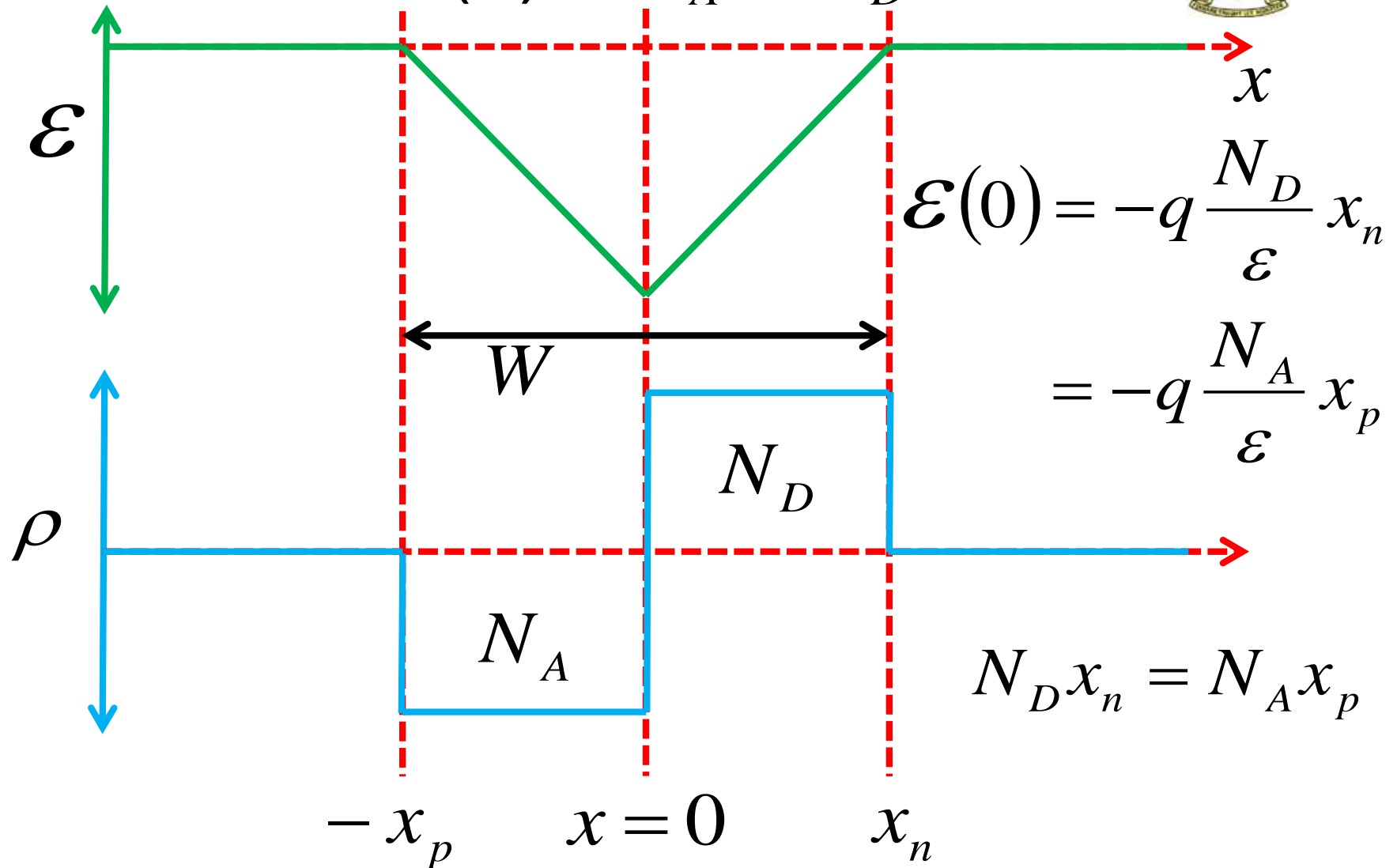
$$\rightarrow W(V_{bi}) = \sqrt{\frac{2\varepsilon V_{bi}}{q} \frac{N_D + N_A}{N_D N_A}}$$

