Physics PY4118
Physics of Semiconductor Devices

9. The Continuity Equation

Focus

So far, we have been looking at equilibrium conditions in a semiconductor.

Here we will start to investigate what happens when something changes.

Majority and Minority Carriers

We have already seen: $np = n_i^2 \qquad n_i \approx 10^{10}\,cm^{-3}$

So in an n-doped semiconductor: $n_n \Rightarrow N_D \quad p_n \Rightarrow \dfrac{n_i^2}{N_D}$

e.g. $n_n = 10^{18}\,cm^{-3} \qquad p_n = \dfrac{10^{20}}{10^{18}} = 100\,cm^{-3}$

Majority carrier >> Minority carrier

If electron hole pairs are injected: $\Delta n = \Delta p = 10^{8}\,cm^{-3}$

$$n_n \approx 10^{18}\,cm^{-3} \quad >> \quad p_n \approx \Delta p = 10^{8}\,cm^{-3}$$

Quasi Fermi level

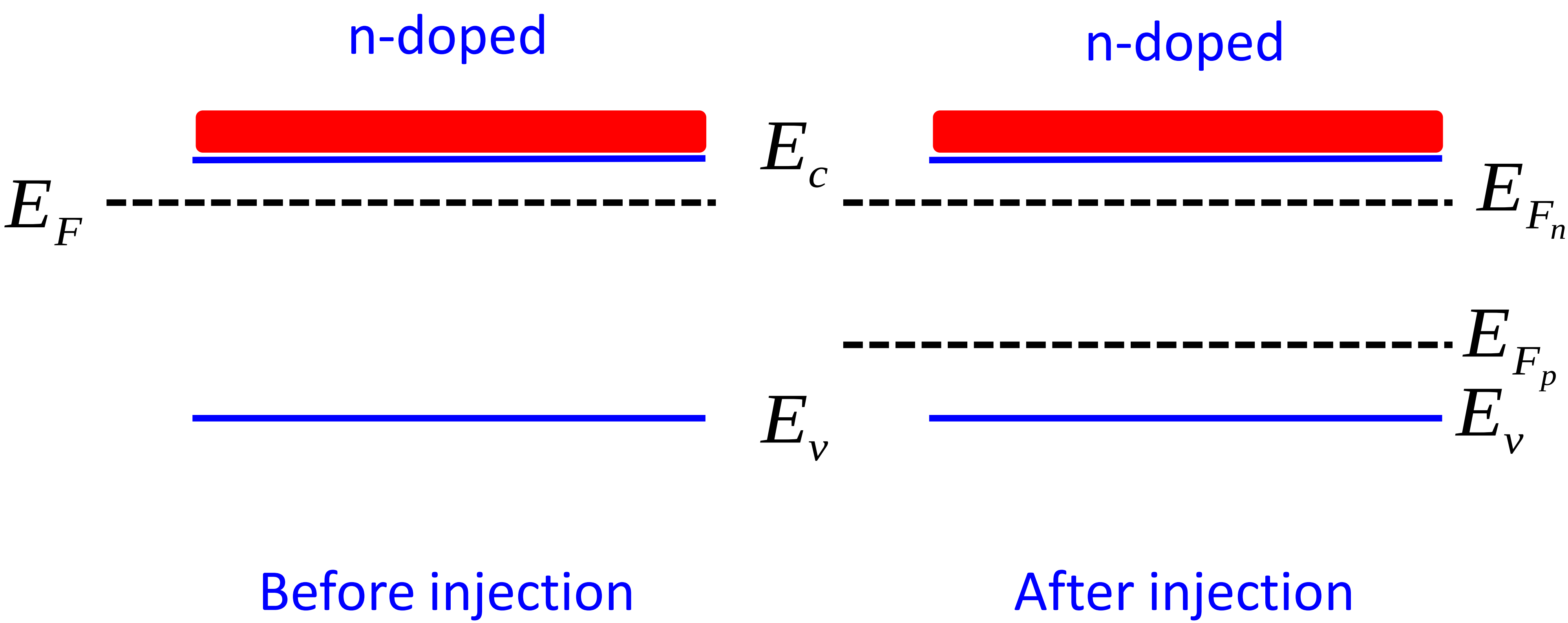
$$E_{Fn} = E_i + kT \ln\left(\frac{n_n}{n_i}\right)$$

