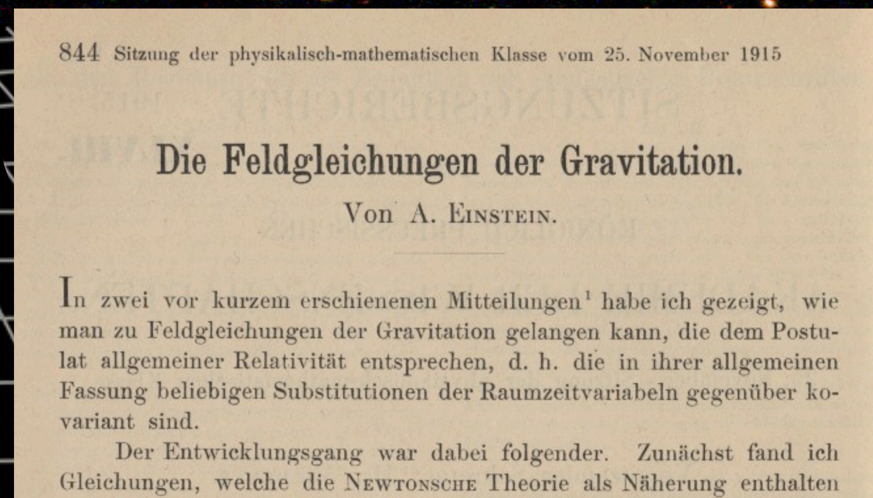
100 years of General Relativity


844 Sitzung der physikalisch-mathematischen Klasse vom 25. November 1915

Die Feldgleichungen der Gravitation.

Von A. Einstein.

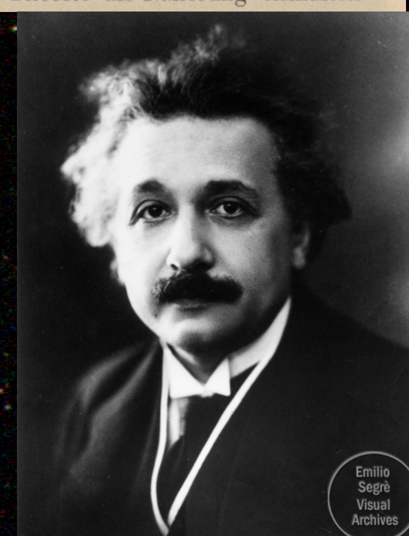
In zwei vor kurzem erschienenen Mitteilungen[1] habe ich gezeigt, wie man zu Feldgleichungen der Gravitation gelangen kann, die dem Postulat allgemeiner Relativität entsprechen, d. h. die in ihrer allgemeinen Fassung beliebigen Substitutionen der Raumzeitvariabeln gegenüber kovariant sind.

Der Entwicklungsgang war dabei folgender. Zunächst fand ich Gleichungen, welche die Newtonsche Theorie als Näherung enthalten

Asaf Pe’er

Dark Energy

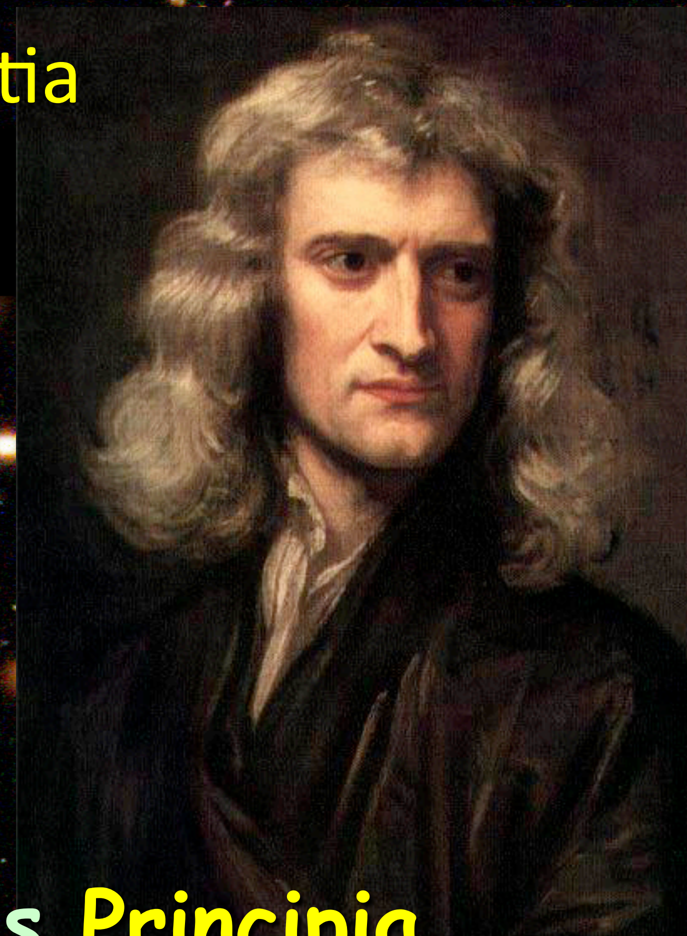
Cosmological constant



Nov. 2015

1632: Galileo Galilei's "Dialogue Concerning the Two Chief World Systems"

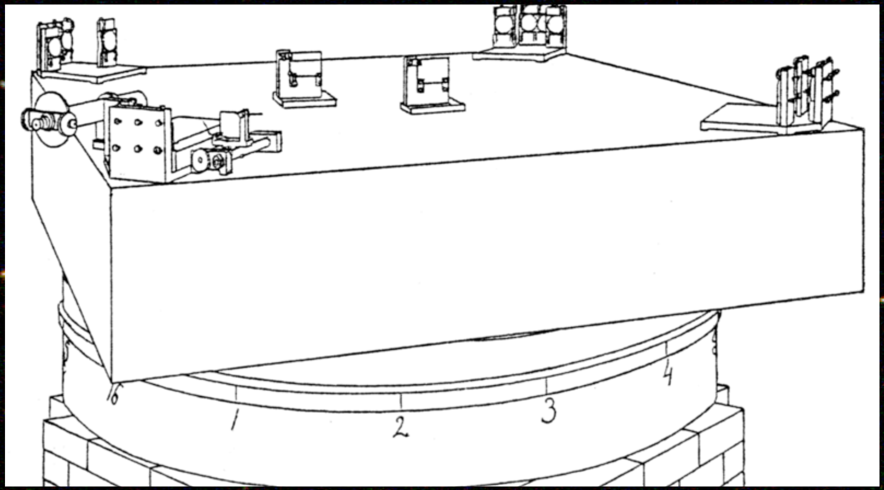
The law of inertia



1687: Isaac Newton's Principia Mathematica: The three laws of motion

Basic laws of classical mechanics

1887: First evidence that classical mechanics is incomplete



Michelson & Morley: Speed of light is constant.

➔Galilean transformation : x’ = x – v t
– and with it the entire Newtonian mechanics,
fail !!

June 1905:
Einstein introduces **special relativity**

* Galilean transformation replaced by Lorentz transformation

* New concept:
space + time merged into **space-time**.

[redacted]

[redacted]

[redacted]

[redacted]

June 1905: Einstein introduces special relativity

* Galilean transformation replaced by Lorentz transformation

* New concept:
space + time merged into **space-time**.

✔ Consistent with Michelson- Morley experiment

✔ Obtain Newtonian mechanics in the limit v <<c

✔ Consistent with Maxwell's theory of EM

June 1905:
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✔ Consistent with Michelson- Morley experiment

✔ Obtain Newtonian mechanics in the limit v <<c

✔ Consistent with Maxwell's theory of EM

Price: space & time become connected into single space-time

June 1905:
Einstein introduces special relativity

Price: space & time become connected into single space-time

Time is relative to the observer !

June 1905:
Einstein introduces special relativity

* Galilean transformation replaced by
Lorentz transformation

* New concept:
space + time merged into **space-time**.

✔ Consistent with Michelson- Morley experiment

✔ Obtain Newtonian mechanics in the limit $v << c$

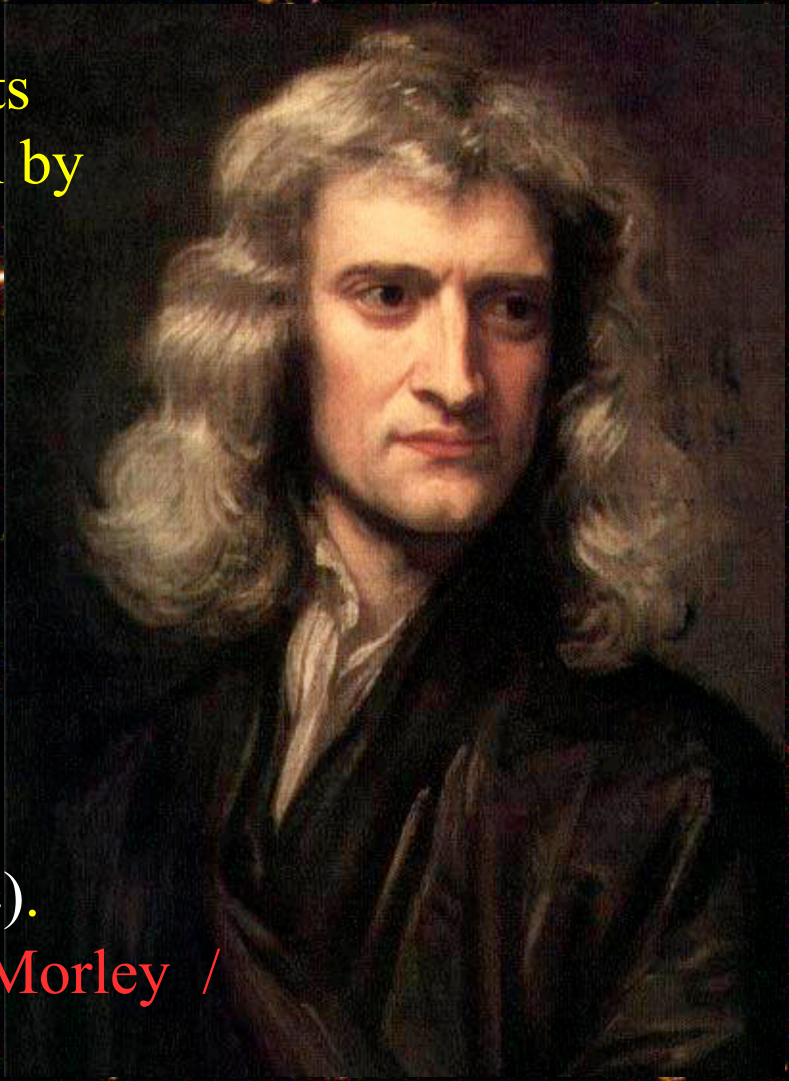
✔ Consistent with Maxwell's theory of EM

✘ Inconsistent with Newton's gravity

1687: Newton's law of universal gravity

Every point mass in the universe attracts any other point mass with a force given by

$$F = G\frac{m_1 m_2}{r^2}$$



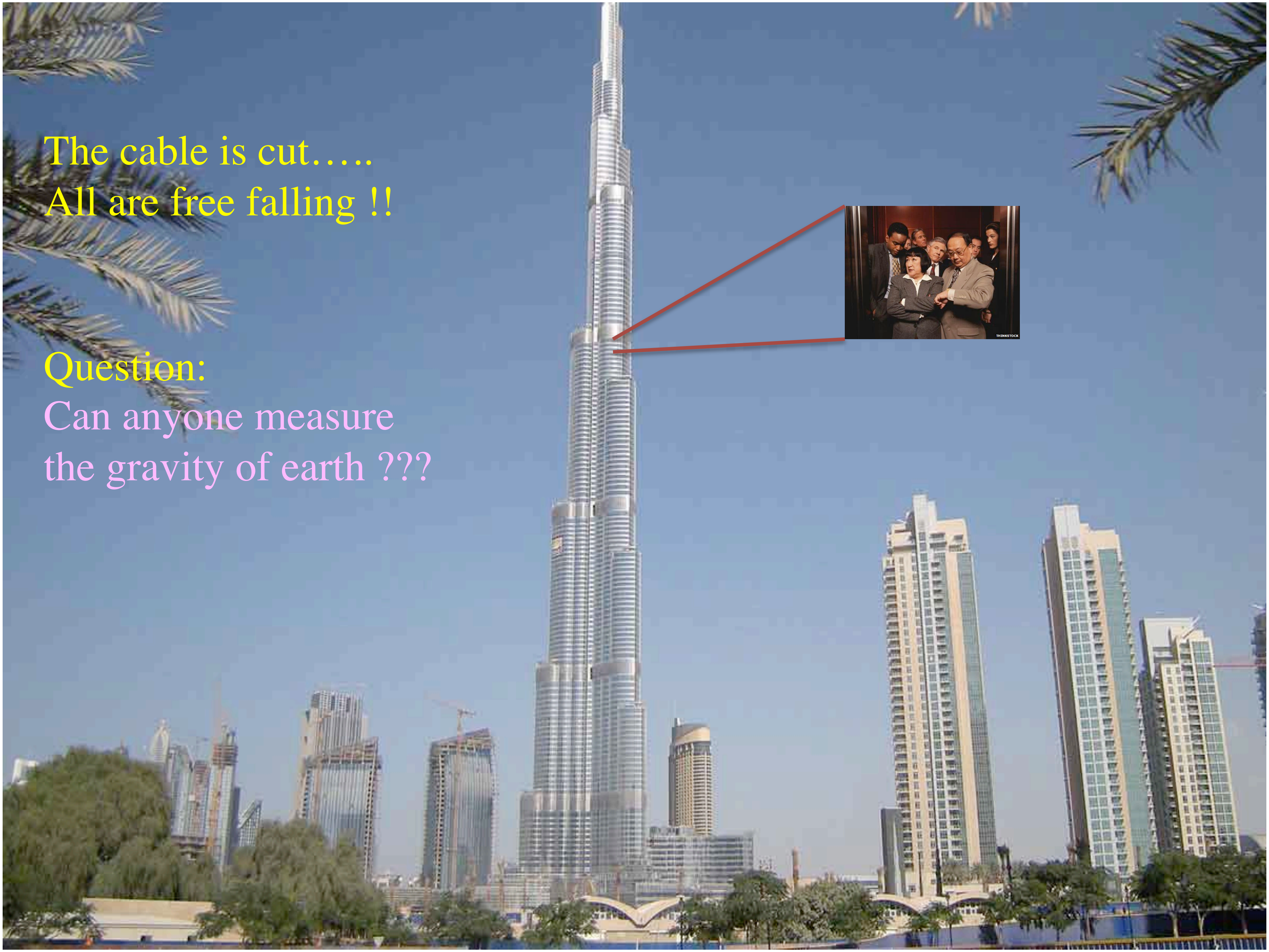
No time in the equation:
Gravity attracts at zero time
➔ at infinitely high speed (greater than c).