PN Junction (8)

$$N_A = N_D$$



UCC

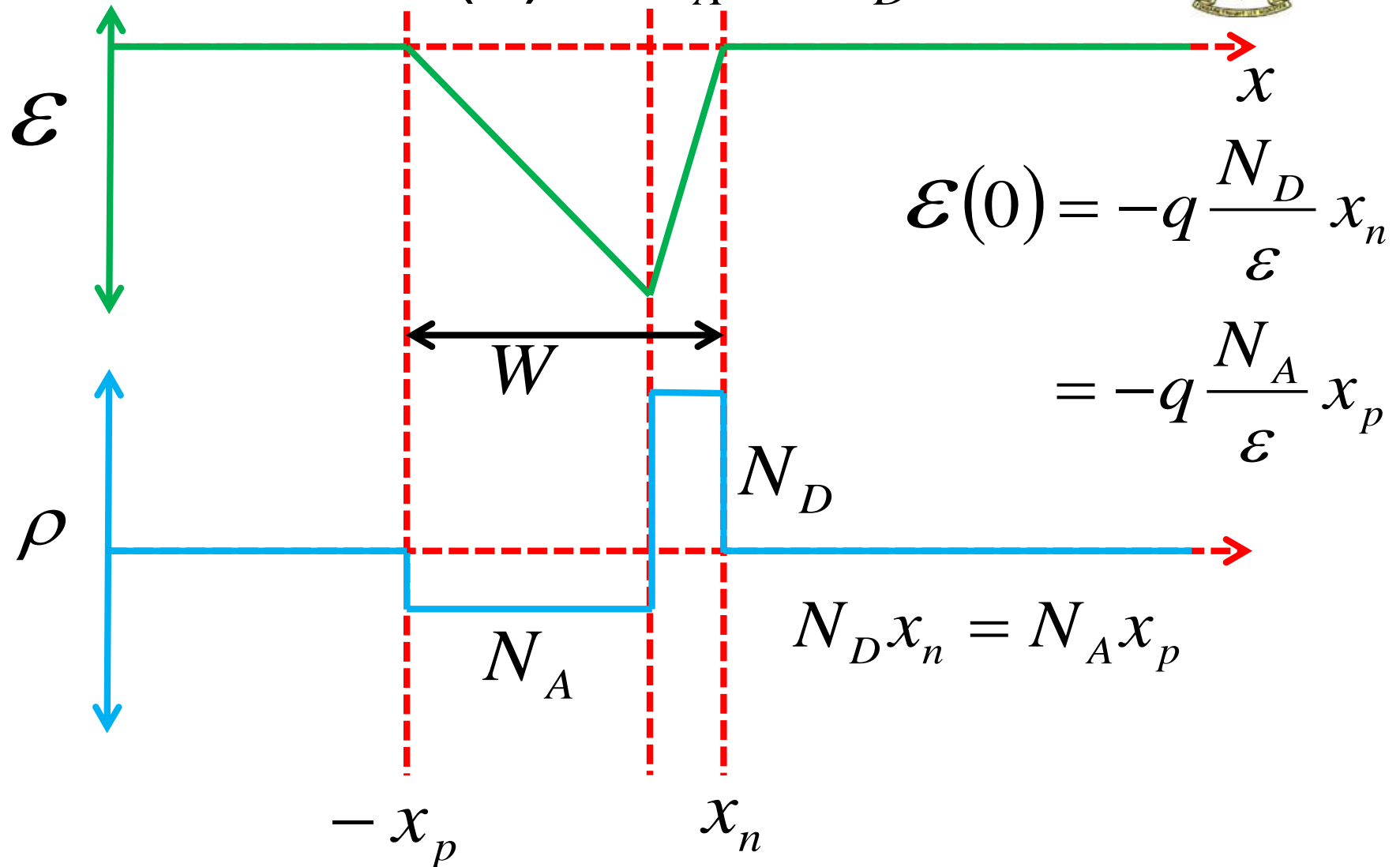


PN Junction (9)