Unchanged…

$$E_{Fp} = E_i - kT \ln\left(\frac{p_n}{n_i}\right) \Rightarrow E_i - kT \ln\left(\frac{\Delta p}{n_i}\right)$$

There is a significant change in the p - quasi Fermi level

Quasi Fermi levels



Movement of minority carriers

Thermal or Light generation results in e-h pairs!

The population of majority carriers ***may*** not be altered.

But, the minority carrier population will be greatly affected, with an enormous increase in population density.

These carriers will then tend to drift (if there is an electric field) diffuse and recombine with the much larger population of majority carriers.

Nomenclature notes

Equilibrium populations: $n_{p0} \quad p_{n0}$

Non-equilibrium populations:

$$n_p = n_{p0} + \Delta n_p \qquad p_n = p_{n0} + \Delta p_n$$

Excess populations: $\Delta n_p \quad \Delta p_n$

Derivatives: $\dfrac{dn_p}{dt} = \dfrac{d(\Delta n_p)}{dt} \qquad \dfrac{dp_n}{dt} = \dfrac{d(\Delta p_n)}{dt}$

Non-equilibrium recombination

In equilibrium, generation equals recombination.

But what if the generation ends – non equilibrium?

The decrease in the number of the excess minority carriers will be proportional to the number of excess minority carriers, thus:

$$\frac{dn_p}{dt} = -\frac{\Delta n_p}{\tau_n} \qquad \frac{dp_n}{dt} = -\frac{\Delta p_n}{\tau_p}$$

$$\Delta n_p(t) = \Delta n_p(0) e^{-\frac{t}{\tau_n}} \qquad \Delta p_n(t) = \Delta p_n(0) e^{-\frac{t}{\tau_p}}$$

2nd year E&M

In 2nd year E&M:

$$\vec{I} = \oint_S \vec{J} \bullet d\vec{a} = \int_V \left(\nabla \bullet \vec{J}\right) d\tau$$

(where the divergence theorem was used)

Charge is always conserved, so for the closed volume:

$$Q = \int_V \rho d\tau$$ so: $$\vec{I} = -\frac{dQ}{dt} = -\int_V \left(\frac{d\rho}{dt}\right) d\tau$$

And, finally:

$$\int_V \left(\nabla \bullet \vec{J}\right) d\tau = -\int_V \left(\frac{d\rho}{dt}\right) d\tau \qquad \nabla \bullet \vec{J} = -\frac{d\rho}{dt}$$

Current Gradients

For electron and hole densities:

$$\frac{dn_p}{dt} = -\frac{1}{q}\frac{d\rho}{dt} = \frac{1}{q}\nabla \bullet J_{n_p} \qquad \frac{dp_n}{dt} = \frac{1}{q}\frac{d\rho}{dt} = -\frac{1}{q}\nabla \bullet J_{p_n}$$

Putting this together with the recombination rate:

$$\frac{dn_p}{dt} = -\frac{n_p - n_{p0}}{\tau_n} + \frac{1}{q}\nabla \bullet J_{n_p} \qquad \frac{dp_n}{dt} = -\frac{p_n - p_{n0}}{\tau_p} - \frac{1}{q}\nabla \bullet J_{p_n}$$

Then we need to consider the current densities:

Add Drift & Diffusion

The drift and diffusion of the minority carriers is unchanged from before:

$$J_{n_p} = qn_p\mu_n\mathcal{E} + qD_n\nabla n_p \qquad \text{or:}$$

$$J_{p_n} = qp_n\mu_p\mathcal{E} - qD_p\nabla p_n$$

Which we insert into the previous equations (for electrons):

$$\frac{dn_p}{dt} = -\frac{n_p - n_{p0}}{\tau_n} + \frac{1}{q}\frac{dJ_{n_p}}{dx} \qquad \text{(the 1D version)}$$

$$= -\frac{n_p - n_{p0}}{\tau_n} + \frac{1}{q}\frac{d}{dx}\left(qn_p\mu_n\mathcal{E} + qD_n\frac{d}{dx}n_p\right)$$