➔ Inconsistent with the Michelson-Morley / relativity

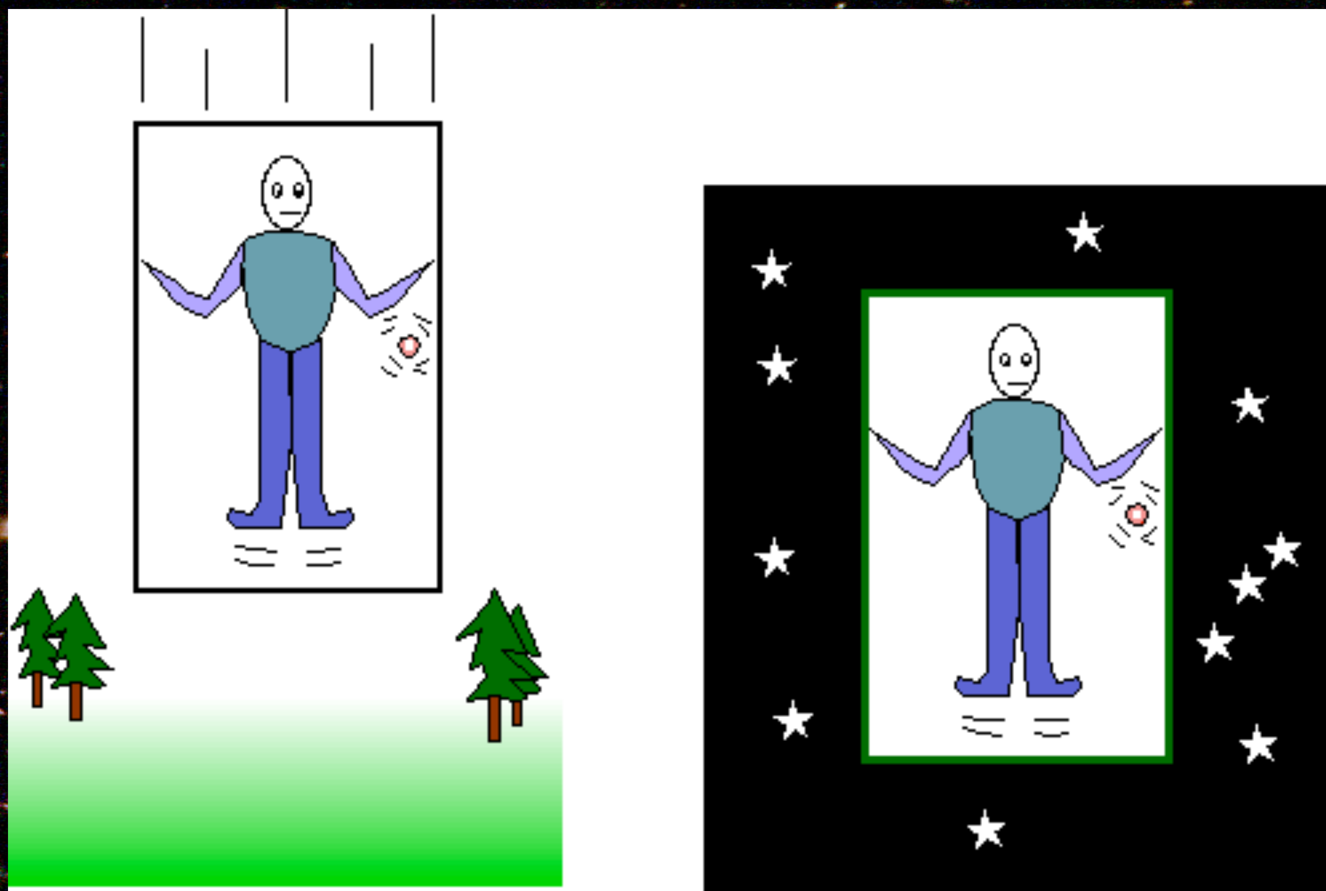
Einstein's elevator gedenkenexperiment (thought experiment) (c. 1907)

A bunch of physicists in an elevator, on their way to a conference….


(A pretty tall building)


The cable is cut…..
All are free falling !!
Question:
Can anyone measure
the gravity of earth ???

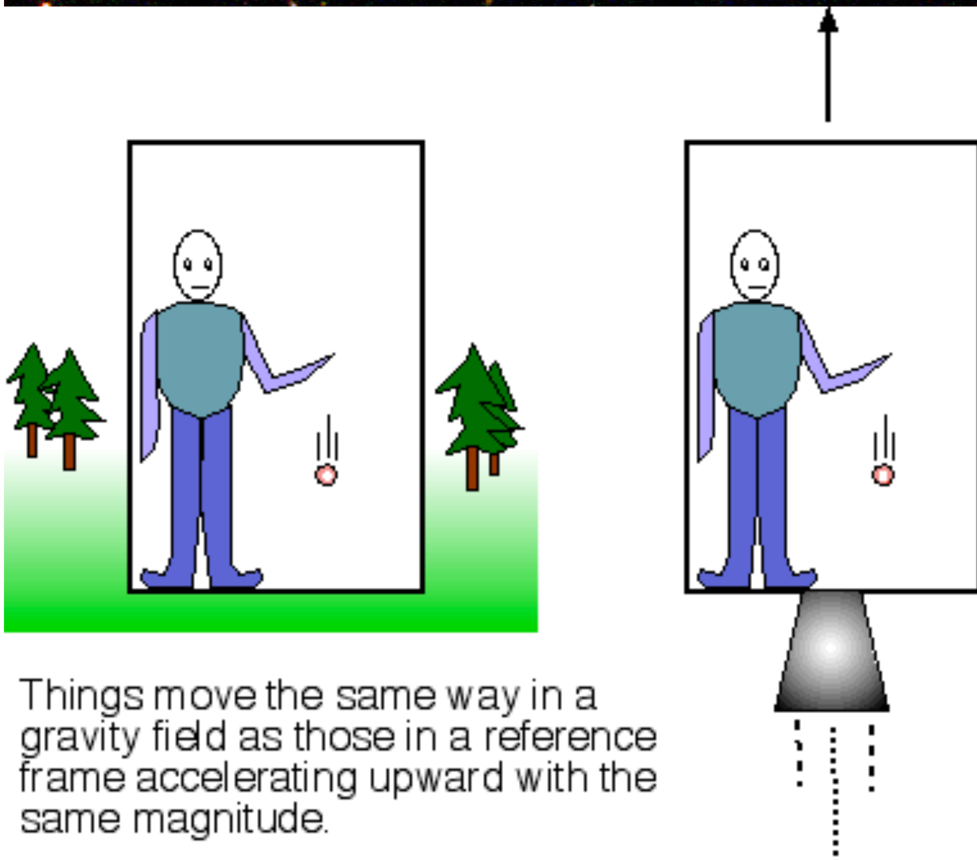
No !!!

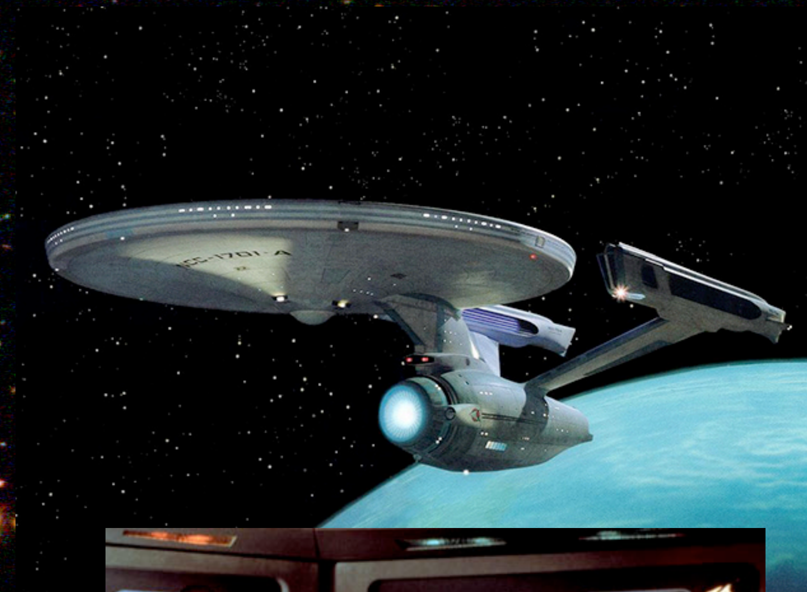


Things falling freely in a gravity field all accelerate by the same amount, so they move the same way as if they were in a region of zero gravity — ``weightlessness''!

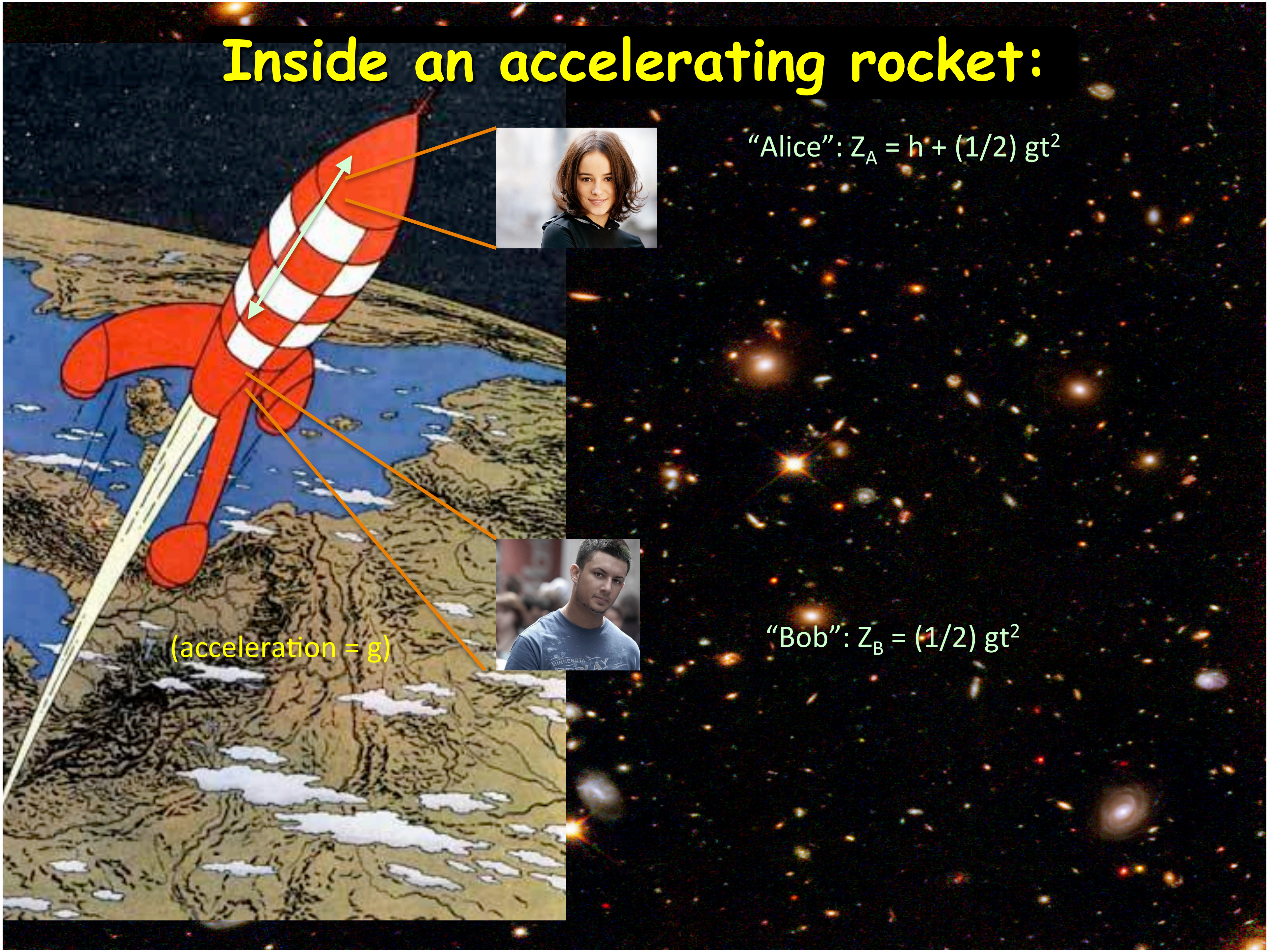
During free fall,
It is impossible (to 1st order) to measure gravity !!

Alternatively:





Every time "gravitational force" appears in the equations,
it can be replaced by "(uniform) acceleration"
This is the "Equivalence Principle"


Inside an accelerating rocket:
"Alice": $Z_A = h + (1/2)\, gt^2$
"Bob": $Z_B = (1/2)\, gt^2$
(acceleration = g)

Inside an accelerating rocket:

At t=0, Bob sends Alice a photon.

Alice receives it at time t_1

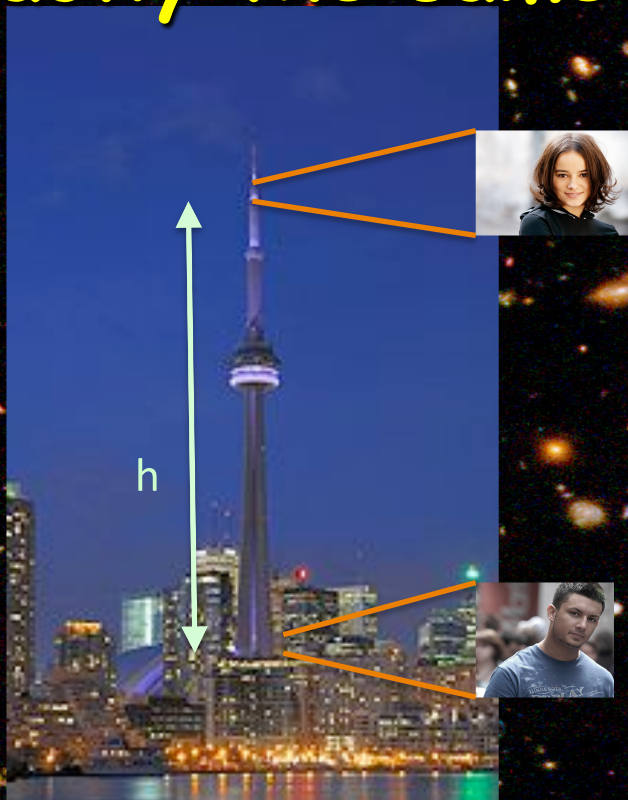
At Δt_B, Bob sends Alice a second photon.

Alice receives at time $t_1+\Delta t_A$

Since the rocket accelerates,
Alice moves faster at later times

→ $\Delta t_A > \Delta t_B$

According to the Equivalence principle, Exactly the same happens here on earth !!



$$\Delta t_A \approx \frac{\Delta t_B}{1 - \frac{gh}{c^2}} > \Delta t_B$$

Gravitational redshift: $\nu_A < \nu_B$

Photons lose energy when “climbing” a grav. potential

Immediate application: GPS !!