$$N_A < N_D$$



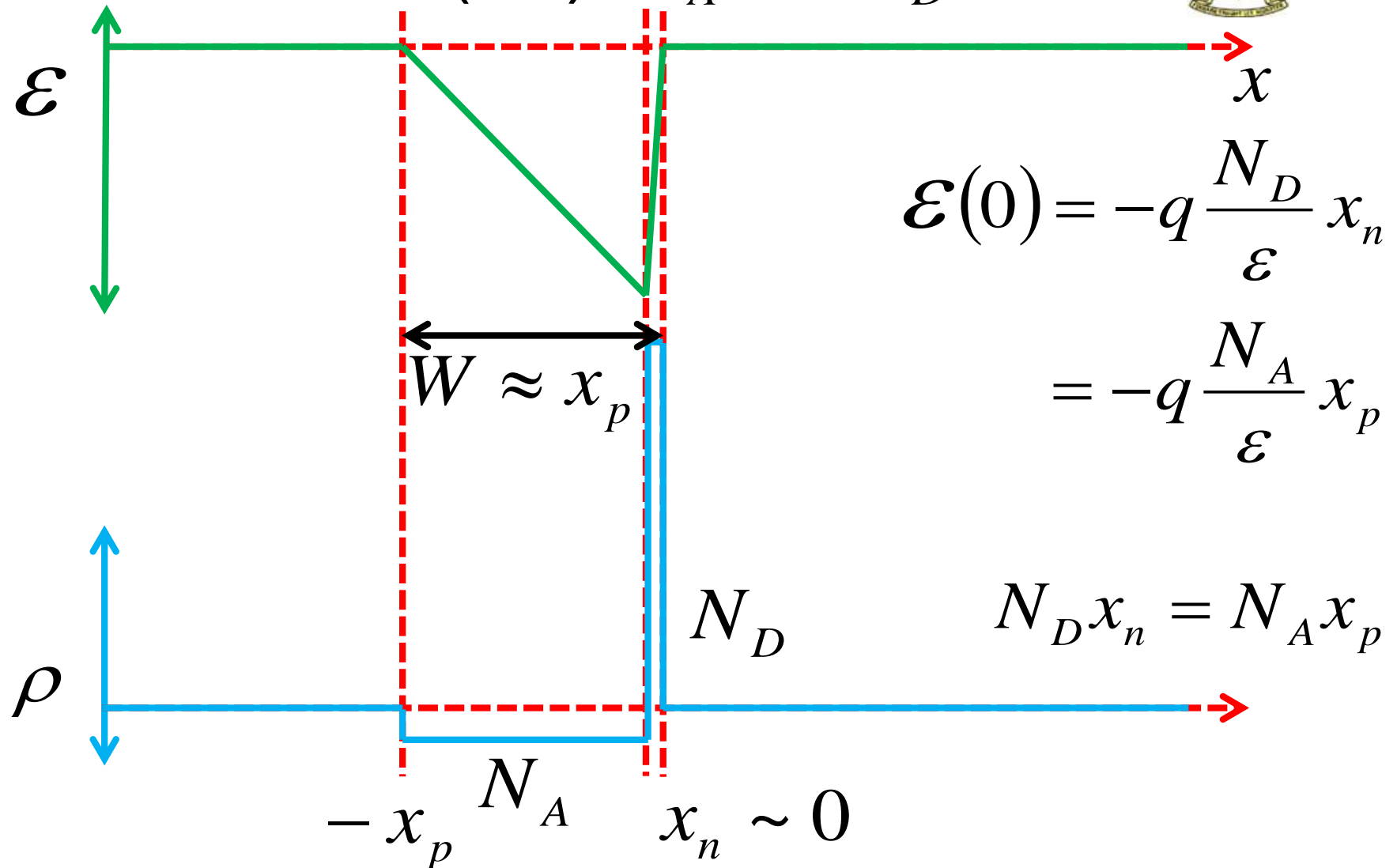
UCC



PN Junction (11) $N_A \ll N_D$



UCC



Depletion Width (2)



So, for a PN junction with high doping on one side:

$$W \rightarrow \sqrt{\frac{2\varepsilon V_{bi}}{q} \frac{N_D + N_A}{N_D N_A}} \quad \text{p-doping}$$

$$W \rightarrow \sqrt{\frac{2\varepsilon V_{bi}}{q} \frac{N_D + N_A}{N_D N_A}} \quad \text{n-doping}$$

The diode (1)



That is, the PN junction under bias.

The bias results in the movement of carriers, so we need to consider the quasi-fermi levels.