Continuity Equations (1)

This simplifies to:

$$\frac{dn_p}{dt} = -\frac{n_p - n_{p0}}{\tau_n} + \mu_n \mathcal{E} \frac{dn_p}{dx} + \mu_n n_p \frac{d\mathcal{E}}{dx} + D_n \frac{d^2 n_p}{dx^2}$$

And we then add generation:

$$\frac{dn_p}{dt} = -\frac{n_p - n_{p0}}{\tau_n} + \mu_n \mathcal{E} \frac{dn_p}{dx} + \mu_n n_p \frac{d\mathcal{E}}{dx} + D_n \frac{d^2 n_p}{dx^2} + G$$

And similarly for holes in n-type materials:

$$\frac{dp_n}{dt} = -\frac{p_n - p_{n0}}{\tau_p} - \mu_p \mathcal{E} \frac{dp_n}{dx} - \mu_p p_n \frac{d\mathcal{E}}{dx} + D_p \frac{d^2 p_n}{dx^2} + G$$

Continuity Equations (2)

These equations can be solved for various condition.

At steady state: $$\frac{dp_n}{dt} = \frac{dn_p}{dt} = 0$$

$$-\frac{n_p - n_{p0}}{\tau_n} + \mu_n \mathcal{E} \frac{dn_p}{dx} + \mu_n n_p \frac{d\mathcal{E}}{dx} + D_n \frac{d^2 n_p}{dx^2} + G = 0$$

$$-\frac{p_n - p_{n0}}{\tau_p} - \mu_p \mathcal{E} \frac{dp_n}{dx} - \mu_p p_n \frac{d\mathcal{E}}{dx} + D_p \frac{d^2 p_n}{dx^2} + G = 0$$

Example A

$$x = 0$$

Steady state carrier Injection at boundary, with no Field/light:

$$\boxed{-\frac{n_p - n_{p0}}{\tau_n}} + \cancel{\mu_n \mathcal{E} \frac{dn_p}{dx}} + \cancel{\mu_n n_p \frac{d\mathcal{E}}{dx}} + D_n \frac{d^2 n_p}{dx^2} + \cancel{G} = 0$$

$$\rightarrow \frac{d^2 n_p}{dx^2} - \frac{n_p - n_{p0}}{D_n \tau_n} = 0$$

With: $L_n = \sqrt{D_n \tau_n}$

$$n_p = n_{p0} + Ae^{-\frac{x}{L_n}} + Be^{\frac{x}{L_n}} \rightarrow n_{p0} + Ae^{-\frac{x}{L_n}}$$

Where: A is the steady state injected carrier density

And: L_n is the diffusion length prior to recombination

Example B (1)

Uniform light generation at: $t > 0$ with no field:

$$\frac{dp_n}{dt} = \boxed{-\frac{p_n - p_{n0}}{\tau_p}} - \cancel{\mu_p \mathcal{E} \frac{dp_n}{dx}} - \cancel{\mu_p p_n \frac{d\mathcal{E}}{dx}} + \cancel{D_p \frac{d^2 p_n}{dx^2}} + G$$

$$\frac{dp_n}{dt} = G - \frac{p_n - p_{n0}}{\tau_p} = -\frac{p_n - p_{n0} - G\tau_p}{\tau_p}$$