Gravitational redshift as probe of GR

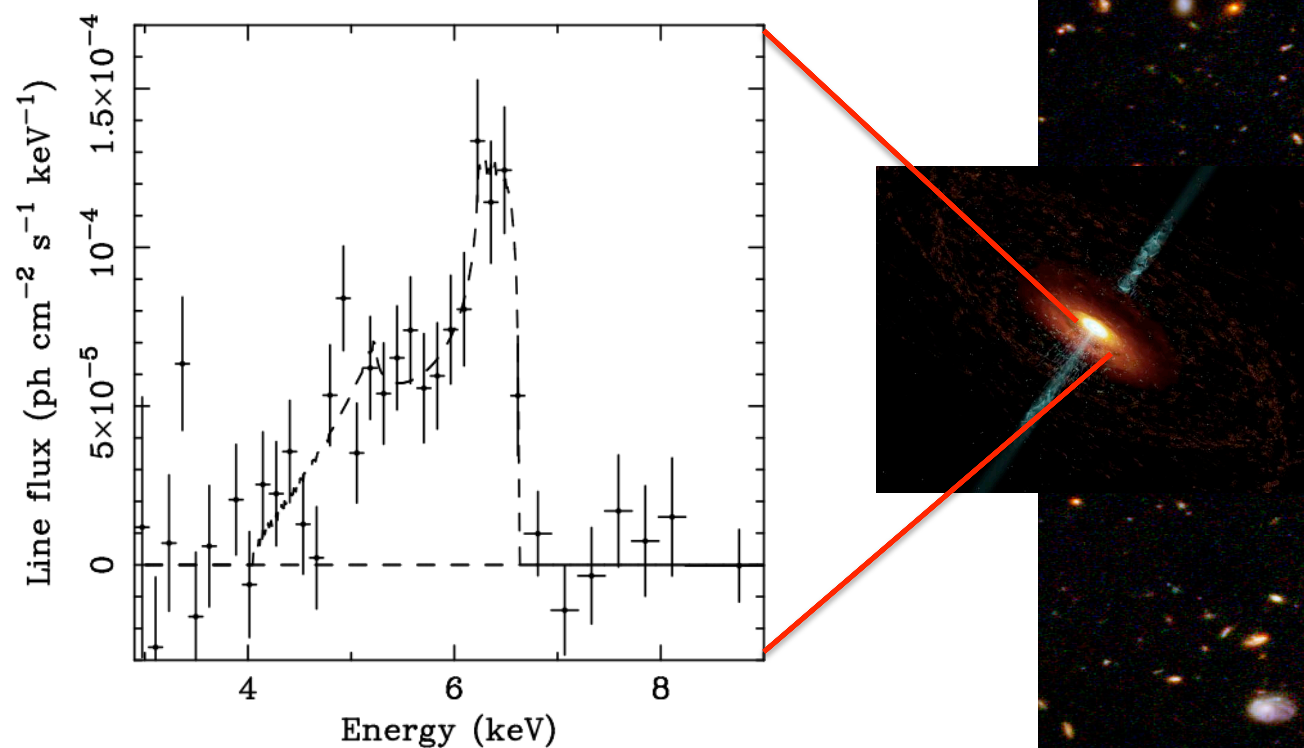
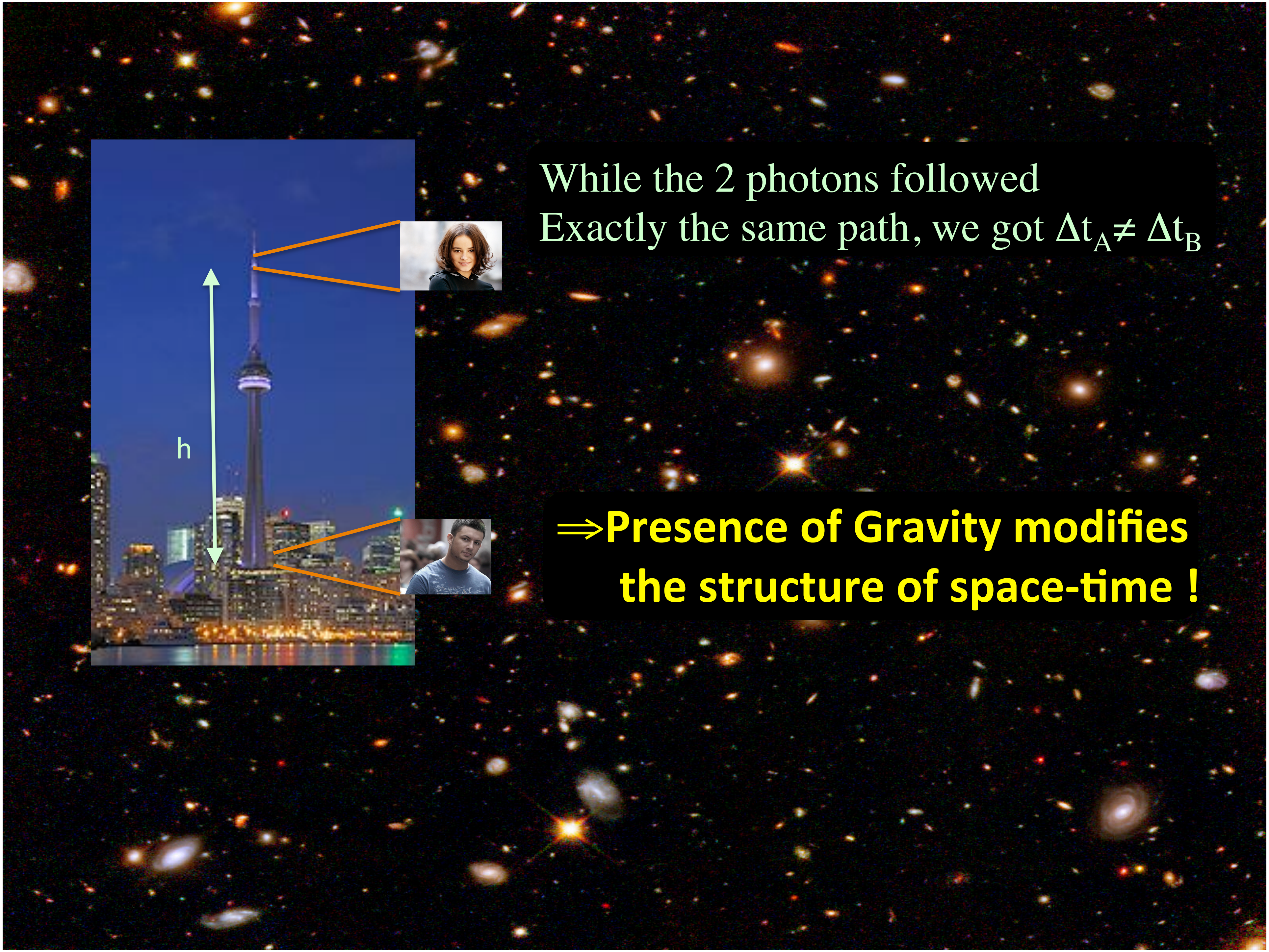
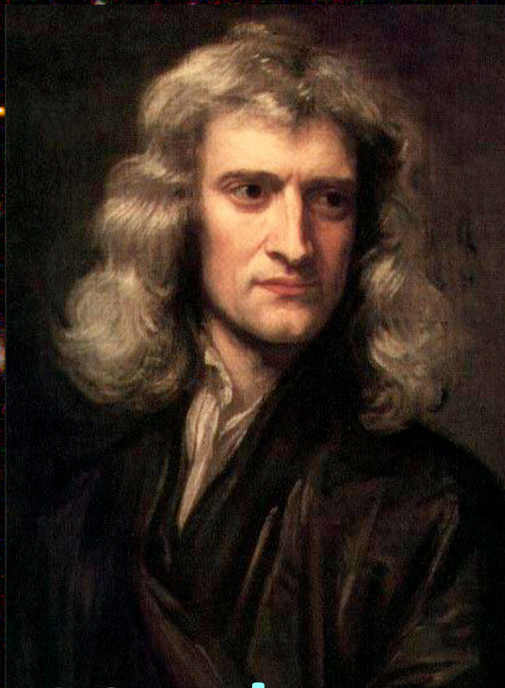


Fig. 1. Continuum subtracted iron line from the long July-1994 *ASCA* observation of the Seyfert-1 galaxy MCG–6-30-15 [20]. The dashed line shows a model consisting of iron line emission from a relativistic accretion disk around a non-rotating (Schwarzschild) black hole, with a disk inclination of $i = 30°$, and an emissivity profile of r^{-3} extending down to the radius of marginal stability ($6\,\mathrm{GM/c^2}$).

Taken from Reynolds & Nowak, 2003


h
While the 2 photons followed
Exactly the same path, we got $\Delta t_A \neq \Delta t_B$
$\Rightarrow$Presence of Gravity modifies
the structure of space-time !



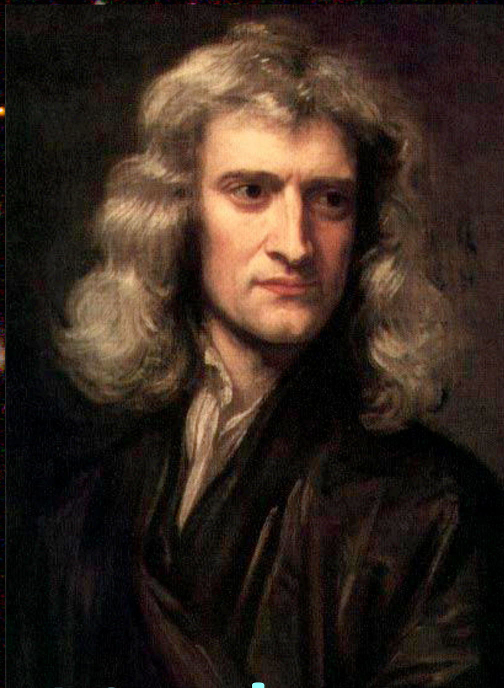
Gravity: basic force of nature.

Operates between massive bodies

No !.

Gravity affects also massless particles

Gravity is <u>not a force!</u>



Gravity: basic force of nature.

Operates between massive bodies

No !.

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Gravity is <u>not a force!</u>

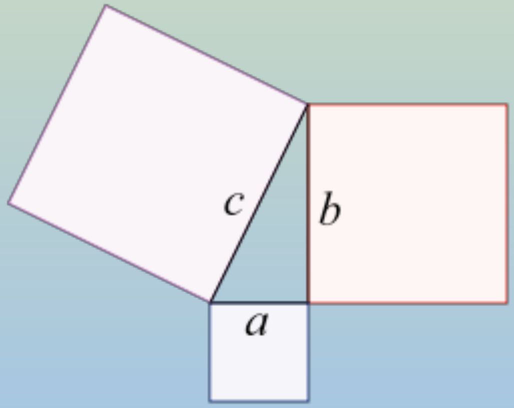
Gravity = Geometry ! (?)!

Euclid of Alexandria (fl. 300BC)

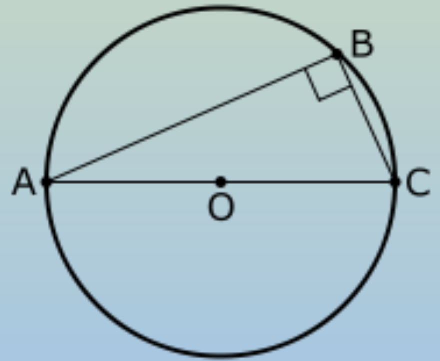
"Father of geometry";

Elements:
A finite set of assumptions (=axioms or postulates).

All geometry is deduced by direct logic.



Pythagorean theorem



Thales' theorem

Euclid's Axioms

Euclid's axioms	
1	Given two points there is one straight line that joins them.
2	A straight line segment can be prolonged indefinitely.
3	A circle can be constructed when a point for its centre and a distance for its radius are given.
4	All right angles are equal.
5	If a straight line falling on two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which the angles are less than the two right angles.
Euclid's common notions	
6	Things equal to the same thing are equal.
7	If equals are added to equals, the wholes are equal.
8	If equals are subtracted from equals, the remainders are equal.
9	Things that coincide with one another are equal.
10	The whole is greater than a part.

René Descartes (1596-1650)

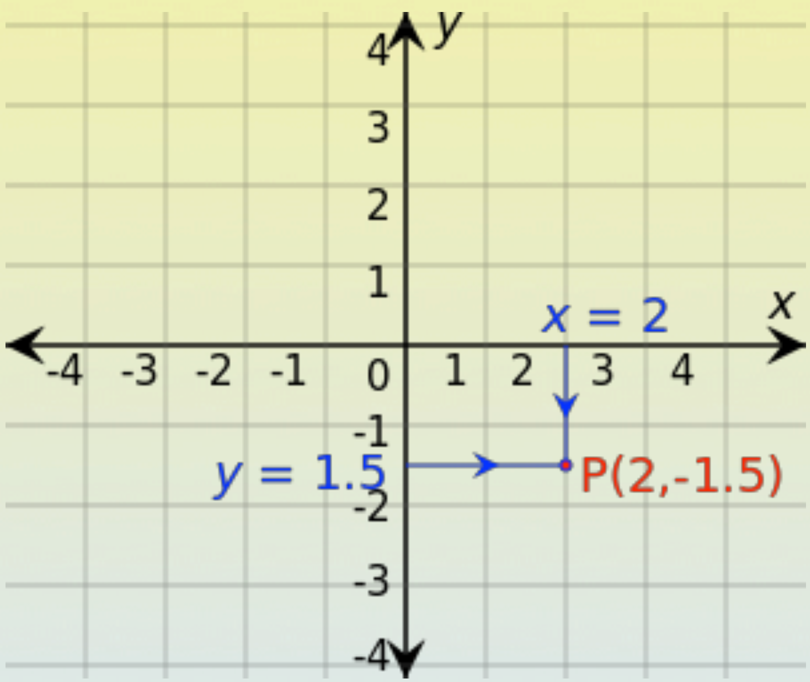
(Latin: Renatus Cartesius)

René Descartes (1596-1650)

(Latin: Renatus Cartesius)

Introduced: Axes !

Analytical (Cartesian) geometry

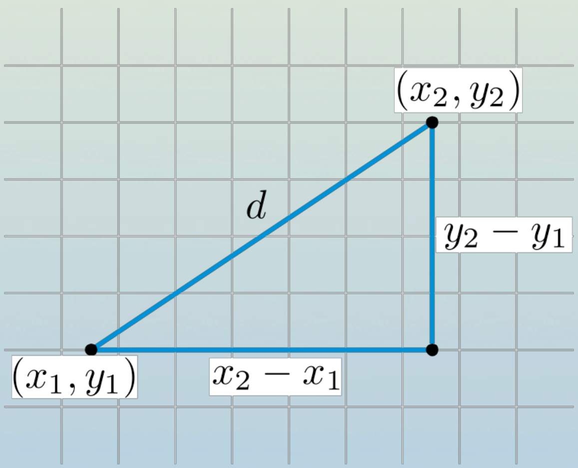


Geometry is combined with Algebra !