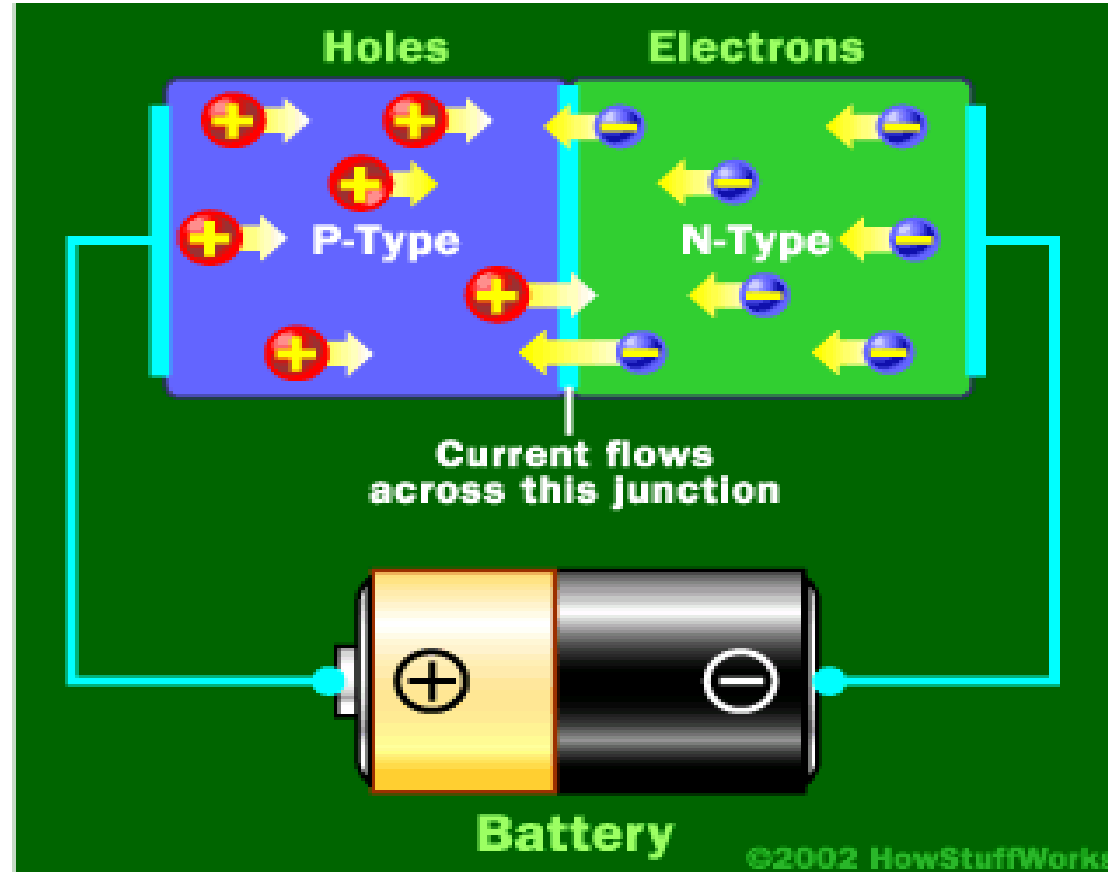
Here we **inject** (or **extract**) carriers into the semiconductor, causing a flow of current, still using the same equations

$$n = n_i e^{\frac{E_{Fn} - E_i}{kT}} \quad p = n_i e^{\frac{E_i - E_{Fp}}{kT}}$$

Quasi Fermi levels

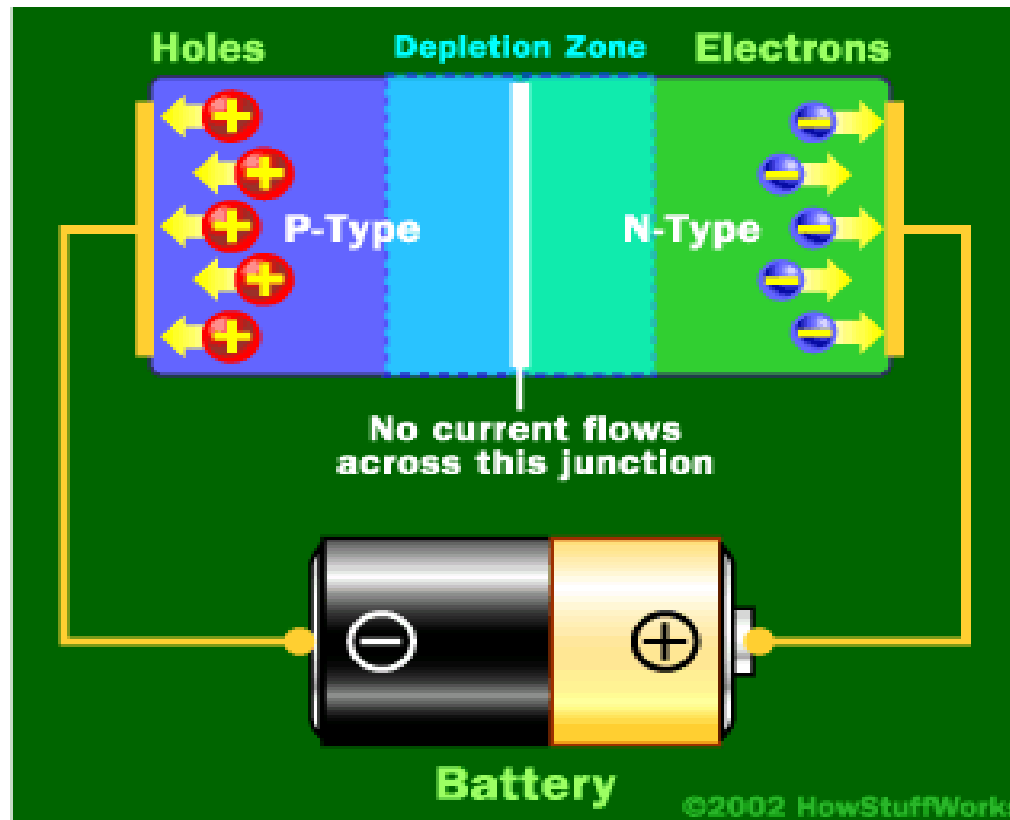
This will either increase or decrease the depletion region.