$$\Downarrow$$

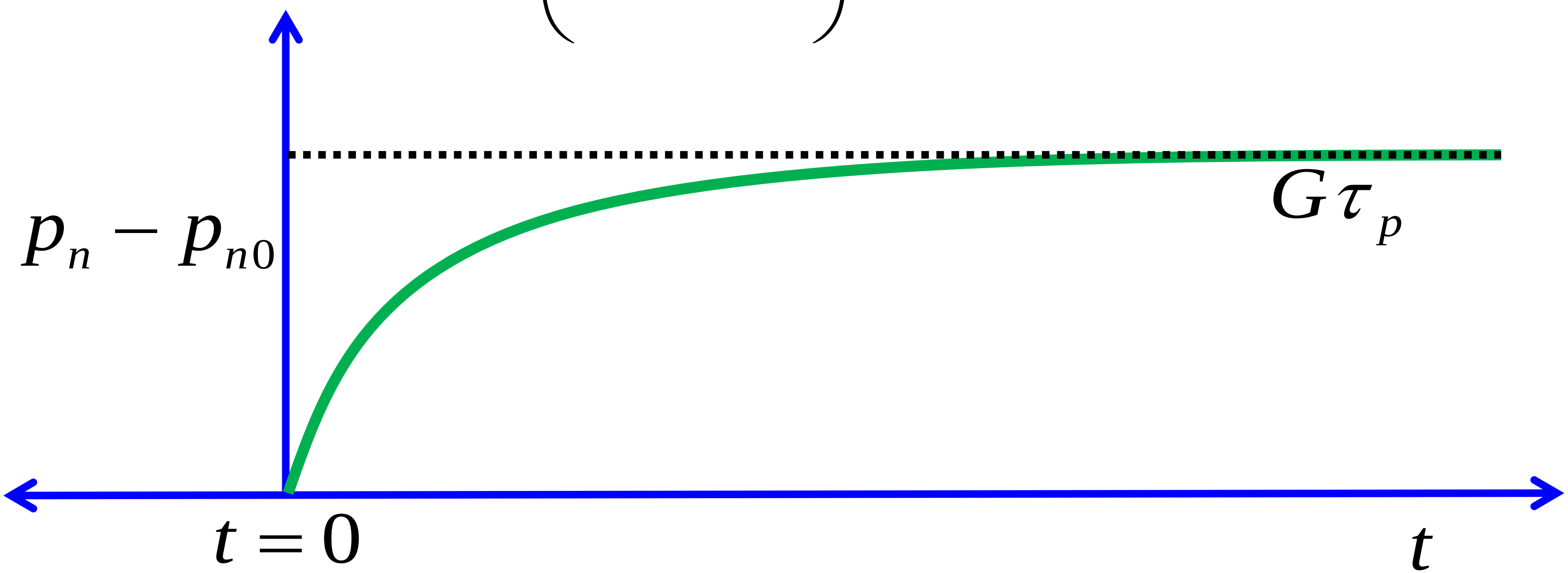
$$\int_{p_{n0}}^{p_n} \frac{dp_n}{p_n - p_{n0} - G\tau_p} = -\int_0^t \frac{dt}{\tau_p}$$

$$\ln\left[p_n - p_{n0} - G\tau_p\right]_{p_{n0}}^{p_n} = \ln\left[\frac{p_n - p_{n0} - G\tau_p}{-G\tau_p}\right] = -\frac{t}{\tau_p}$$

Example B (2)