René Descartes (1596-1650)

(Latin: Renatus Cartesius)

Analytical (Cartesian) geometry

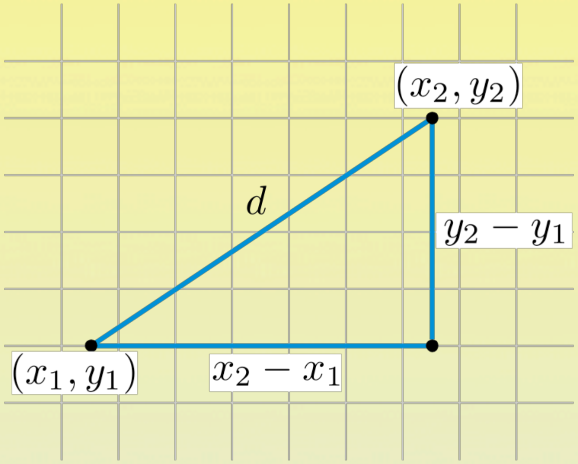


$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

(Pythagorean theorem)

Modern version of Pythagorean theorem

$$d^2 = \Delta x^2 + \Delta y^2 (+\Delta z^2)$$



Using a new notation:

$$\Delta x^1 \equiv \Delta x$$

$$\Delta x^2 \equiv \Delta y$$

$$\Delta x^3 \equiv \Delta z$$

$$d^2 = \sum_{i=1}^{3} \left(\Delta x^i\right)^2$$

$$d^2 = \sum_{i,j=1}^{3} A_{ij} \left(\Delta x^i\right)\left(\Delta x^j\right)$$

Where

$$A_{ij} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Euclid's Axioms

Euclid's axioms	
1	Given two points there is one straight line that joins them.
2	A straight line segment can be prolonged indefinitely.
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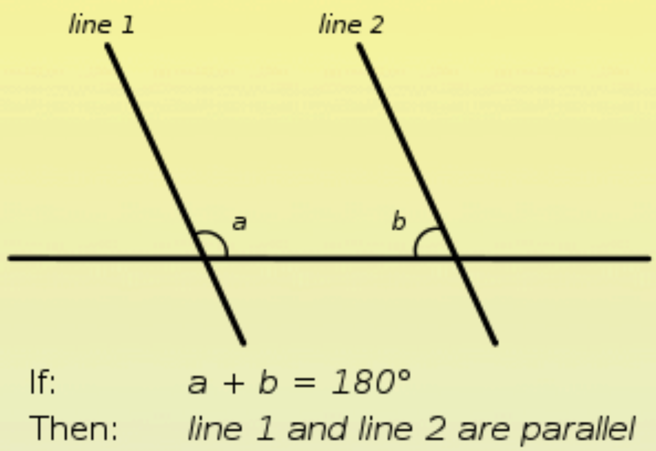
10	The whole is greater than a part.

Progress by Adrien-Marie Legendre (1752-1833):

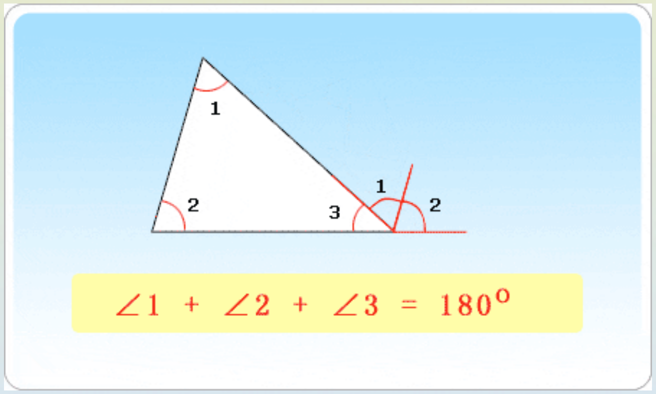
1800:

Euclid's 5th postulate,
"That, if a straight line falling on two straight lines make the interior angles on the same side less than two right angles, if produced indefinitely, meet on that side on which are the angles less than the two right angles."



Is equivalent to:

"The sum of the angles of a triangle is equal to two right angles."



Rise of non-Euclidean geometry

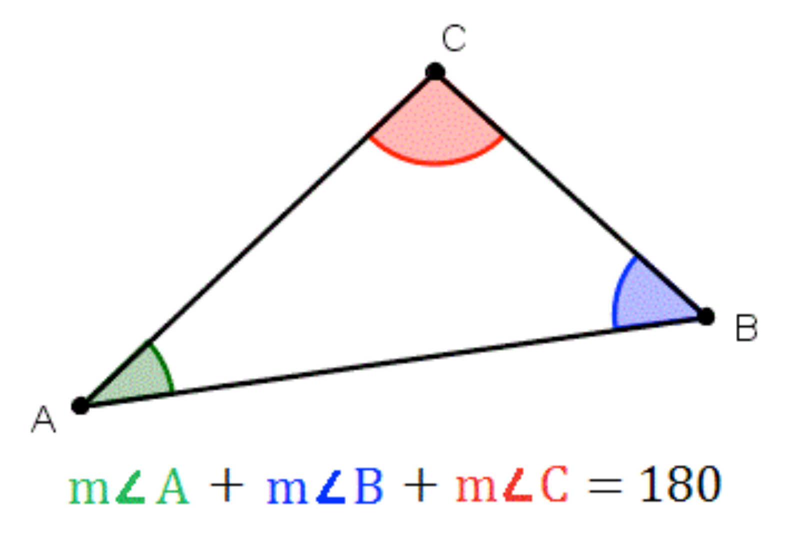
Carl Friedrich Gauss
(1777 – 1855)
Developed in 1813 !
(but not published !!)

Janos Bolyai

Nikolai Lobachevski

(c. 1830)

What is the sum of angles in a triangle ?

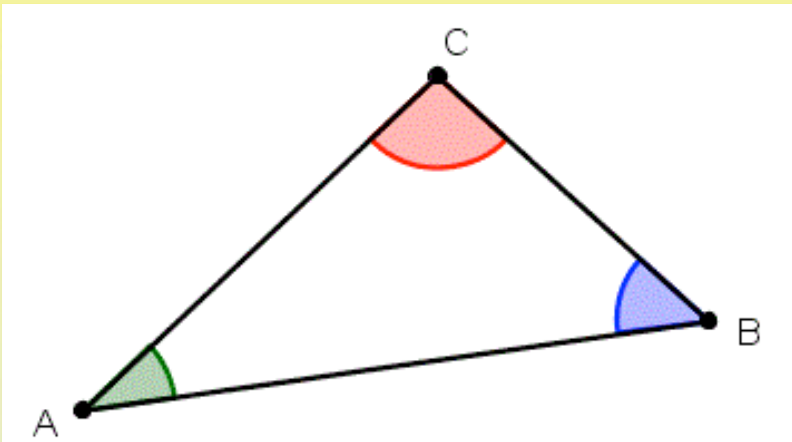


A_{ij}

What is the sum of angles in a triangle ?

Non-Euclidean geometries:

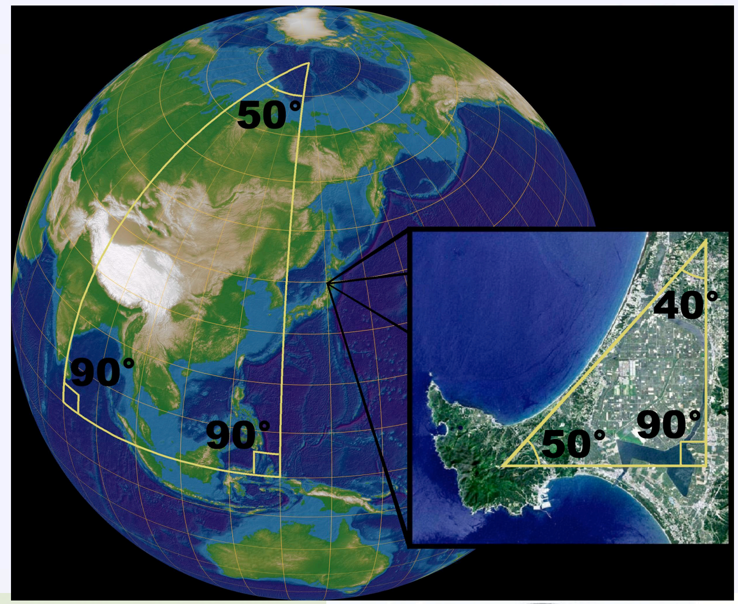
All of Euclid's assumptions are satisfied – but the 5th !



$m\angle A + m\angle B + m\angle C = 180$

A_{ij}

What about on a sphere?



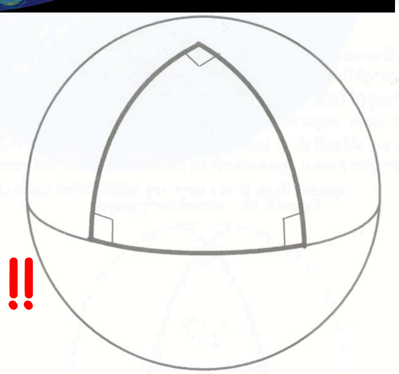


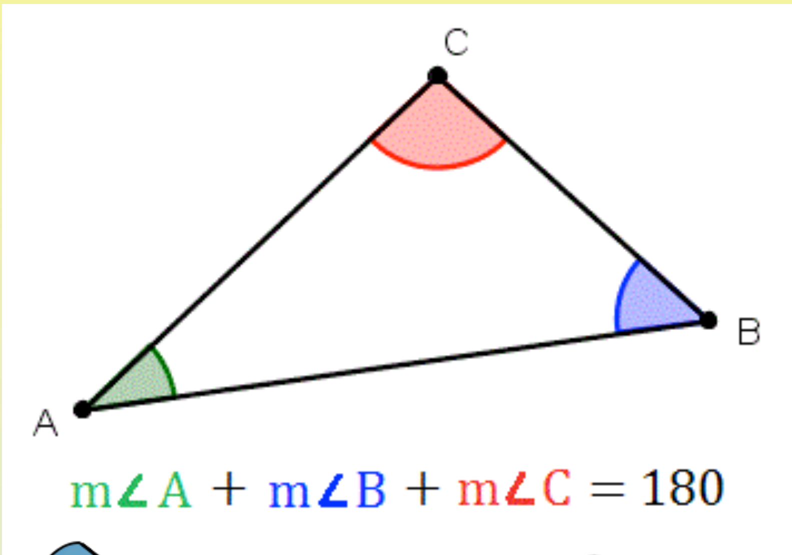
Figure 6.13 Triple-right triangle on a sphere

Sum of angles on a sphere is always > 180°!!

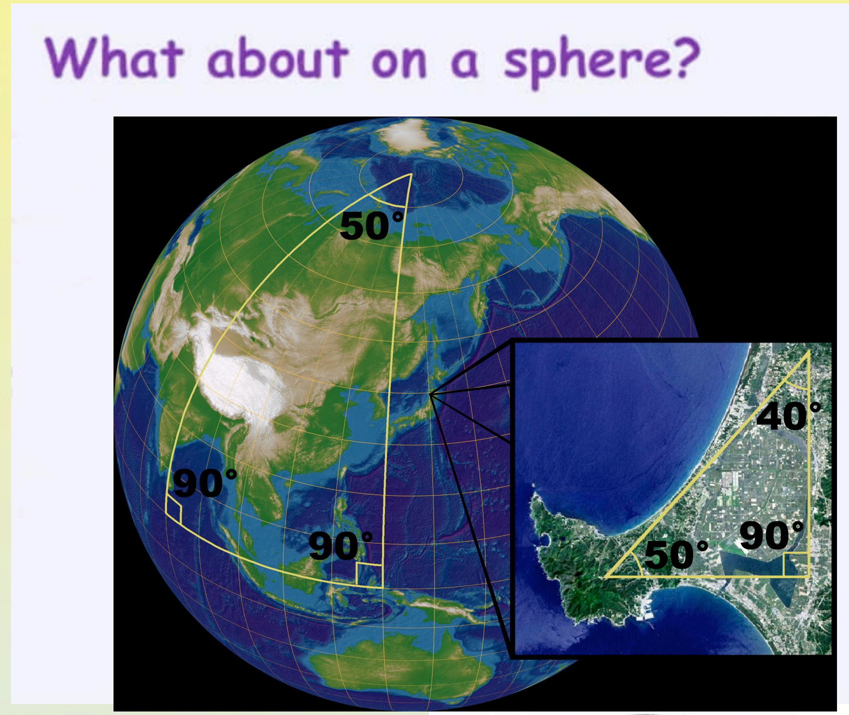
What is the sum of angles in a triangle ?

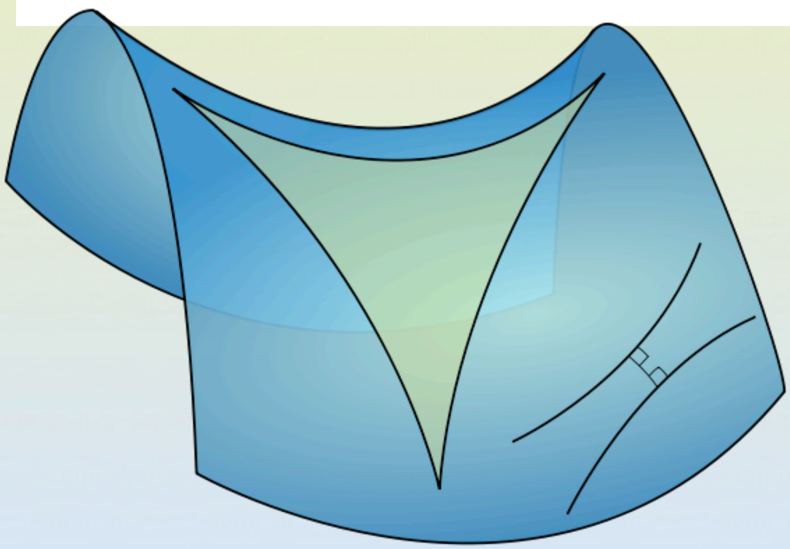
Non-Euclidean geometries:

All of Euclid's assumptions are satisfied – but the 5th !