Forward Biased Diode



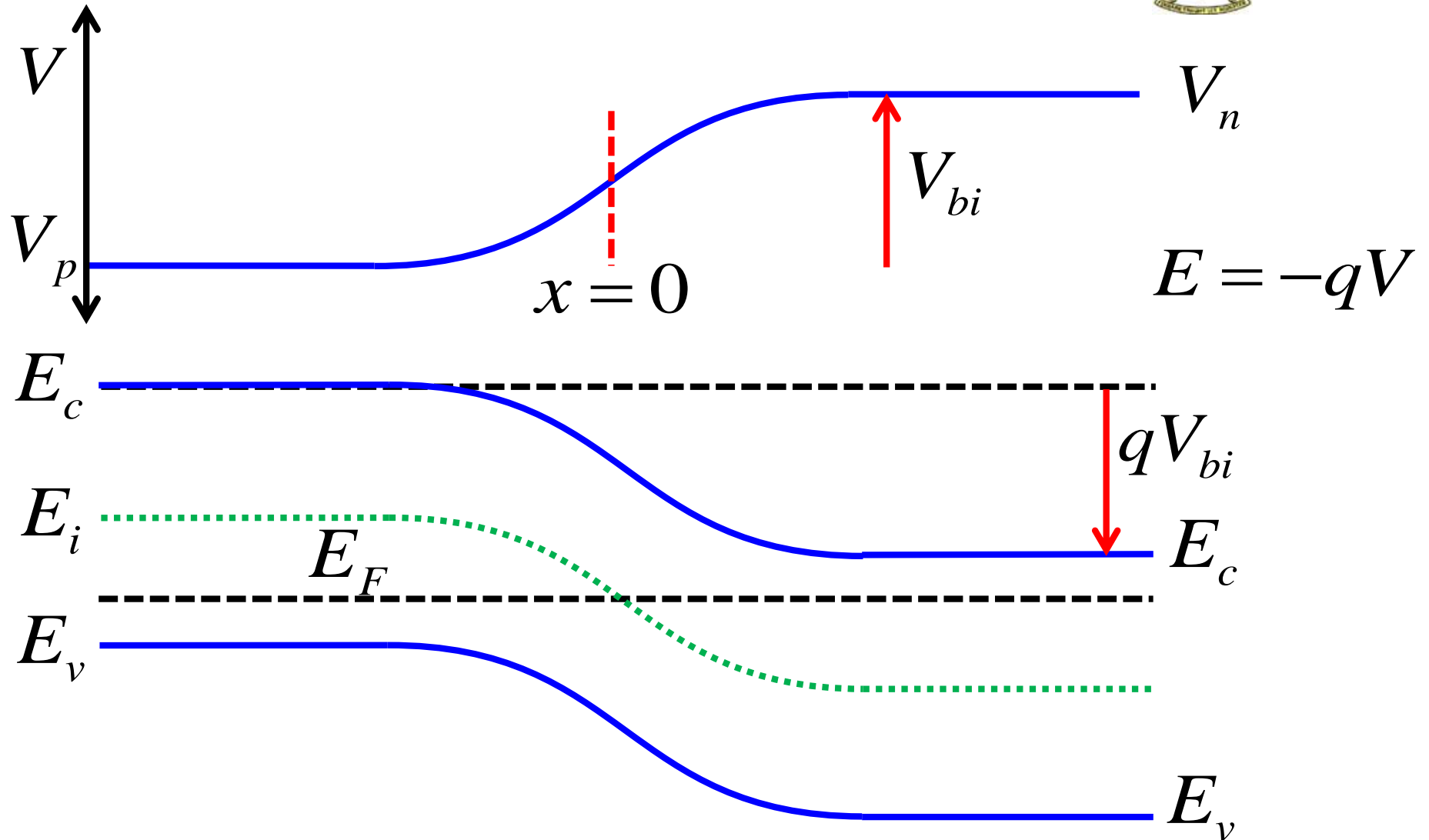
When the negative end of the circuit is hooked up to the N-type layer and the positive end is hooked up to P-type layer, electrons and holes start moving and the depletion zone disappears.