$$p_n - p_{n0} - G\tau_p = -G\tau_p e^{-\frac{t}{\tau_p}}$$

$$p_n = p_{n0} + G\tau_p\left(1 - e^{-\frac{t}{\tau_p}}\right)$$

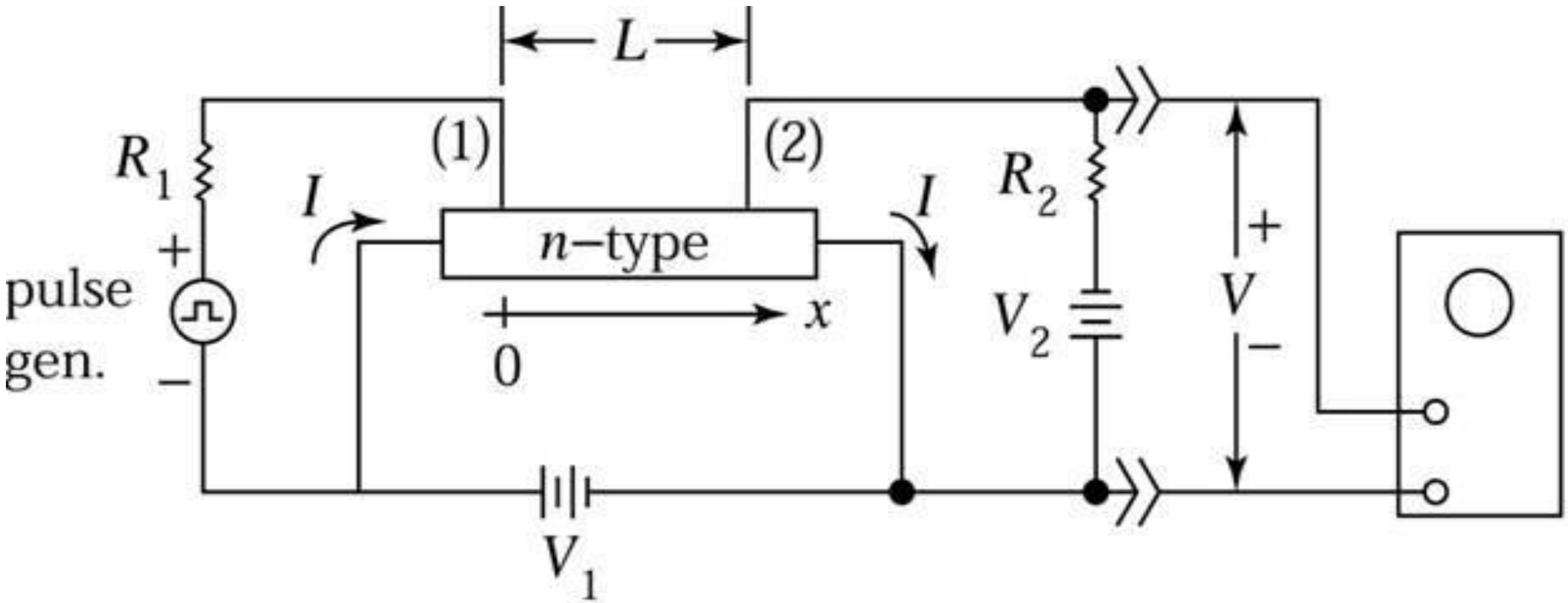


Example C (1)

William Shockley – 1956 Nobel Laureate in Physics

1948 Shockley – Haynes Experiment

Independent measurement of drift and diffusion.



Example C (2)

After a pulse is injected:

$$\frac{dp_n}{dt} = -\frac{p_n - p_{n0}}{\tau_p} - \mu_p \mathcal{E} \frac{dp_n}{dx} + D_p \frac{d^2 p_n}{dx^2}$$

This can be partly simplified with: $p_n - p_{n0} = \phi e^{-\frac{t}{\tau_p}}$

$$\frac{dp_n}{dt} = \frac{d}{dt}\left(\phi e^{-\frac{t}{\tau_p}}\right) = \frac{d\phi}{dt} e^{-\frac{t}{\tau_p}} - \frac{\phi}{\tau_p} e^{-\frac{t}{\tau_p}}$$

$$\frac{dp_n}{dx} = \frac{d\phi}{dx} e^{-\frac{t}{\tau_p}} \qquad \frac{d^2 p_n}{dx^2} = \frac{d^2 \phi}{dx^2} e^{-\frac{t}{\tau_p}}$$

Example C (3)

So, the PDE simplifies to: $$\frac{d\phi}{dt} = -\mu_p \mathcal{E} \frac{d\phi}{dx} + D_p \frac{d^2\phi}{dx^2}$$

With no field: $$\frac{d\phi}{dt} = D_p \frac{d^2\phi}{dx^2}$$

This is the heat or diffusion equation

Whose solution is: $$\phi = \frac{A}{\sqrt{4\pi D_p t}} \exp\left(-\frac{x^2}{4D_p t}\right)$$

Example C (4)

Proof: $$\frac{d\phi}{dt} = -\frac{\phi}{2t} + \frac{x^2\phi}{4D_p t^2}$$

$$\frac{d\phi}{dx} = -\frac{2x\phi}{4D_p t} \rightarrow D_p \frac{d^2\phi}{dx^2} = -\frac{\phi}{2t} - \frac{x^2\phi}{4D_p t^2}$$