A_{ij}





G-B-L hyperbolic metric:
Sum of angles: < 180°

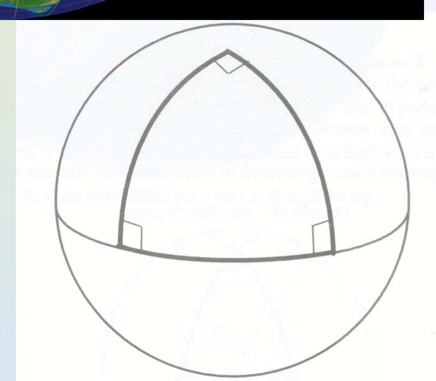
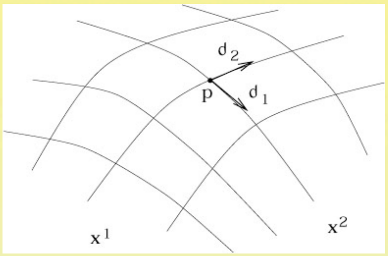
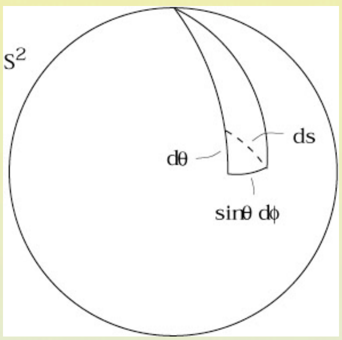


Figure 6.13 Triple-right triangle on a sphere

Straight lines (or: shortest paths) in curved space: geodesics

Internal properties of space: the metric
(how can you measure space- without leaving it ?)





Carl Friedrich Gauss
(1777 – 1855)

Metric: a mathematical function that describes distances, intrinsic to the space

Distance: $ds^2 = d\theta^2 + \sin^2\theta d\phi^2$

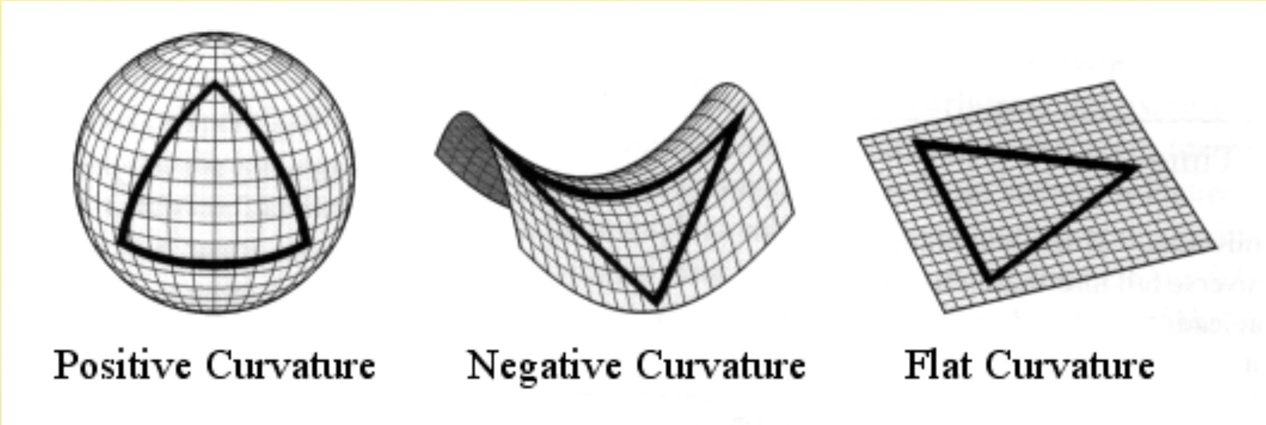
$$ds^2 = \sum_{i,j=1}^{2} g_{ij}\left(dx^i\right)\left(dx^j\right) \qquad g_{ij} = \begin{pmatrix} 1 & 0 \\ 0 & \sin^2\theta \end{pmatrix}$$

$dx^1 \equiv d\theta$

$dx^2 \equiv d\phi$

g_{ij}: Metric

Curvature as an inner property of space
(how can you tell that a sphere is a sphere ?)

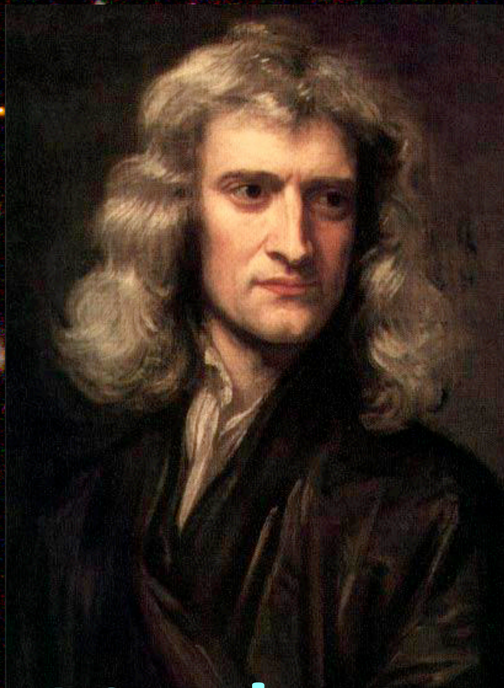


Examples:
shortest path in curved space – geodesic
(1st derivative of the metric)

2nd derivative of the metric –
intrinsic curvature (Riemann Tensor)

Bernhard Riemann (1826 – 1866)

We can measure the properties of space – without leaving it !



Gravity: basic force of nature.

Operates between massive bodies

No !.

Gravity affects also massless particles

Gravity is <u>not a force!</u>

Gravity = Geometry ! (?)!

Gravity = Geometry ! (?)!

1. How to describe particle motion in the presence of gravity ?

2. How does the presence of gravity affects the geometry of space-time ?

Gravity = Geometry ! (?)!

1. How to describe particle motion in the presence of gravity ?

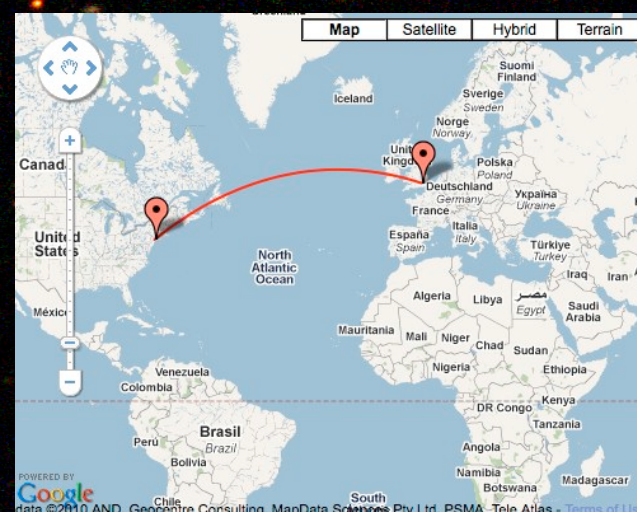
In the presence of gravity ….

Objects move in straight lines (geodesics)

Exactly what they do in the absence of gravity !!

(and clearly at v<=c)

It is the space-time that is curved !



Gravity = Geometry ! (?)!

2. How does the presence of gravity affects the geometry of space-time ?

In an analogue way to Newtonian potential:

$$\nabla^2 \Phi = 4\pi G \rho$$

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Curvature
(geometrical part)

Stress-energy tensor:
describes the **energy & momentum** content & flux of a system
(physical part)

Gravity = Geometry ! (?)!

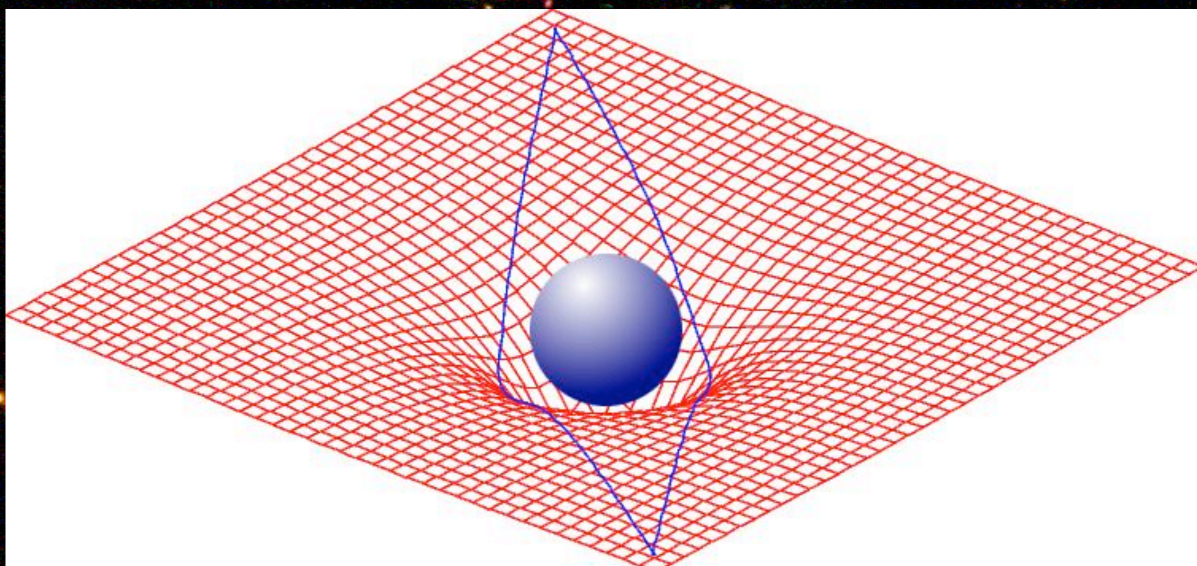
Einstein's field equation (1915)

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} (+ g_{\mu\nu} \Lambda) = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Set of 10 nonlinear equations;
Contain 2nd derivatives of the metric

Gravity = Geometry !!! → Light deflection

(though photons have no mass !!)

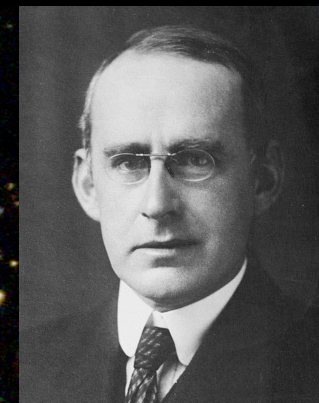


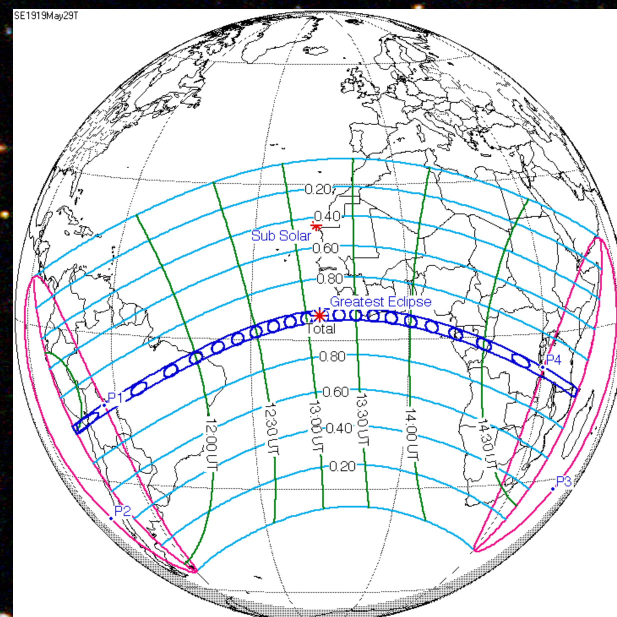
Gravity curves space-time

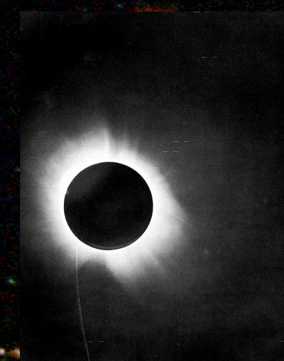
When passing close to a massive objects, light rays are deflected

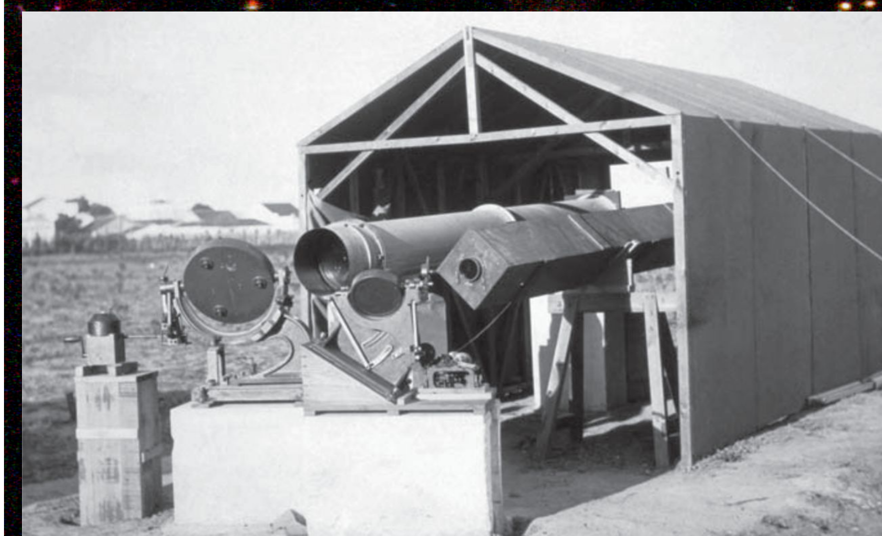
Arthur Eddington (1882-1944): Light deflection during solar eclipse (1919)