Reverse Biased Diode

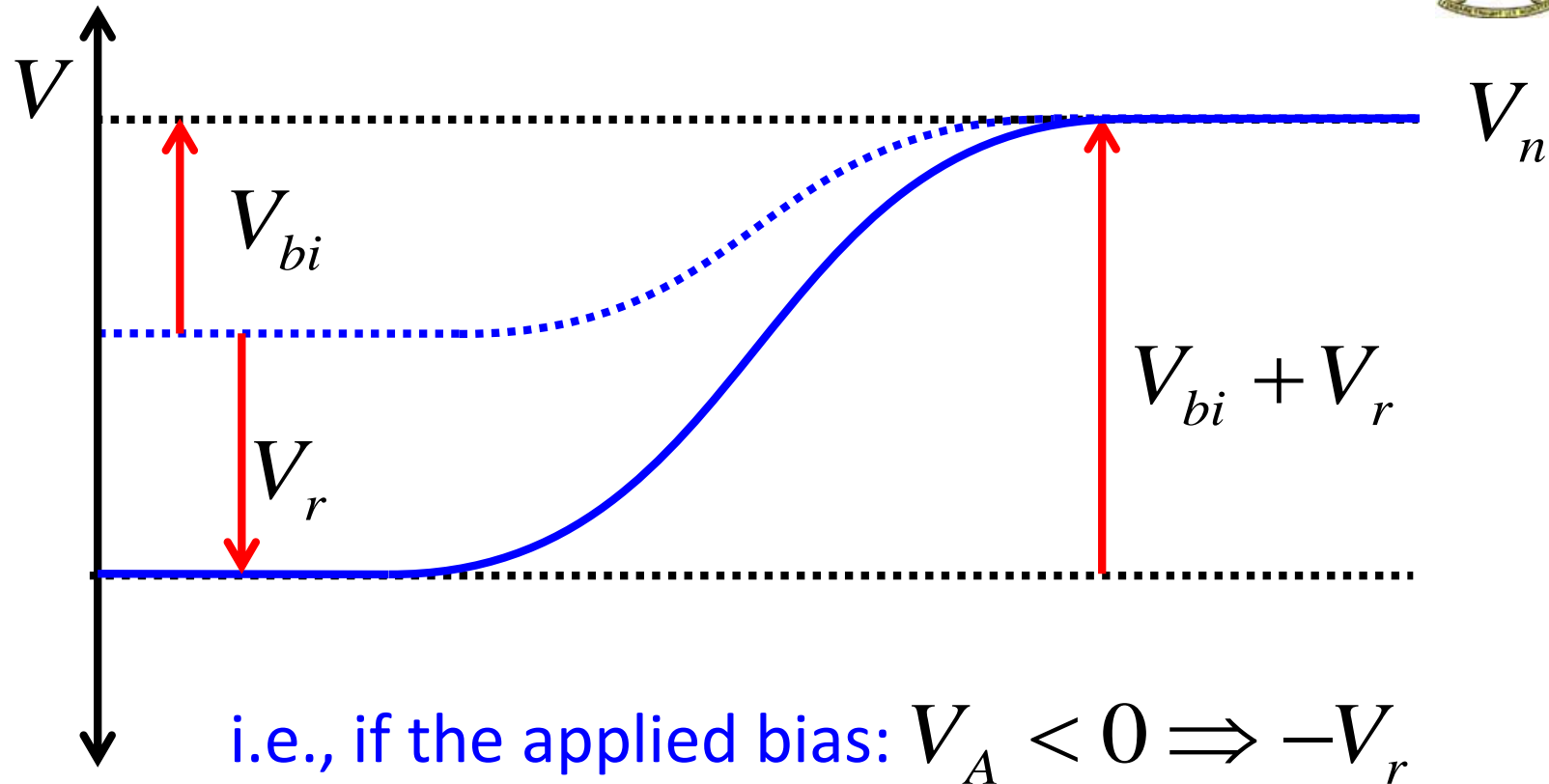


When the negative end of the circuit is hooked up to the N-type layer and the positive end is hooked up to the P-type layer, free electrons collect on one end of the diode and holes collect on the other. The depletion zone gets bigger.