Thus: $$\frac{d\phi}{dt} = D_p \frac{d^2\phi}{dx^2}$$

Example C (5)

The full solution (with field) is: $x \rightarrow x - \mu_p \mathcal{E} t$

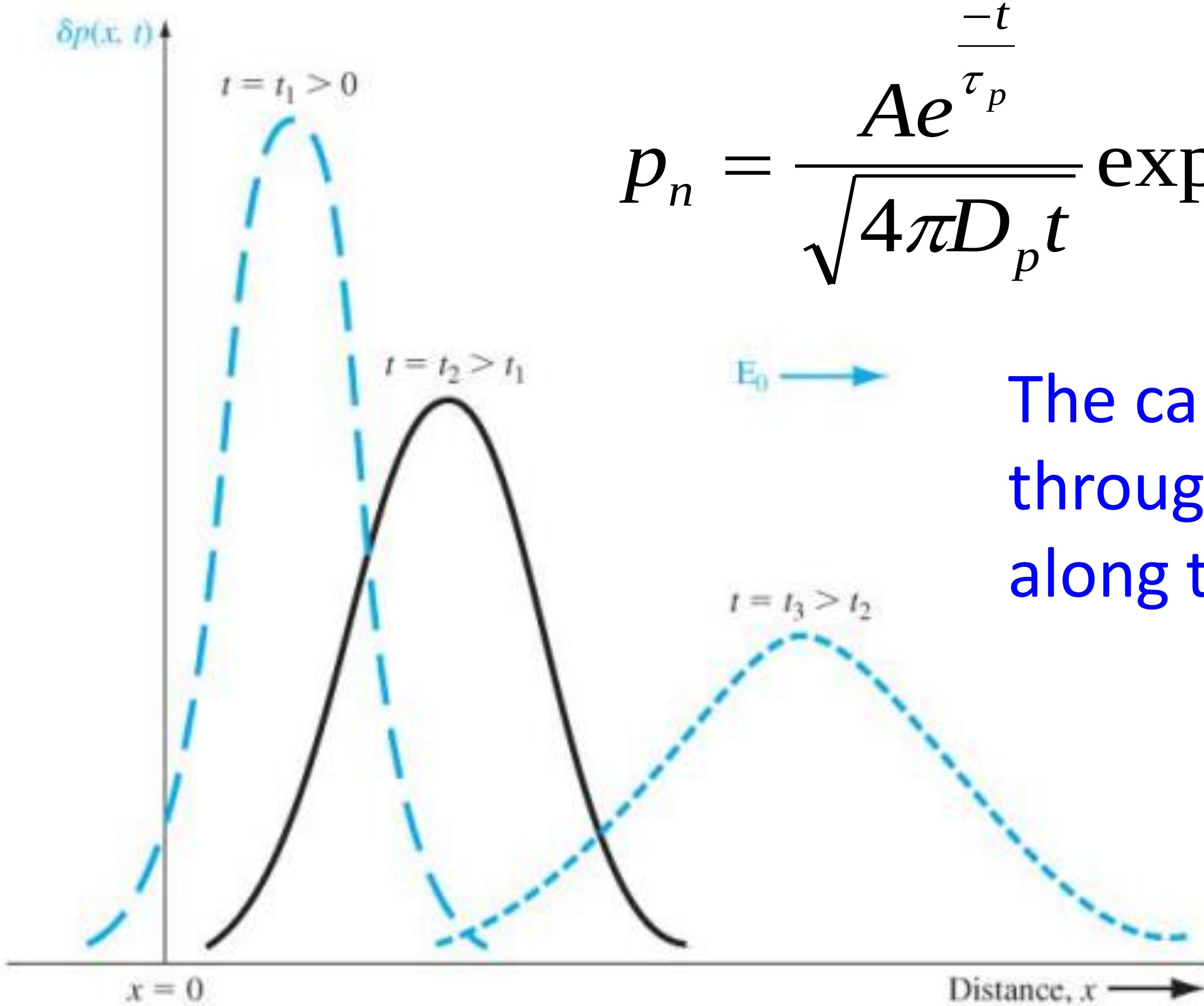
$$p_n = \frac{A e^{\frac{-t}{\tau_p}}}{\sqrt{4\pi D_p t}} \exp\left(-\frac{\left(x - \mu_p \mathcal{E} t\right)^2}{4 D_p t} \right)$$

And, different parameters can be extracted from the experiment.

