Optical Instruments

\[
\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}
\]

The eye, magnifying glass, microscope and telescope.
Determine the focal length of your eye when looking at an object far away.

\[
\frac{1}{\infty} + \frac{1}{25 \text{ mm}} = \frac{1}{f} \quad f_{\text{relaxed}} = 25 \text{ mm}
\]
Determine the focal length of your eye when looking at an object up close (25 cm).

\[
\frac{1}{250 \text{ mm}} + \frac{1}{25 \text{ mm}} = \frac{1}{f}
\]

\[f_{tense} = 22.7 \text{ mm}
\]

\[f_{relaxed} = 25 \text{ mm}
\]
The Eye – Near and Far Points

- The **near point** is the closest distance for which the lens can accommodate to focus light on the retina
  - Typically at age 10, this is about 18 cm
  - It increases with age, ~ 25 cm for an adult

- The **far point** of the eye represents the largest distance for which the lens of the relaxed eye can focus light on the retina
  - Normal vision has a far point of infinity
Farsightedness

- Also called hyperopia
- The image focuses behind the retina
- Can usually see far away objects clearly, but not nearby objects
Correcting Farsightedness

- A converging lens placed in front of the eye can correct the condition
- The lens refracts the incoming rays more toward the principle axis before entering the eye
  - This allows the rays to converge and focus on the retina
Nearsightedness

- Also called myopia
- In *axial myopia* the nearsightedness is caused by the lens being too far from the retina
- In *refractive myopia*, the lens-cornea system is too powerful for the normal length of the eye
Correcting Nearsightedness

- A diverging lens can be used to correct the condition
- The lens refracts the rays away from the principle axis before they enter the eye
  - This allows the rays to focus on the retina
If you are nearsighted - myopic...

Want to have (virtual) image of distant object, $d_o = \infty$, at the far point, $-d_{\text{far}}$. 

\[
\frac{1}{d_o} + \frac{1}{-d_{\text{far}}} = \frac{1}{f_{\text{lens}}} \quad \Rightarrow \quad \frac{1}{\infty} + \frac{1}{-d_{\text{far}}} = \frac{1}{f_{\text{lens}}} \quad \Rightarrow \quad f_{\text{lens}} = -d_{\text{far}}
\]
If you are farsighted - hyperopic...

Virtual image due to convex lens

Want the near point to be at \( d_o \).

When object is at \( d_o \), lens must create an image at \(-d_{\text{near}}\).

\[
\frac{1}{d_o} + \frac{1}{-d_{\text{near}}} = \frac{1}{f_{\text{lens}}}
\]

\[
\frac{1}{25 \text{ cm}} + \frac{1}{-50 \text{ cm}} = \frac{1}{f}
\]

\( f = 50 \text{ cm} \)
Angular Size

- The larger the angular size of the object is, the larger the image is on your retina, and the bigger it appears to be.
- How small of font can you read?
Angular Size Question

The closer the object the larger the image is on the retina.
Unaided Eye

How big the object looks with unaided eye.

Bring object as close as possible (near point N)

\[
\tan(\theta) = \frac{h_o}{N}
\]

\[
\theta \approx \frac{h_o}{N}
\]

If \( \theta \) is small and expressed in radians.
Simple Magnifier

- A simple magnifier consists of a single converging lens
- This device is used to increase the apparent size of an object
- The size of an image formed on the retina depends on the angle subtended by the eye
- When the object is placed near the focal point of a converging lens, the lens forms a virtual, upright, and enlarged image
  - Arrange it so that it is at near point (25 cm)
Magnifying glass produces virtual image behind object, allowing you to bring object to a closer $d_o$: and larger $\theta'$

$$\theta' = \frac{h_i}{d_i} = \frac{h_o}{d_o}$$

Compare to unaided eye

$$\theta = \frac{h_o}{N}$$

Ratio of the two angles is the **angular magnification** $M$:

$$M = \frac{\theta'}{\theta} = \frac{h_o/d_o}{h_o/N} = \frac{N}{d_o}$$

**Angular Magnification**
Angular Magnification \( M = \frac{N}{d_o} \)

For the lens:
\[
\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \quad \Rightarrow \quad \frac{1}{d_o} = \frac{1}{f} - \frac{1}{d_i}
\]

\(-\infty < d_i < -N\) So \( \frac{1}{d_o} \) is in the range \( \frac{1}{f} \leq \frac{1}{d_o} \leq \frac{1}{f} + \frac{1}{N} \)

\( \therefore M \) lies between \( \frac{N}{f} \) and \( \frac{N}{f} + 1 \) and the shorter the focal length, the greater the magnification \( M \).