Potential with no bias

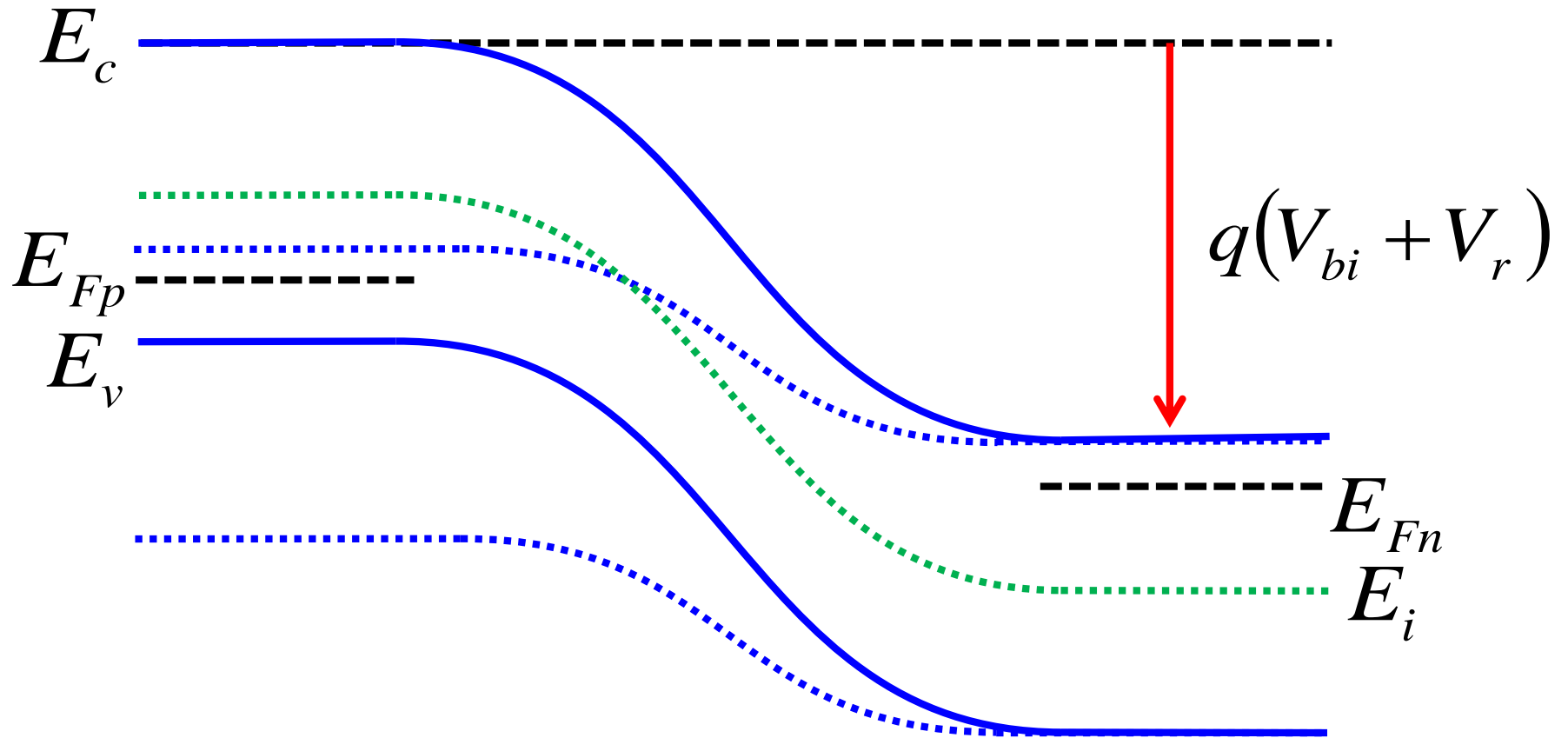


Potential with reverse bias

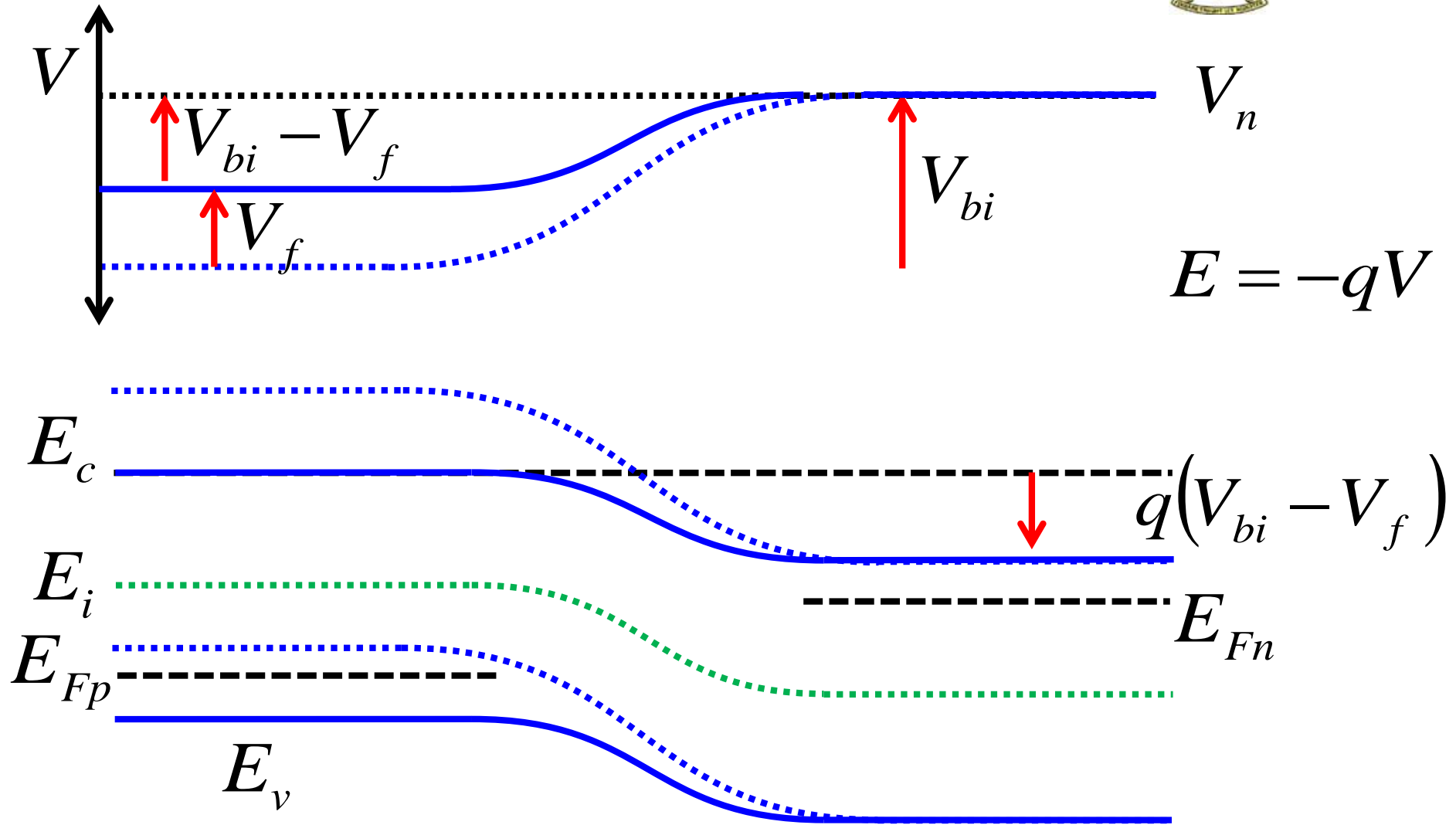


Potential with reverse bias

$$E = -qV$$



Potential with forward bias



Depletion Width (3)



Under bias: $V_{bi} \rightarrow V_{bi} - V_A$ ← applied

$$W \rightarrow \sqrt{\frac{2\epsilon(V_{bi} - V_A)}{q} \frac{N_D + N_A}{N_D N_A}}$$

Thus: $W \rightarrow 0$ as $V_A \rightarrow V_{bi}$

Reverse Bias (1)



Under Reverse bias: $V_{bi} \rightarrow V_{bi} + V_R$

$$W = \sqrt{\frac{2\epsilon(V_{bi} + V_R)}{q} \frac{N_D + N_A}{N_D N_A}} \propto \sqrt{V_{bi} + V_R}$$

The capacitance of the diode (parallel plate): $C = \frac{\epsilon A}{d} \rightarrow \frac{\epsilon A}{W}$

$$C = A \sqrt{\frac{\epsilon q}{2(V_{bi} + V_R)} \frac{N_D N_A}{N_D + N_A}}$$

The capacitance gets smaller with increasing bias