See: http://en.wikipedia.org/wiki/Heat_equation

Example C (6)

$$p_n = \frac{A e^{\frac{-t}{\tau_p}}}{\sqrt{4\pi D_p t}} \exp\left(-\frac{\boxed{\left(x - \mu_p \mathcal{E} t\right)}^2}{4 D_p t}\right)$$

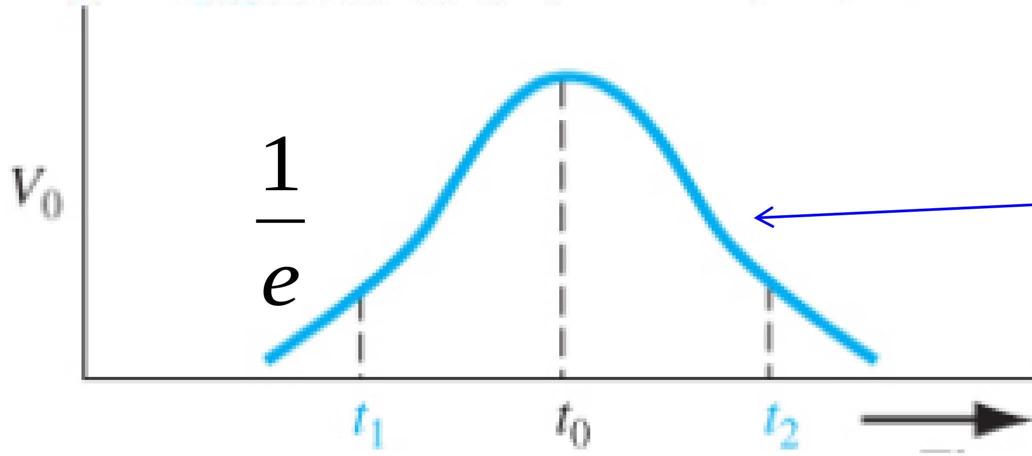


The carrier pulse broadens through diffusion as it drifts along the semiconductor.

$$x = L = \mu_p \mathcal{E} t$$

$$\rightarrow \mu_p = \frac{L}{\mathcal{E} t} = \frac{v_d}{\mathcal{E}}$$

Example C (7)



$$\exp\left(-\frac{\left(x-\mu_p\mathcal{E}t\right)^2}{4D_p t}\right)$$

At a given: $x = L$ the peak will occur at: $t = t_0 = \dfrac{L}{\mu_p\mathcal{E}}$

The $\dfrac{1}{e}$ points exist at: $\left(L-\mu_p\mathcal{E}\left(t_0 \pm \dfrac{\Delta t}{2}\right)\right)^2 = 4D_p t_0$

Thus: $$D_p = \frac{\left(\mu_p\mathcal{E}\right)^2\left(\Delta t\right)^2}{16t_0}$$

Final Note

Using the Einstein Relations (Slide 7.18)…

$$\frac{dn_p}{dt} = -\frac{n_p - n_{p0}}{\tau_n} + \mu_n \mathcal{E} \frac{dn_p}{dx} + \mu_n n_p \frac{d\mathcal{E}}{dx} + \frac{\mu_n kT}{q} \frac{d^2 n_p}{dx^2} + G$$

$$\frac{dp_n}{dt} = -\frac{p_n - p_{n0}}{\tau_p} - \mu_p \mathcal{E} \frac{dp_n}{dx} - \mu_p p_n \frac{d\mathcal{E}}{dx} + \frac{\mu_p kT}{q} \frac{d^2 p_n}{dx^2} + G$$