29 May, 1919: total solar eclipse seen from
Sobral, Brazil & island of Principe (near Guinea, Africa)

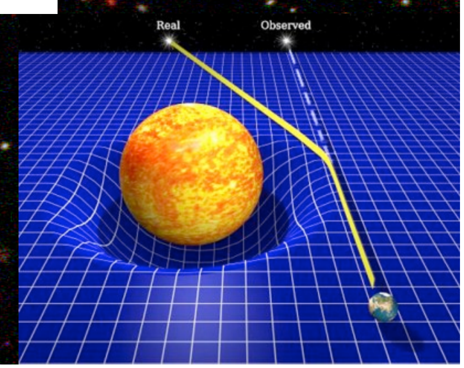








$\Delta\vartheta = 1.75\ (R_{sun}/r)$ arcsec,



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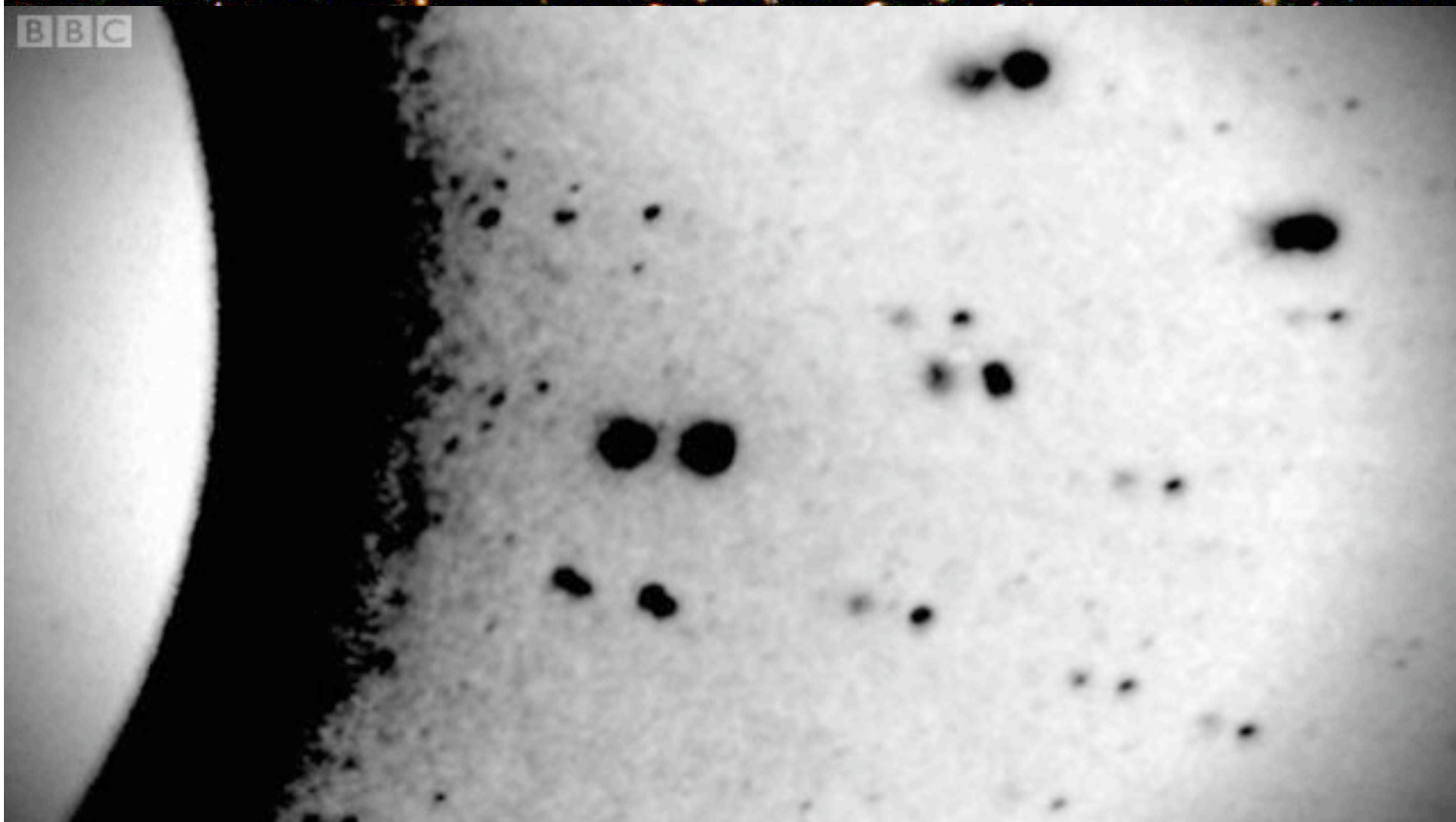
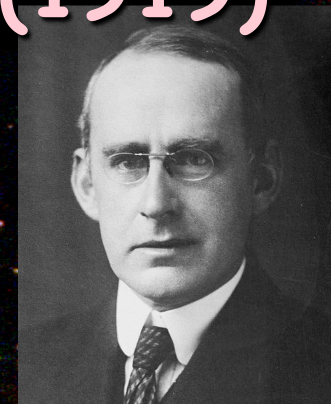
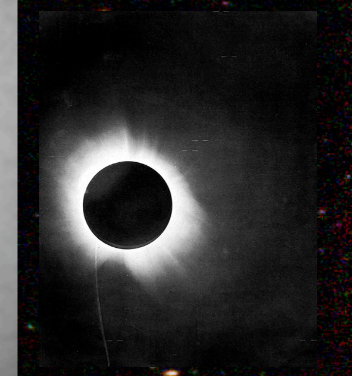
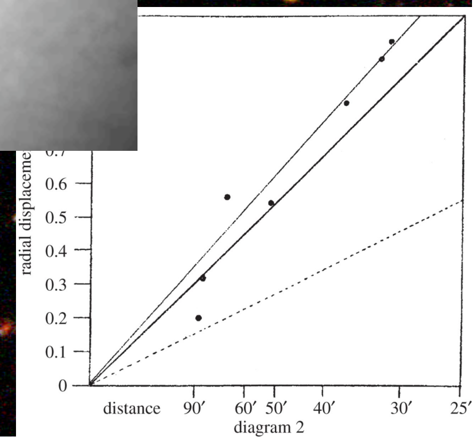






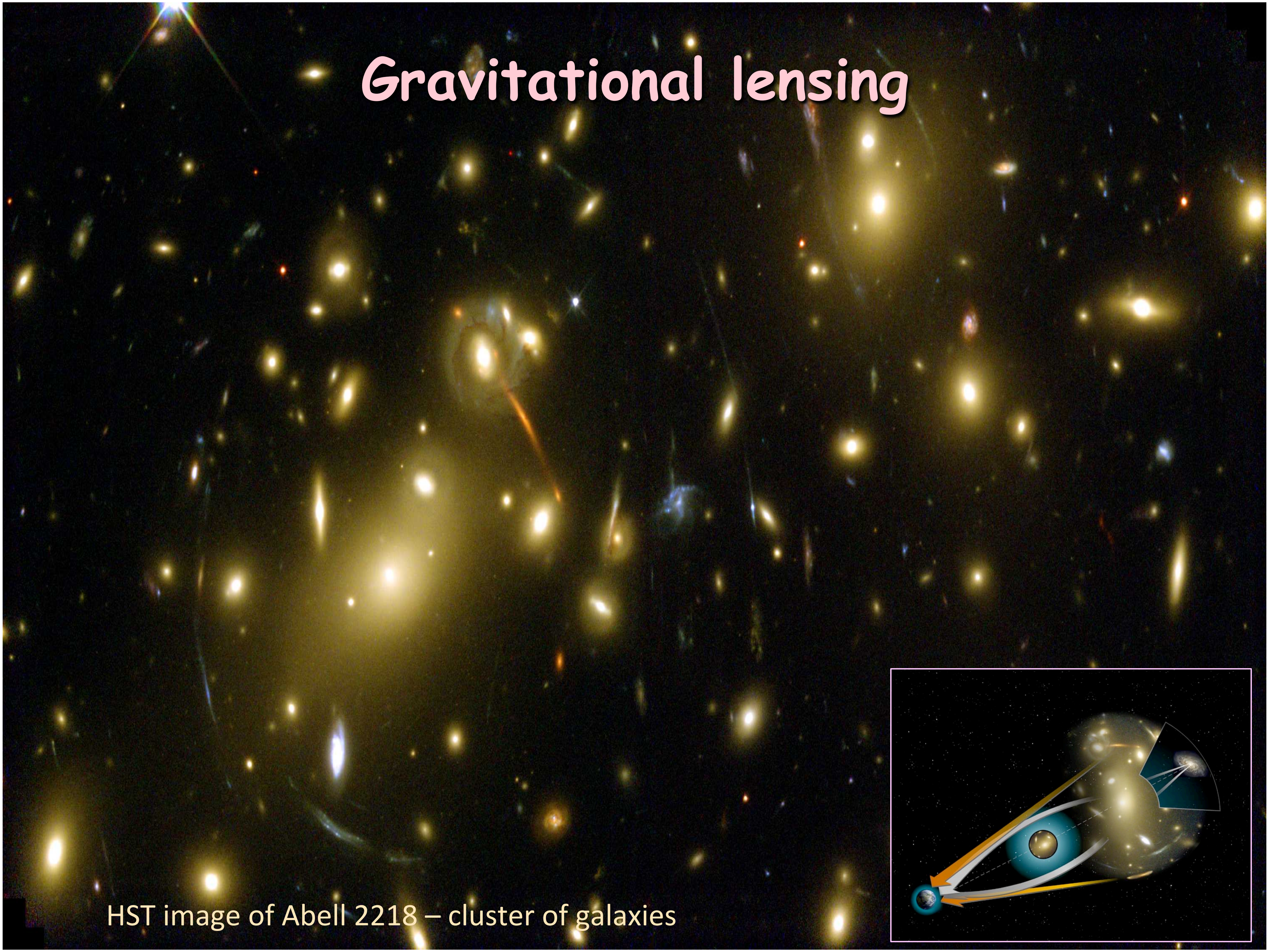


Theory: $\Delta\vartheta = 1.75\ (R_{sun}/r)$ arcsec

Data: $\Delta\vartheta = 1.61 \pm 0.31$ arcsec (Eddington);

1.92+-0.12 arcsec (Davidson & Furner)

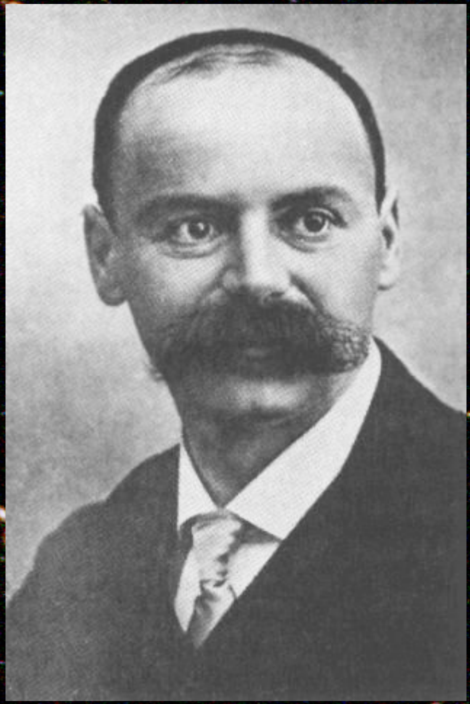
Gravitational lensing

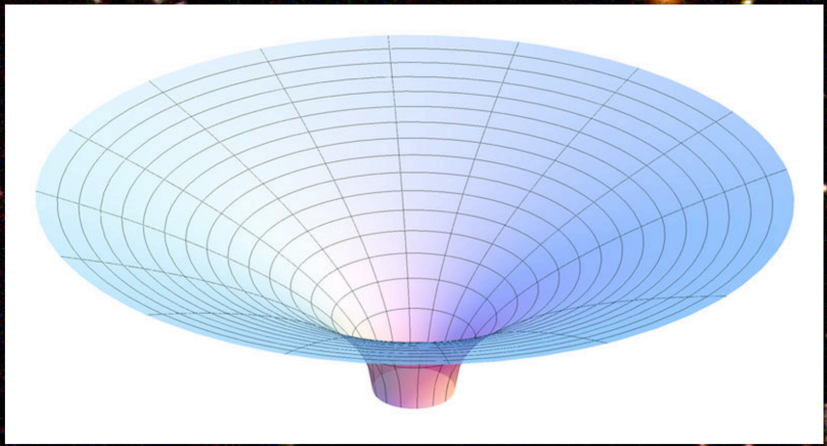


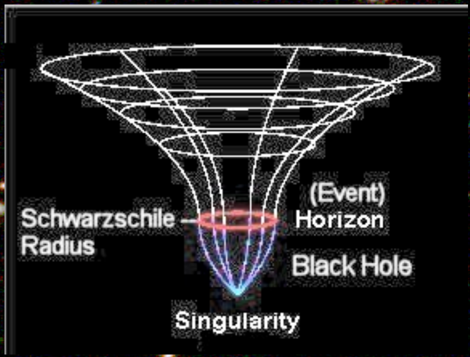
HST image of Abell 2218 – cluster of galaxies

Karl Schwarzschild (1873-1916)
1915: Solution to Einstein's equations in vacuum

$$ds^2 = -\left(1 - \frac{2GM}{rc^2}\right)dt^2 + \left(1 - \frac{2GM}{rc^2}\right)^{-1} dr^2 + r^2 d\Omega^2$$





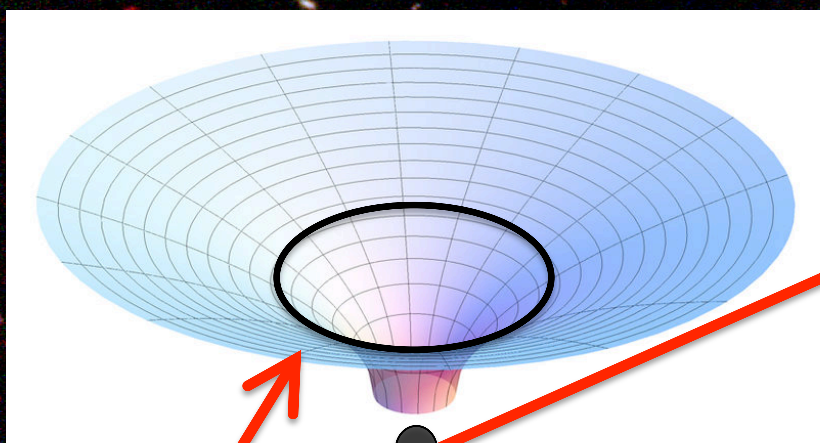


At $r=0$ – singularity

For $r \le \frac{2GM}{c^2}$ there is no "turning back" at v<c (event horizon)

Schwarzschild's black holes

$$ds^2 = -\left(1 - \frac{2GM}{rc^2}\right)dt^2 + \left(1 - \frac{2GM}{rc^2}\right)^{-1} dr^2 + r^2 d\Omega^2$$



At *r=0* - "Singularity"
Space-time curves to infinity;
$\varrho \rightarrow \infty$