\( M = \frac{25}{10} + 1 = 3.5 \)
Magnification by a Lens

- With a single lens, it is possible to achieve angular magnification up to about 4 without serious aberrations.
- With multiple lenses, magnifications up to about 20 can be achieved.
  - The multiple lens can correct for aberrations.
- Note: this is still for an single eyepiece.
- A typical microscope eyepiece is 10x.
Compound Microscope

- A compound microscope consists of two lenses
  - Gives greater magnification than a single lens
  - The objective lens has a short focal length, \( f_o < 1 \, \text{cm} \)
  - The ocular lens (eyepiece) has a focal length, \( f_e \) of a few cm
Compound Microscope, cont

- The lens are separated by a distance $L$
  - $L$ is much greater than either focal length
- The approach to analysis is the same as for any two lenses in a row
  - The image formed by the first lens becomes the object for the second lens
- The image seen by the eye, $I_2$, is virtual, inverted and very much enlarged
Telescopes

- Two fundamental types of telescopes
  - Refracting telescope uses a combination of lens to form an image
  - Reflecting telescope uses a curved mirror and a lens to form an image

- Telescopes can be analyzed by considering them to be two optical elements in a row
  - The image of the first element becomes the object of the second element
Refracting Telescope

- The two lenses are arranged so that the objective forms a real, inverted image of a distance object.
- The image is near the focal point of the eyepiece.
- The two lenses are separated by the distance \( f_o + f_e \) which corresponds to the length of the tube.
- The eyepiece forms an enlarged, inverted image of the first image.
- Largest Refracting Telescope (1m diameter) in the world is in Wisconsin (Yerkes Observatory).
Disadvantages of Refracting Telescopes

- Large diameters are needed to study distant objects
- Large lenses are difficult and expensive to manufacture
- The weight of large lenses leads to sagging which produces aberrations
Reflecting Telescope

- Helps overcome some of the disadvantages of refracting telescopes
  - Replaces the objective lens with a mirror
  - The mirror is often parabolic to overcome spherical aberrations
- In addition, the light never passes through glass
  - Except the eyepiece
  - Reduced chromatic aberrations
Reflecting Telescope, Newtonian Focus

- The incoming rays are reflected from the mirror and converge toward point A
  - At A, a photographic plate or other detector could be placed
- A small flat mirror, M, reflects the light toward an opening in the side and passes into an eyepiece
- Largest in the world are 10 m diameter Keck telescopes on Mauna Kea in Hawaii
- Largest single mirror telescope in US has 5 m diameter mirror on Mount Palomar in California
Resolution

- The ability of an optical system to distinguish between closely spaced objects is limited due to the wave nature of light.
- If two sources of light are close together, they can be treated as non-coherent sources.
- Because of diffraction, the images consist of bright central regions flanked by weaker bright and dark rings.
Rayleigh’s Criterion

- If the two sources are separated so that their central maxima do not overlap, their images are said to be resolved.
- The limiting condition for resolution is Rayleigh’s Criterion.
  - When the central maximum of one image falls on the first minimum of another image, they images are said to be just resolved.
  - The images are just resolved when their angular separation satisfies Rayleigh’s criterion.
Barely Resolved (Left) and Not Resolved (Right)
Just Resolved

- If viewed through a slit of width $a$, and applying Rayleigh’s criterion, the limiting angle of resolution is

$$\theta_{\text{min}} = \frac{\lambda}{a}$$

- For the images to be resolved, the angle subtended by the two sources at the slit must greater than $\theta_{\text{min}}$
Resolution with Circular Apertures

- The diffraction pattern of a circular aperture consists of a central, circular bright region surrounded by progressively fainter rings.
- The limiting angle of resolution depends on the diameter, $D$, of the aperture.

$$\theta_{\text{min}} = 1.22 \frac{\lambda}{D}$$