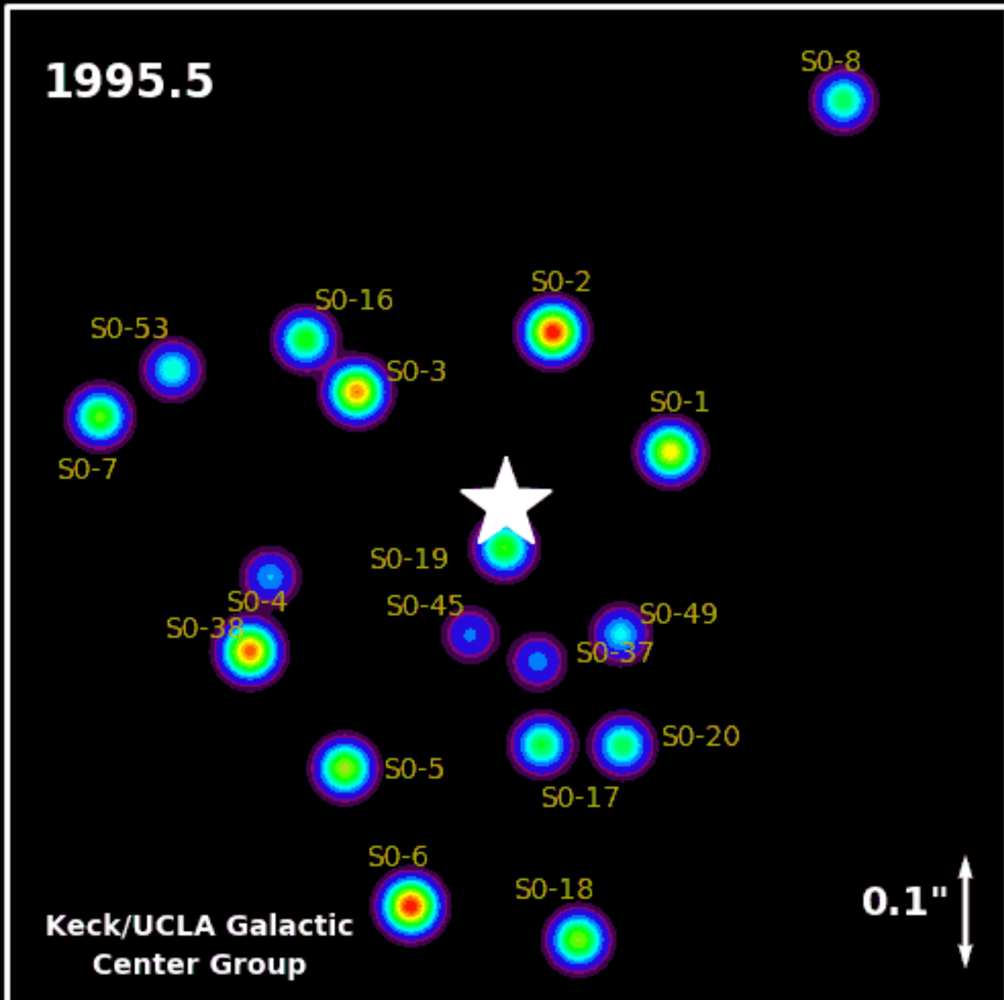
At $r \le \frac{2GM}{c^2}$ - there is no "turning back" at v<c;

thus nothing can escape.
(Schwarzshield radius, "event horizon")

A journey into a black hole

Taken from the website of Andrew Hamilton, JILA (Colorado)

The black hole at the center of our galaxy



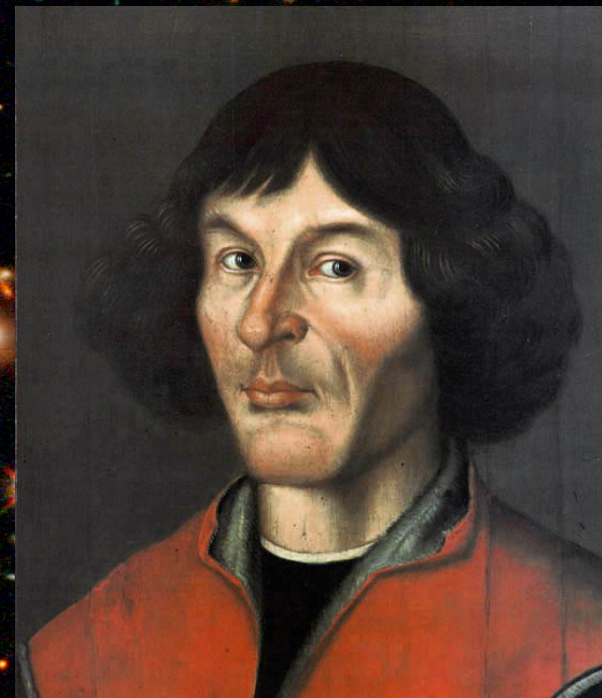
Inferred indirectly, by looking at trajectories of stars

Taken from the website of Andrea Ghez, UCLA

GR and modern cosmology

Nicolaus Kopernikus (1473 – 1543):

We are NOT at the center of the universe !



The cosmological principle:

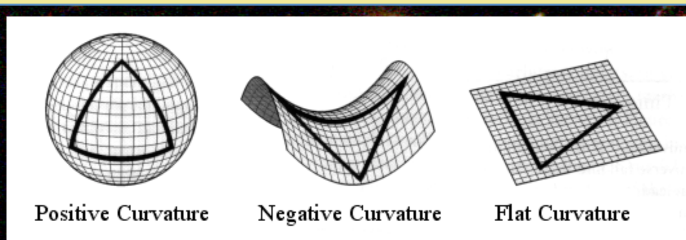
On a large scale, the universe is **homogeneous and isotropic**

GR and modern cosmology

How to describe a **homogeneous** and **isotropic** universe ?

Friedmann-(Lemaitre)-
Robertson-Walker metric:

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right]$$



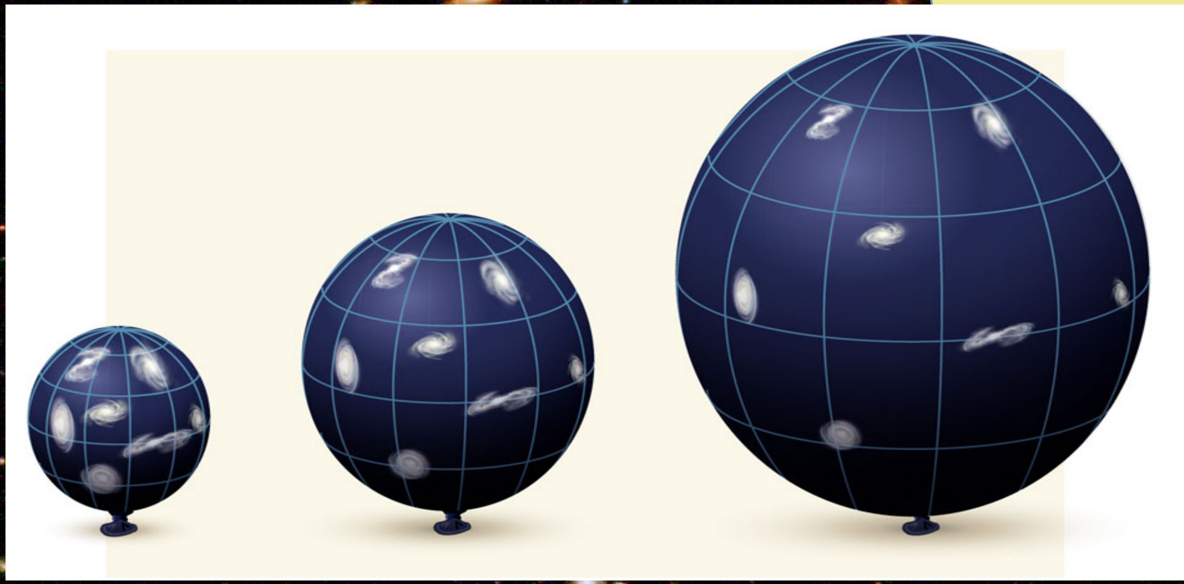
Alexander Friedmann (1922)

$a(t)$: "Scale factor"; $\quad k = \begin{cases} -1 & \text{“}\textit{open}\text{” (negative curvature)} \\ 0 & \text{“}\textit{flat}\text{”} \\ 1 & \text{“}\textit{Closed}\text{” (positive curvature)} \end{cases}$

The scale factor a(t)

Multiplies all spatial coordinates: $d(t) = d_0\, a(t)$

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right]$$



a(t) evolves in time !

Friedmann Equations:

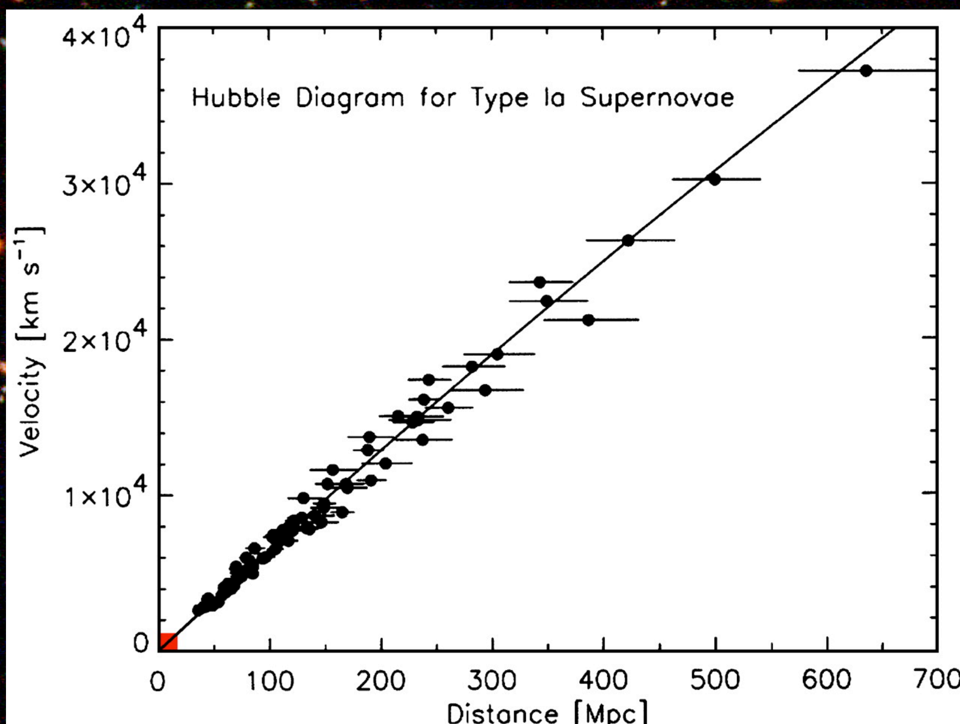
$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}$$

The expanding universe

"Scale factor": $d(t) = d_0\, a(t)$; $a(t)$ **grows with time**

The universe expands;
The most distant galaxies recede faster from us
(Huble's law)

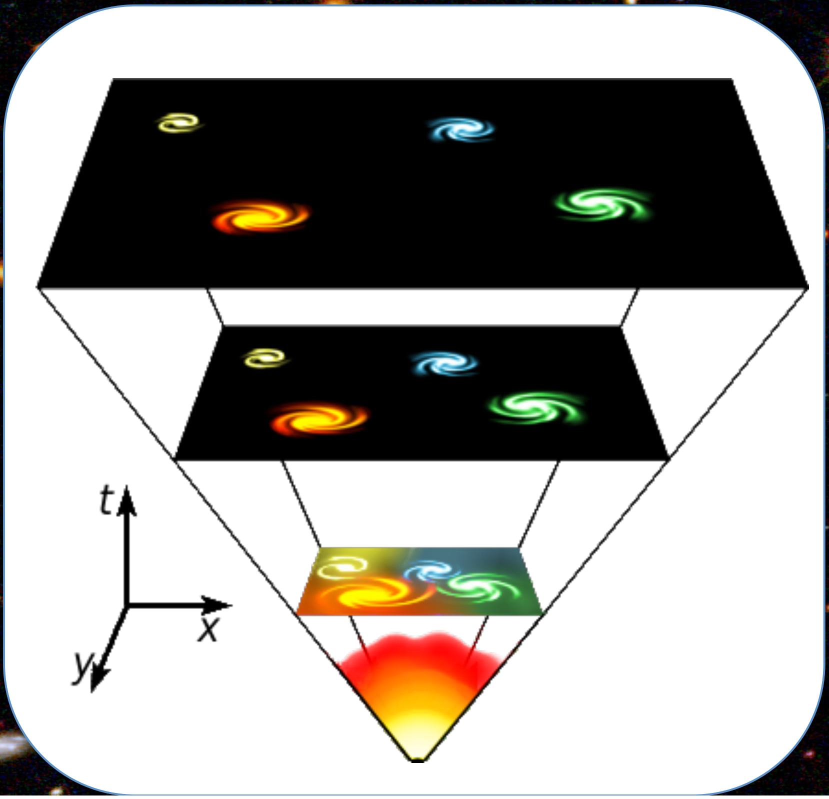


Edwin Hubble (1929)

The expanding universe

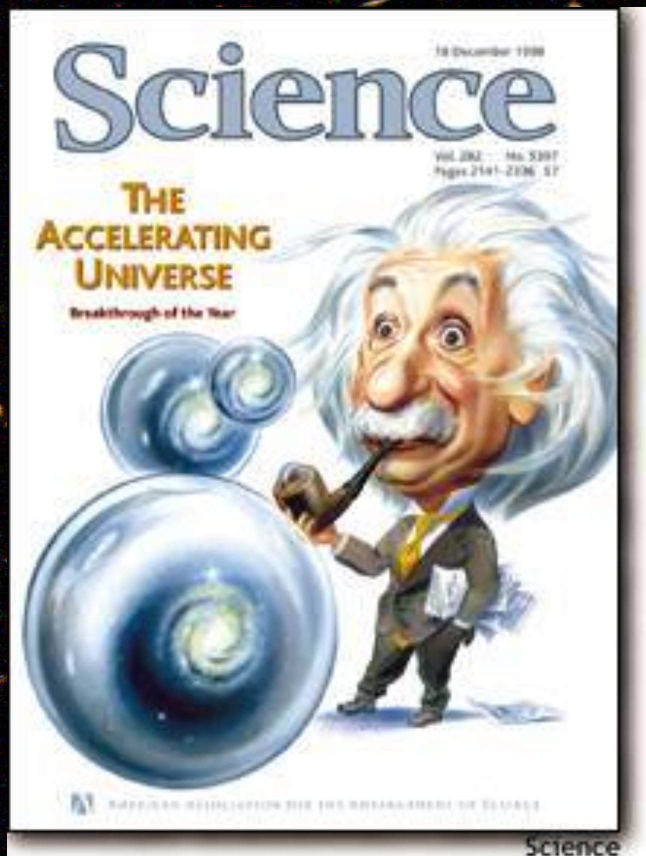
Hubble's law: the universe expands - *a(t)* **grows with time**

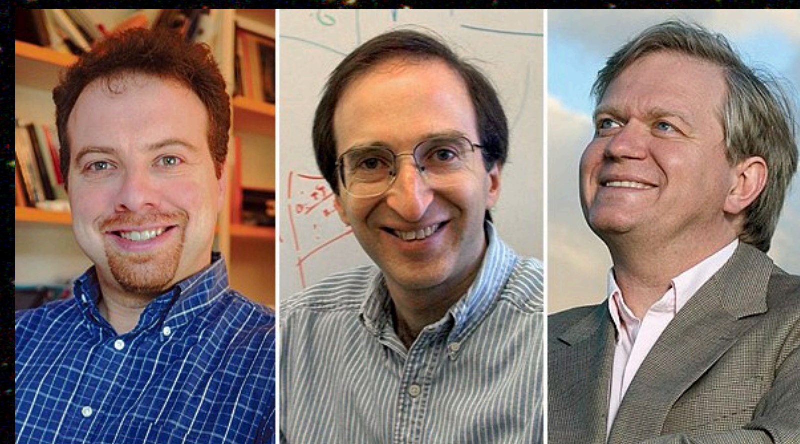
Some time in the past, *a(t)*=0 **"The big bang"**



Edwin Hubble (1929)

1998: The accelerating universe





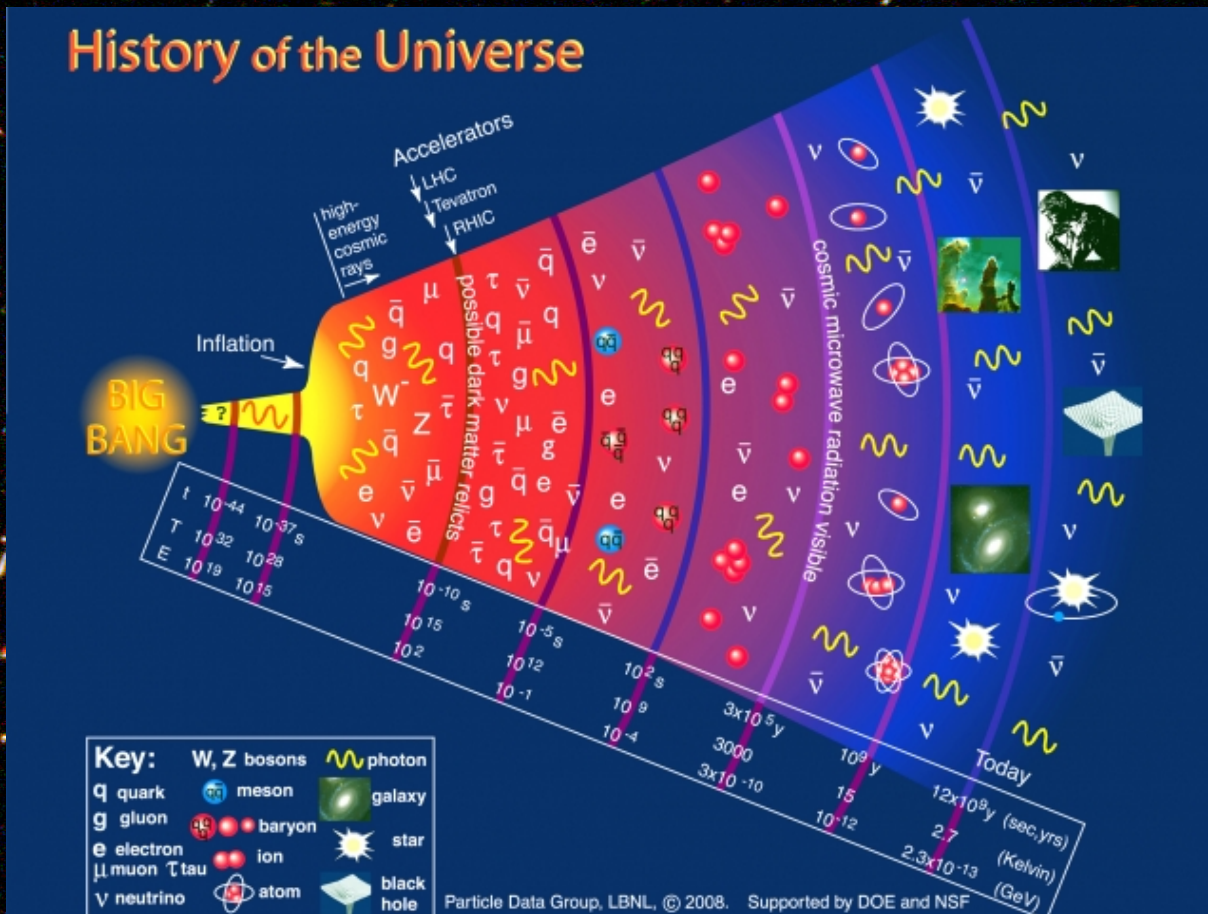
Johns Hopkins University; University Of California At Berkeley; Australian National University

From left, Adam Riess, Saul Perlmutter and Brian Schmidt shared the Nobel Prize in physics awarded Tuesday.

Cosmological constant =
dark energy (74%)!

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} (+ g_{\mu\nu} \Lambda) = \frac{8\pi G}{c^4} T_{\mu\nu}$$

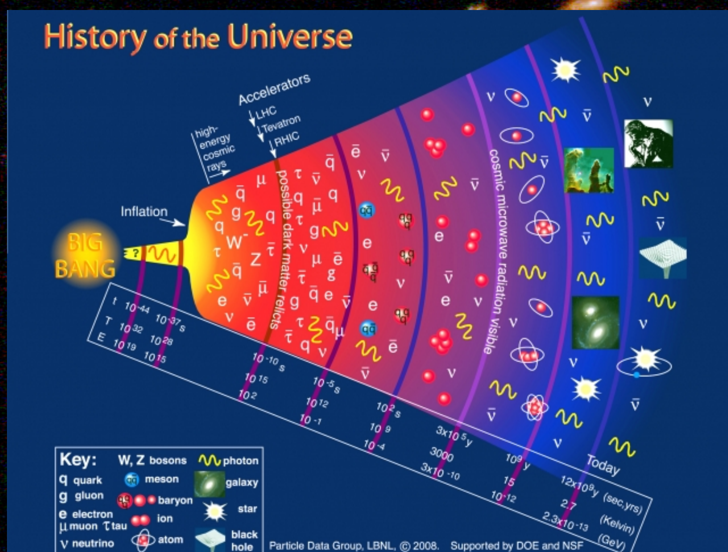
1998: The expanding universe: back in time

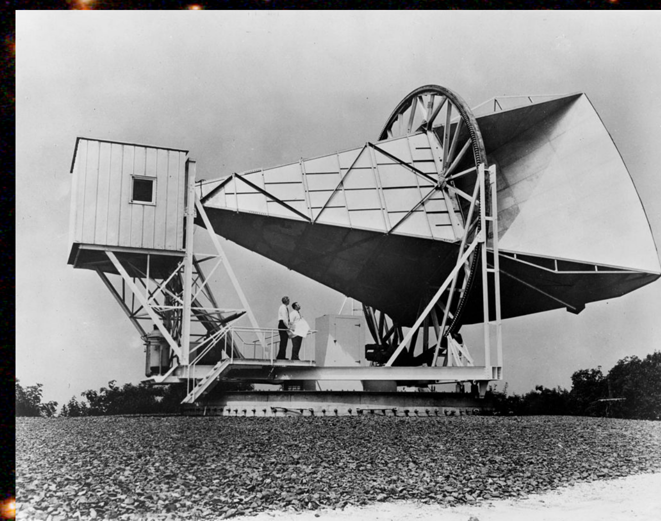


Initially, the universe was so hot…
no atoms could exist

1998: The expanding universe: back in time

When the universe expanded, it cooled…
Atoms formed, and photons escaped

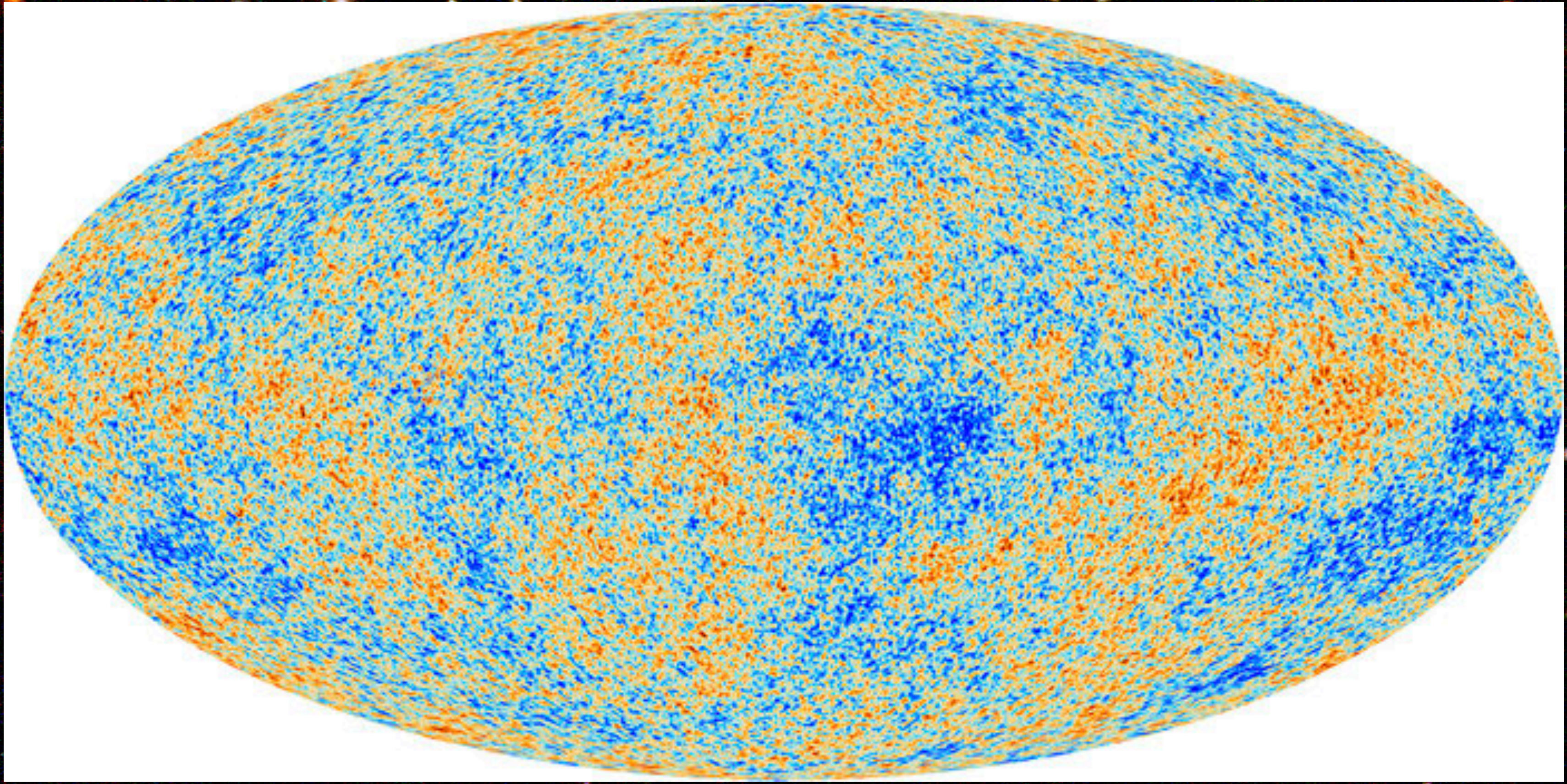




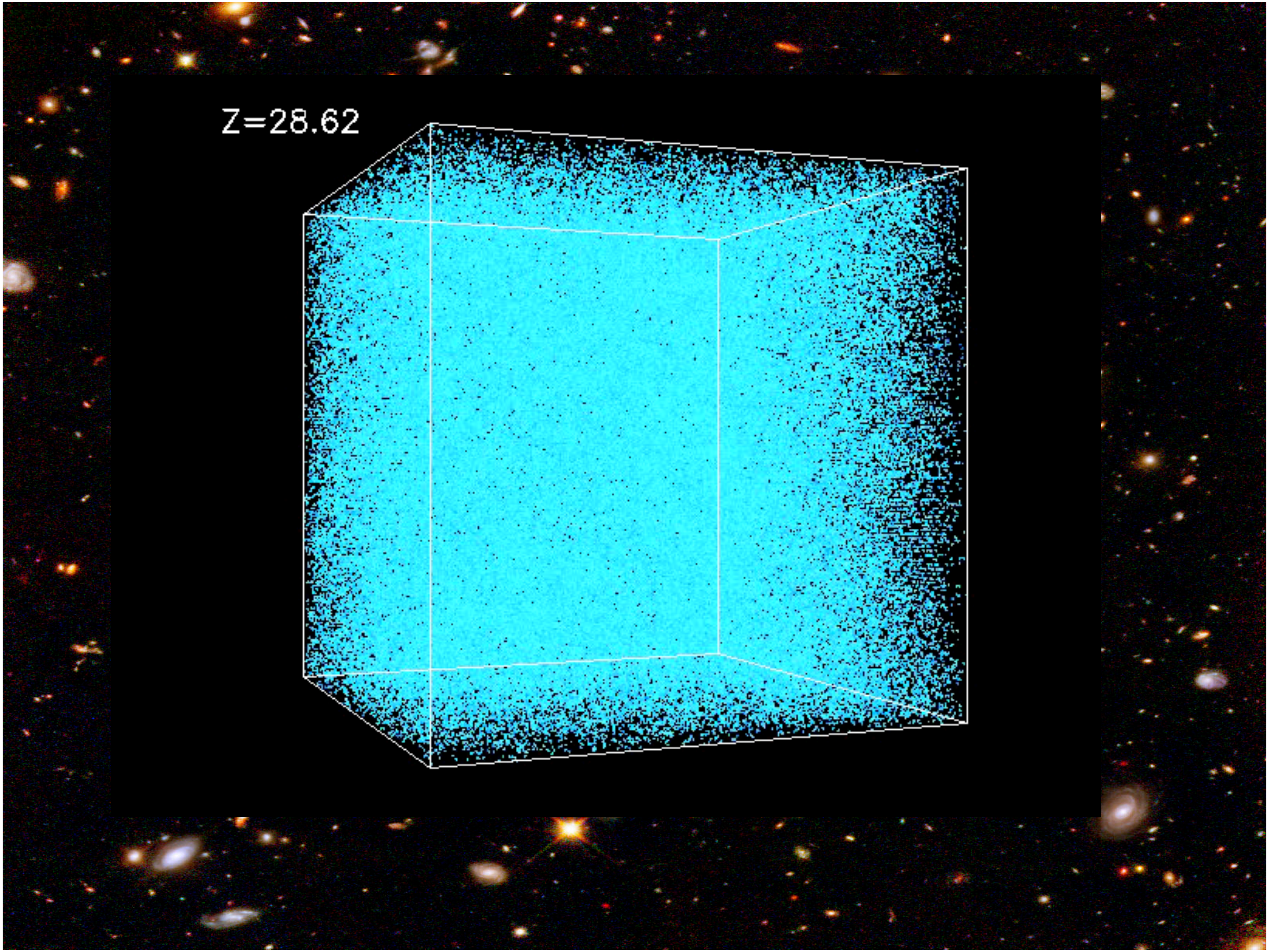
Penzias & Wilson, 1964

These photons are all around us.
We call it the cosmic microwave background (CMB)

CMB fluctuations: imprints of galaxies to be born



ESA's "Planck" mission (2013)


Z=28.62

Summary

(Together with quantum mechanics),
general relativity
is a cornerstone of modern physics.

100 years after its discovery,
there are more questions than answers.

Physics is exciting !!

Asaf Pe'er, Nov. 2015